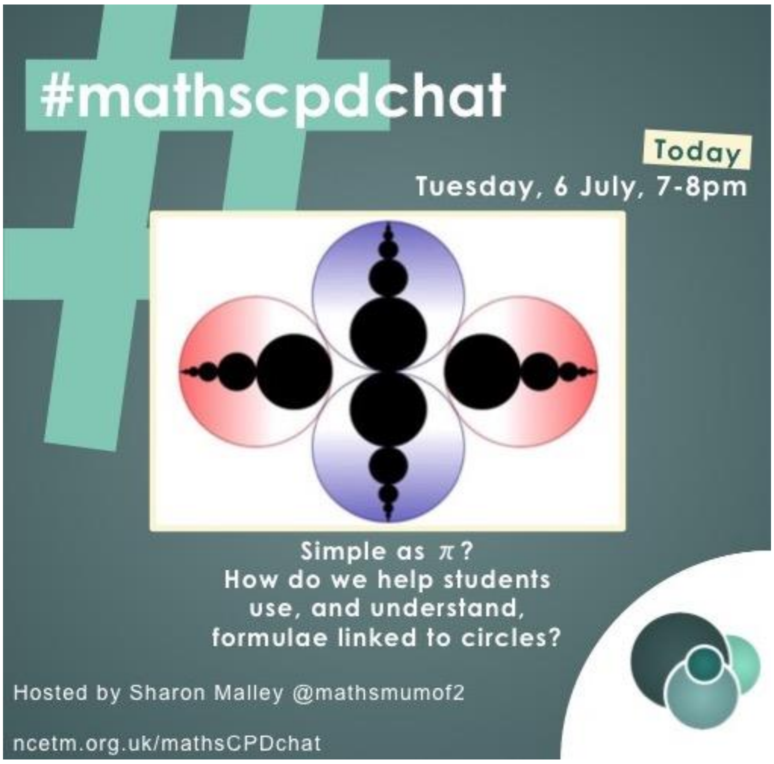
#mathscpdchat 6 July 2021

Simple as π? How do we help students use, and understand, formulae linked to circles?
Hosted by Sharon Malley

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*



Among the links shared during the discussion were:

Mastery Professional Development: 6.2 Perimeter, area and volume which is a Key Stage 3 guidance document from the NCETM. It offers key ideas to help guide teacher planning, including the idea that no matter how large or small a circle is, the ratio between its circumference and its diameter is always the same. Effective ways of addressing common difficulties and misconceptions are suggested. It was shared by Sharon Malley

pi development which is material created by Don Steward, consisting of a sequence of carefully designed, attractive image-sheets, each of which might be an effective focus for thought, discussion and learning. It was shared by Charlotte Hawthorne

[Measuring Circumference and Diameter](#) which is a GeoGebra applet by Jennifer Silverman. Users explore the relationship between the diameter and circumference of a circle by setting different values for the diameter of a circle, and 'unrolling' it along a number line. It was shared by Aaron

Calculating Pi with real Pies which is a Numberphile video in which Matt Parker calculates a value for π by counting hundreds of identical pies after he has carefully arranged them to form a huge circle and one diameter. It was shared by Peter Williams

Parts of a circle which is a worksheet in which students are challenged to name the centre, radius and circumference in diagrams of circles with some line segments shown and points marked with upper-case letters. It was shared by Sharon Malley

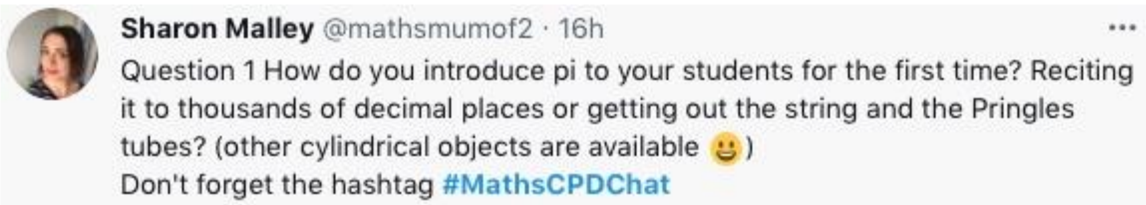
Mathematical Hooks which is a large collection of resources, such as videos and photographs, that are likely to engage students. They can provide starting points for student explorations of mathematical ideas in order to gain deeper understanding of them. It was shared by Julia Smith

Circumference: Teacher Notes which is an illustrated short document created by Barrie Galpin and Jay Timotheus in which they explain how a program on the TI-Nspire calculator might be used by students to explore the relationship between the radius, diameter and circumference of a circle. It was shared by Jay Timotheus

Circle in a Semicircle, Roll On, Two Paths and Running Race which are all NRICH problems taken from the UKMT Mathematical Challenges. They were shared by Mary Pardoe

Track Design which is an extended NRICH problem. It was shared by Mary Pardoe

The screenshots below, of chains of tweets posted during the chat, show parts of several conversations about various ways of starting to focus pupils' attention on lengths and areas in circles. **Click on any of these screenshots-of-a-tweet to go to that actual tweet on Twitter.** The conversations were generated by this tweet from Sharon Malley:



NCETM
NATIONAL CENTRE FOR EXCELLENCE IN THE TEACHING OF MATHEMATICS

67,890 digits

In 1981, an Indian man named Rajan Mahadevan accurately **recited** 31,811 digits of **pi** from memory. In 1989, Japan's Hideaki Tomoyori **recited** 40,000 digits. The current **Guinness World Record** is held by Lu Chao of China, who, in 2005, **recited** 67,890 digits of **pi**.

Mar 13, 2015

and included these from Peter Williams, Charlotte Hawthorne, Sharon Malley and Jay Timotheus:

Peter Williams @MathsImpact · 16h
Replying to @mathsmumof2
I like starting with estimating the circumference, then some rough measuring, usually by marking the distance for a full roll of something.

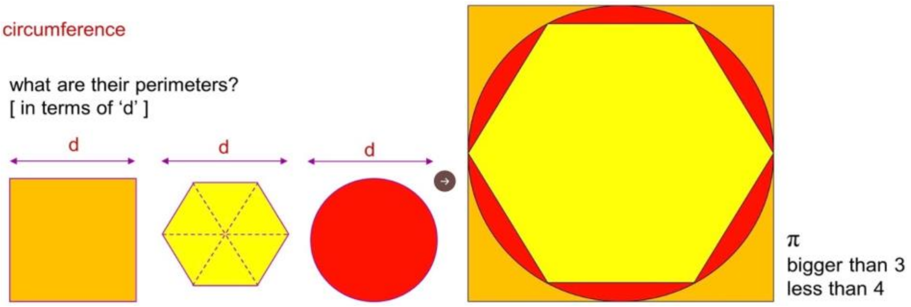
I want to establish "it's a bit more than 3" before we go on to more precision.

I also love this numberphile video

Calculating Pi with Real Pies - Numberphile
Our Pi Playlist (more videos): http://bit.ly/PiPlaylistHow accurately can we calculate Pi using hundreds of REAL...
youtube.com

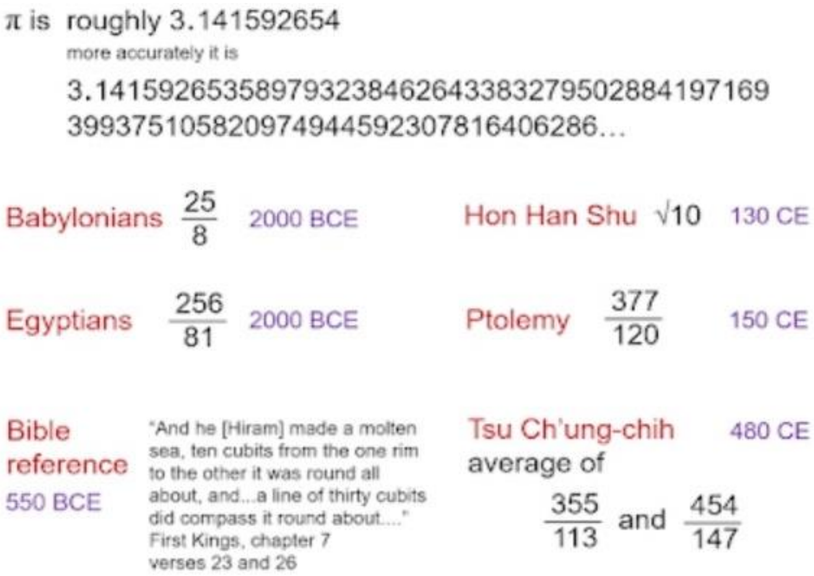
Charlotte Hawthorne @mrshawthorne7 · 17h
Replying to @mathsmumof2
I wouldn't try to get them to discover it. Depending on time I may get them to measure a few objects to get a feel for it being roughly three but I use the tools on mathspad and the Don steward resources. I'll find the links to the ones I use.

Charlotte Hawthorne @mrshawthorne7 · 17h
donsteward.blogspot.com/2015/12/pi-dev... #mathscpdchat



Sharon Malley @mathsmumof2 · 17h

#mathscpdchat I've got that one saved too! I love this slide from within it



Jay Timotheus @jaytimotheus · 16h

Replying to @mrshawthorne7 and @mathsmumof2

I think the old SMP books asked pupils to calculate (perimeter/diagonal) for regular polygons of increasing numbers of sides. Nice way to get a sense that the ratio is tending towards a limit. @mathscpdchat #mathscpdchat

Jay Timotheus @jaytimotheus · 39m

Replying to @jaytimotheus @mrshawthorne7 and 2 others

And this also works really well using dynamic geometry software, which also provides accurate calculations. @mathscpdchat **#mathscpdchat**

these from [Pete Mattock](), [Sharon Malley](), [Peter Gates](), [Aaron]() and [Atul Rana]():

Mr Mattock FCCT NPQSL @MrMattock · 15h
Replying to @mathsmumof2
I always thought the measuring activity was backwards. It shouldnt be to discover pi, but to practice working with it once they have been shown pi as the ratio of Circumference and Diameter surely? #mathscpdchat

2 3

Sharon Malley @mathsmumof2 · 15h
How would you do it in that way? To show what the measurements should be? I agree that there is a risk that misconceptions can be formed about pi at this point if it is done as an initial activity. #mathscpdchat

1

Mr Mattock FCCT NPQSL @MrMattock · 15h
So I think it is important to separate out learning about pi as the constant ratio between the circumference and diameter of circles from using it to find circumference and diameter. So for me, I introduce pi as the ratio and then would see the measuring activity as a way of...

2

Peter Gates @petergates3 · 4h
Why should they think there is a ratio which is constant? It reads as if you "introduce" and I am not sure what that looks like ....is that like telling? It is a fascinating piece of mathematics...how the human brain moves from personal practical experiences to irrationality.

Mr Mattock FCCT NPQSL @MrMattock · 3h
Normally I would introduce through use of dynamic geometry software, either through them being able to manipulate the circle and see the constant ratio, or through demonstration if I dont have the facility to get them in front of the software themselves. I find the "tell them"...

Mr Mattock FCCT NPQSL @MrMattock · 3h
...part tends to come more from the measuring activity - they get something near 3 and hey presto, by the way kids, that is actually pi.
I follow up the dynamic geometry software work with a discussion (usually teacher led) around the irrationality of pi by looking at fitting...

Mr Mattock FCCT NPQSL @MrMattock · 3h

...a rational length onto a curve, showing how if we break it down into smaller and smaller units, the straight line distance still wont perfectly match the curve, hence have to go to infinity (whilst technically knowing this creates a good argument for irrationality without...

1 1

Mr Mattock FCCT NPQSL @MrMattock · 3h

...actually proving it). The measuring activity then becomes part of their work getting comfortable with pi and the idea of a constant ratio, seeing how close they get to pi for different circular objects.

1 1

Peter Gates @petergates3 · 1h

That's really interesting as it isn't how I would do it, will see if i can squeeze in time between packing to respond

Peter Gates @petergates3 · 39m

I personally prefer enactive engagement rather than looking at a computer generated simulation. Not too bothered initially about π that can come later. The big ideas for me are invariance and three-ish-ness. π can come later as a research task. Maybe applets etc at end.

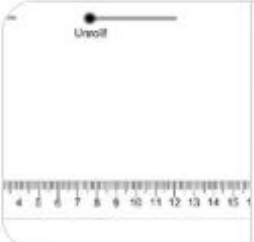
Aaron @MrBroMaths · 16h

Replying to @mathsmumof2

This applet, calculators and/or an Excel spreadsheet.

"Sir, it's the same, again.. 🤯"

Area is my favourite. We deduce intuitively that it must be between 2pi^2 and 4pi^2 by inscribing the circle in a square. Sensible to suggest pi lots of r^2.



Measuring Circumference and Diameter
Measuring Circumference and Diameter
geogebra.org

Atul Rana @atulrana · 16h
Nice applet!
1

Peter Gates @petergates3 · 4h
It is clever, but surely not as good as physical activity?

these from [Charlotte Hawthorne](), [Gemma Scott]() and [Sharon Malley]():

Charlotte Hawthorne @mrshawthorne7 · 16h
Replying to @mathsmumof2
I like looking up their birthdays in pi at the end of the lesson...they often stay after the bell to find out where theirs is. Some are rather proud of theirs is earlier on! 😂 One student remembered what my birthday was and then wished me happy birthday the other week ☺️
2 · 8

Director of Maths @DirectorMaths · 16h
I do something very similar, talk about this curious number pi (they've usually heard of it) and look for birthdays, phone numbers etc then link to its geometric properties. I've tried the measuring but they can't do it accurate enough 😂 #mathscpdchat
1 · 4

Sharon Malley @mathsmumof2 · 16h
Do you look at the link between lengths first or between areas? #mathscpdchat
1

Director of Maths @DirectorMaths · 16h
I would naturally go for lengths having established pi = circumference/diameter but reading how others introduce I might explore areas next time #mathscpdchat

these from [Jay Timotheus](), [Sharon Malley]() and [Mary Pardoe]():

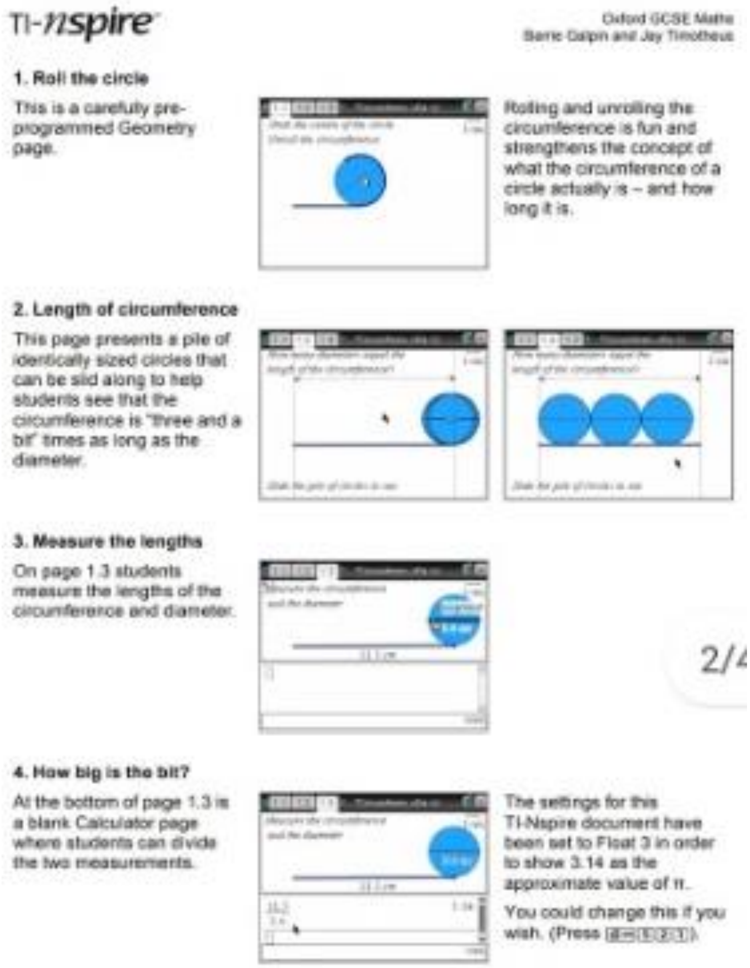
Jay Timotheus @jaytimotheus · 17h
Replying to @mathscpdchat and @mathsmumof2
Using Geogebra to 'unroll' a circle. Then we can compare the unrolled length with the original circle and see that it's 'three and a bit' diameters. @mathscpdchat **#mathscpdchat**

Sharon Malley @mathsmumof2 · 17h
I had one of those proper wow moments with a class when they saw this and realised that circumference was a length **#mathscpdchat**

Jay Timotheus @jaytimotheus · 17h

Replying to @jaytimotheus @mathscpdchat and @mathsmumof2

I co-wrote this resource many years ago. Technology has changed but the ideas still work! Available here: google.com/url?sa=t&sourc... @mathscpdchat #mathscpdchat



TI-nspire

Oxford GCSE Maths
Barrie Galpin and Jay Timotheus

1. Roll the circle

This is a carefully pre-programmed Geometry page.

Rolling and unrolling the circumference is fun and strengthens the concept of what the circumference of a circle actually is – and how long it is.

2. Length of circumference

This page presents a pile of identically sized circles that can be slid along to help students see that the circumference is "three and a bit" times as long as the diameter.

3. Measure the lengths

On page 1.3 students measure the lengths of the circumference and diameter.

4. How big is the bit?

At the bottom of page 1.3 is a blank Calculator page where students can divide the two measurements.

The settings for this TI-Nspire document have been set to Float 3 in order to show 3.14 as the approximate value of π.

You could change this if you wish. (Press [illegible]).

Mary Pardoe @PardoeMary · 17h

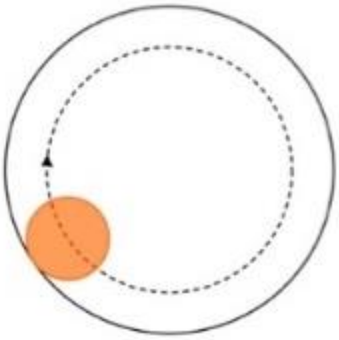
Replying to @jaytimotheus @mathscpdchat and @mathsmumof2

This is a nice kind of problem once they're getting the hang of the relationship between diameter and circumference ...

nrich.maths.org/4737

#mathscpdchat

A circular disc of diameter d rolls without slipping around the inside of a ring of internal diameter $3d$, as shown in the diagram. By the time that the centre of the inner disc returns to its original position for the first time, how many times will the inner disc have turned about its centre? What if the disc rolls around the outside of the ring?



This problem is taken from the UKMT Mathematical Challenges.

https://nrich.maths.org/4737

and these from Sharon Malley, Jay Timotheus, Pete Mattock, Mary Pardoe and Charlotte Hawthorne:

Sharon Malley @mathsmumof2 · 17h

What problems do you find students have with circles and the relationships within them? Do they know what a chord is or why the diameter must go through the centre? **#mathscpdchat**

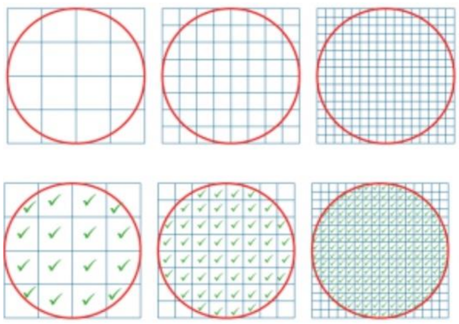
Jay Timotheus @jaytimotheus · 17h

Replying to @mathsmumof2

Problem... Thinking that pi must be an incredibly large number because it "goes on forever". @mathscpdchat #mathscpdchat

Sharon Malley @mathsmumof2 · 17h

Agreed, I think it is really important to do estimates of circumferences and areas with pi as 3 first, before using pi in a more accurate form #mathscpdchat exploring activities like this with an estimate of an area compared to the known radius.



Mr Mattock FCCT NPQSL @MrMattock · 17h

Replying to @mathsmumof2

Dont people teach the nomenclature for circles explicitly? #mathscpdchat

1

Sharon Malley @mathsmumof2 · 17h

We would hope so but I think there is a focus on radius, diameter and circumference and diameter as a special chord can be overlooked. #mathscpdchat

1

Mr Mattock FCCT NPQSL @MrMattock · 17h

May as well get them knowing the names of all the parts before they actually learn about them. I have an activity that in my (free) TES shop on circle nomenclature that I have used for years. Segments, sectors, arc, radii, all included. #mathscpdchat

Mary Pardoe @PardoeMary · 17h

Replying to @MrMattock and @mathsmumof2

You can bring in the 'nomenclature' when they feel-a-need-for/see-the-point-of knowing it?
e.g. ... when tackling a task like this ...
nrich.maths.org/7359
#mathscpdchat

Track Design

https://nrich.maths.org/7359

Age 14 to 16
Challenge Level ★

- Warm-up
- Try this next
- Think higher
- Read: mathematics
- Read: technology
- Explore further

Imagine you are building a new Olympic stadium and you are responsible for designing and marking out the running track. The track needs to fulfil the following specifications:

- The distance around the inside edge of the inner lane should be 400m.
- There should be 8 lanes.
- Each lane should be 1.25m wide.
- The track should consist of two straight sections joined by two semi-circular sections.
- The straight sections should each be 85m in length (a straight section is extended over the curve for the 100m race, as shown below).

Charlotte Hawthorne @mrshawthorne7 · 17h

Replying to @PardoeMary @MrMattock and @mathsmumof2

We name things we care about. ☺️

I'm all for the approach of getting them to the point where they are asking "what can I call this bit?", "so HOW do we do this?"

(to read the discussion sequence generated by any tweet look at the 'replies' to that tweet)

Other areas where discussion focused were:

a tweeted comment made as a reply to a reminder (posted 15 minutes before the chat) of the topic to be discussed, generated two more tweets (links to the NRICH task and the NCETM materials are provided above):

Mary Pardoe @PardoeMary · 16h

What about starting with a problem ... asking students what they think will need to know/be-able-to-calculate in order for there to be any possibility that they can answer it?

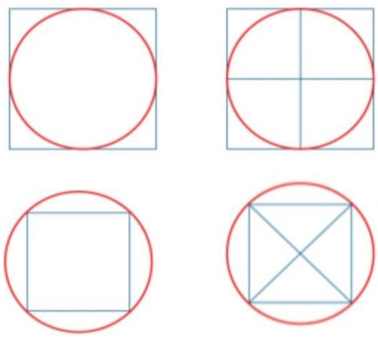
The diagram shows a semi-circle containing a circle which touches the circumference of the semicircle and goes through its centre. What fraction of the semicircle is shaded?

https://nrich.maths.org/2166

Sharon Malley @mathsmumof2 · 15h

That is an excellent image I like this set from the @NCETM materials so many questions to ask about the relationships. #mathscpdchat

ncetm.org.uk/media/1qabpyac...



although most of the discussion prompted by the host's first question is shown in the sequence of screenshots of tweets, these useful replies were also tweeted …

Tessmaths @tessmaths · 15h

Replying to @mathsmumof2

A1 #mathscpdchat Sorry I'm late...I've moved all my favourite circle and pi hooks to the top of my hooks padlet for you to view here... padlet.com/tessmaths1/hoo...

Start off by showing the World Champ...love the teeshirts too - several things there to start off a pi discussion

Mathematical Hooks

Made with a touch of glamour

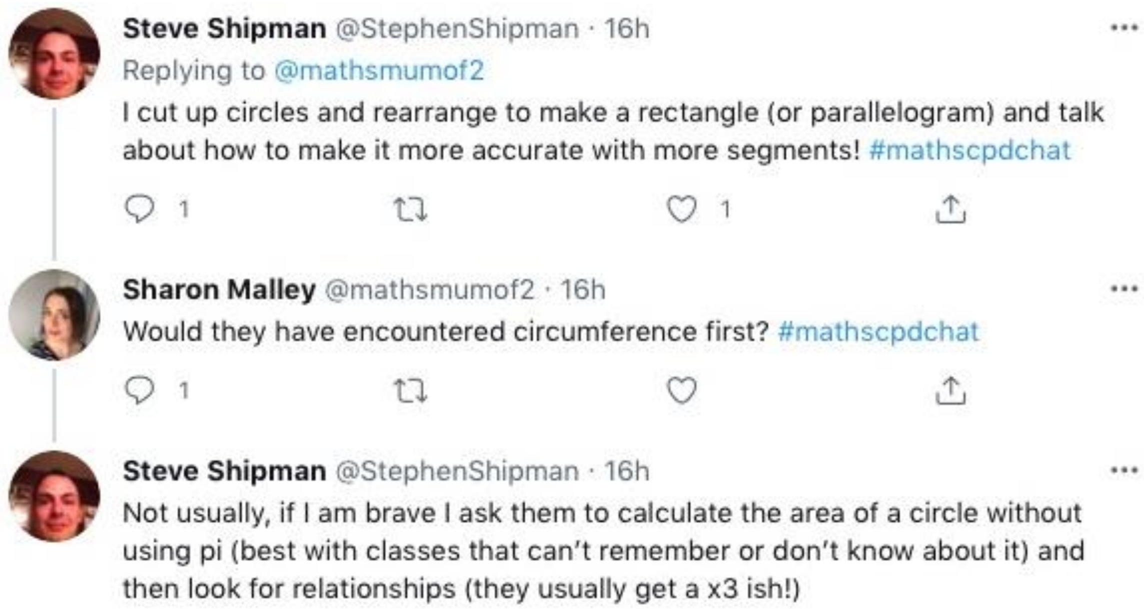
padlet.com

(link provided above)

Atul Rana @atulrana · 16h

Replying to @mathsmumof2

I made a video on that! They cut out strips of paper, put across diameter and mark the length. Wrap the strip around circumference and mark again. See how many folds of the diameter make a circumference. Repeat for other circular shapes at home; tumblers, lids etc. #MathsCPDchat



Steve Shipman @StephenShipman · 16h
Replying to @mathsmumof2
I cut up circles and rearrange to make a rectangle (or parallelogram) and talk about how to make it more accurate with more segments! #mathscpdchat
1 1

Sharon Malley @mathsmumof2 · 16h
Would they have encountered circumference first? #mathscpdchat
1

Steve Shipman @StephenShipman · 16h
Not usually, if I am brave I ask them to calculate the area of a circle without using pi (best with classes that can't remember or don't know about it) and then look for relationships (they usually get a x3 ish!)

… and a light-hearted conversation developed about the number of digits in the decimal expression of π that teachers can themselves remember/recite, for example:

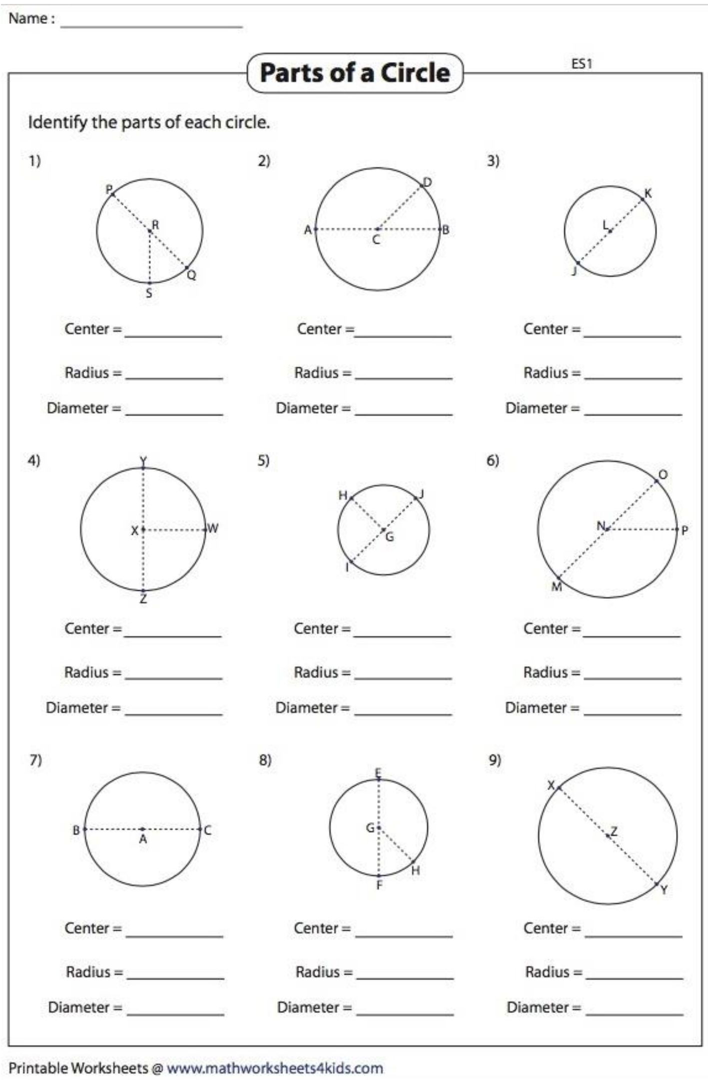
Charlotte Hawthorne @mrshawthorne7 · 16h
Replying to @mathsmumof2
I can do 32 digits... kids seem pretty impressed by that. 😂

Becca @Red_Maths · 16h
Replying to @mrshawthorne7 and @mathsmumof2
I learnt to the first zero too! 😂

- several teachers commented that, owing to the **inaccuracy of students' measurements**, it is not sensible to set tasks in which the aim is for students to deduce empirically a value for circumference ÷ diameter that is more accurate than 3;

in addition to the discussion generated by the host's question about difficulties 'students have with circles and the relationships within them' that is also shown in the sequence of screenshots of tweets, the host made this comment:

Sharon Malley @mathsmumof2 · 17h
Thanks for signposting that Pete. I liked this resource as it encourages the use of two letter notation for line segments and reinforced that diameters are made from two radii in a straight line passing through the centre #mathscpdchat


Name : ____________

Parts of a Circle ES1

Identify the parts of each circle.

1)

Center = ________
Radius = ________
Diameter = ________

2)

Center = ________
Radius = ________
Diameter = ________

3)

Center = ________
Radius = ________
Diameter = ________

4)

Center = ________
Radius = ________
Diameter = ________

5)

Center = ________
Radius = ________
Diameter = ________

6)

Center = ________
Radius = ________
Diameter = ________

7)

Center = ________
Radius = ________
Diameter = ________

8)

Center = ________
Radius = ________
Diameter = ________

9)

Center = ________
Radius = ________
Diameter = ________

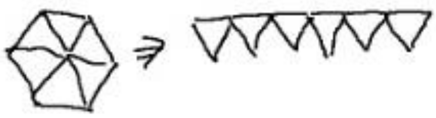
Printable Worksheets @ www.mathworksheets4kids.com

(the link to this worksheet is provided above)

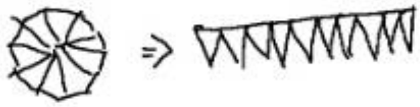
towards the end of the chat the host asked this interesting question (illustrated with an image from a recent tweet by Kathy Murdoch) ...


Sharon Malley @mathsmumof2 · 17h

So does anybody think we should scrap pi and use Tau as the constant? #mathscpdchat if you "open up" any polygon, it is made of many triangles:

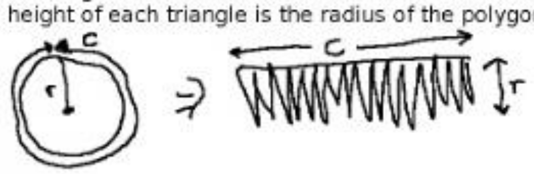


as the number of edges grow, you get closer to a circle:

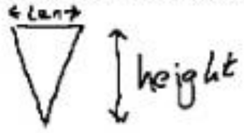


so a circle is essentially a line with infinite triangles hanging off it

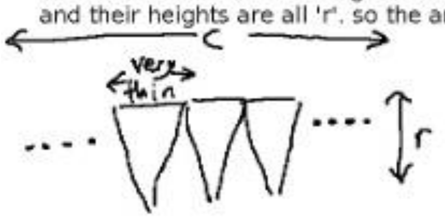
the length of the line is the circumference in all cases, and the height of each triangle is the radius of the polygon in all cases



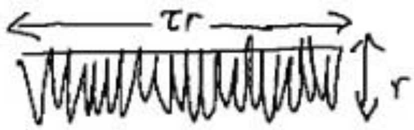
we know that the area of a triangle is ½ x height x length.



in a circle there are 'c' triangles of infinitely small width, and their heights are all 'r'. so the are



now, the circumference of the circle is defined as τ (tau) radiuses



so the area of the unfolded strip (which is the area of the circle) is

$$\frac{1}{2} \times \tau r \times r$$

ie: $\underline{\underline{\frac{1}{2} \tau r^2}}$

… to which there were no replies.