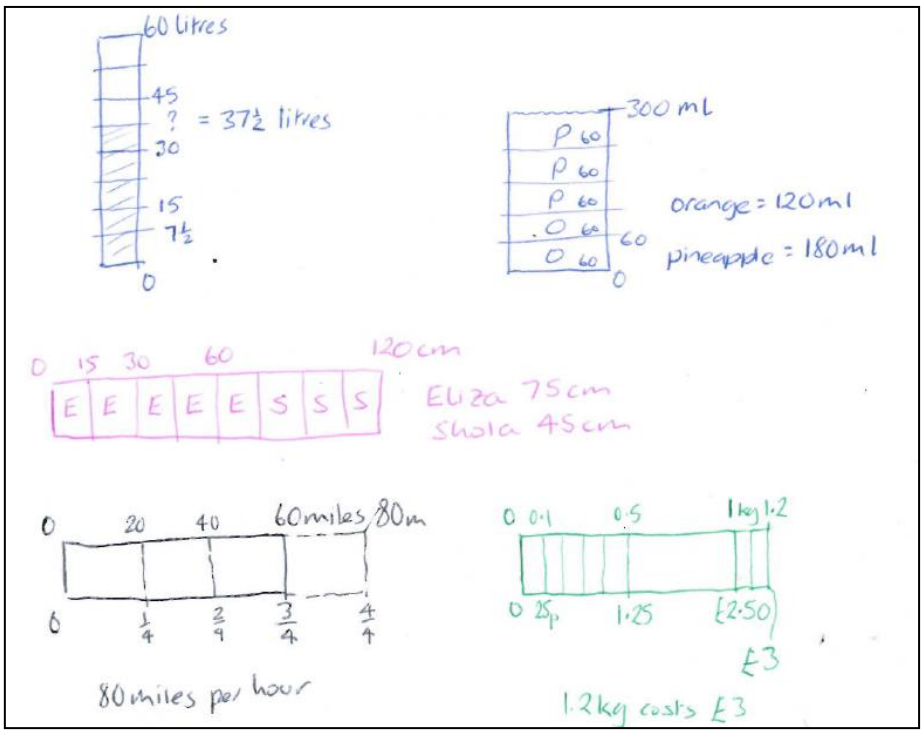
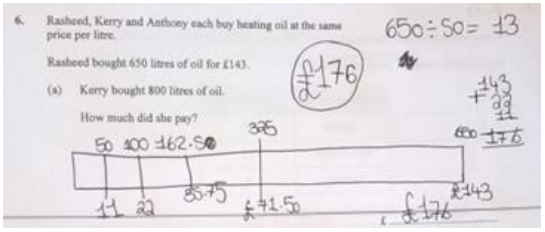
National Centre for Excellence in the Teaching of Mathematics

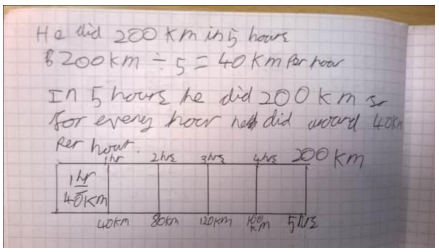
NCETM report on the KS3 Multiplicative Reasoning Project

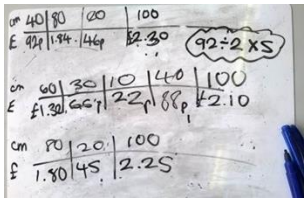
September 2013 – July 2014

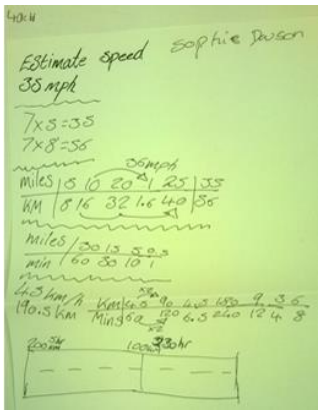
Pupils making sense of proportional problems

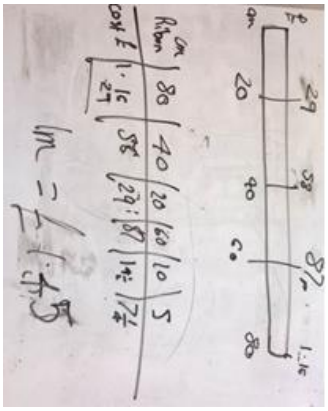


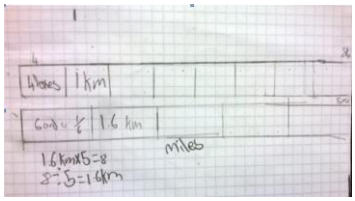












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Contents

Section 1: Introduction

The KS3 Multiplicative Reasoning project was a one year DfE-funded pilot delivered through the NCETM. It was a professional development project for teachers, focusing on pupils' learning of multiplicative reasoning.

Schools' involvement with the project ran from October 2013 until July 2014. As a pilot, it aimed to investigate what could be learnt through linking research into how pupils learn this area of mathematics with teachers' classroom practice. The results will inform possible support packages for wider use by schools.

This document reports NCETM's internal evaluation of the project. It draws on a range of feedback and observation data collected over the period of the pilot.

The project was also accompanied by a randomised control trial (RCT) to test impact on pupils' mathematical learning. Sheffield Hallam University were appointed by the DfE to carry out the RCT and report on it as part of an external evaluation of this aspect of the project. The Sheffield Hallam report can be found using the following link: https://www.gov.uk/government/publications/multiplicative-reasoning-professional-development-programme

Partnerships between schools and local universities have been at the heart of the project. This evaluation also reports on the model employed in the delivery of this project and considers its suitability as a model for effective PD[1] delivery across other areas of practice and the curriculum.

[1] ***PD = Professional Development***

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Multiplicative reasoning

A variety of terms have been utilised in recent years to refer to mathematical content including: multiplication by integers, decimals and fractions, percentages, ratio, proportion, enlargement, scaling and rates of change. The term 'Multiplicative reasoning' is being used here to encompass the understanding of the structures within these content areas and the appropriate use of models, algorithms and techniques to solve problems related to these content areas.

Many situations encountered in life and work involve contexts and questions that are based on proportionality. Many, seemingly different, strands of the mathematics curriculum involve multiplicative reasoning (MR), as do many questions in a typical GCSE exam paper.

The project looked at the effect on teachers (subject and pedagogical knowledge) and pupils learning of:

- **Making connections in mathematics** where the underlining structure is multiplicative.
- **Deepening the understanding of the mathematics related to solving problems** where the underlining structure is multiplicative.

Consideration of the full range of MR across the mathematics curriculum would have required a larger scale project. This project aimed to see what could be achieved as a starting point by helping teachers to engage with some of the issues and teaching approaches that have proved effective in practice and are supported by research. As such, the project focused primarily on teachers' professional development related to developing pupils' understanding of these aspects of the curriculum; the lessons were designed as a professional development experience for teachers, rather than as a coherent curriculum experience for pupils.

Importantly, the project attempted to provide a strong foundation from which further work within school mathematics teams and between schools could be undertaken by teachers.

Purpose of this Evaluation

This document attempts:

- to share teacher's responses to the project materials and processes in order to assess the different aspects of the project and its impact on teachers' CPD.
- to include teacher's more detailed findings, together with consideration of the effectiveness of the **TIME**[1] **team model** used to deliver the project, in order to contribute to a user guide to support groups who may wish to undertake PD through using a similar methodology in the future. In particular, such groups may be of interest to the work of Maths Hubs and could include secondary school mathematics departments, primary schools and mathematics teams in sixth form and FE colleges.

Focus for internal evaluation

This internal evaluation focuses on the following key areas:

- **The impact on teachers' subject knowledge, pedagogy, classroom and professional practice**
- **The effectiveness of the professional development delivery model**
- **Teachers' reported impact on pupil learning**

[1] ***TIME = Teachers Improving Mathematics Education***

Main evidence sources for evaluation

Evaluation evidence was collected during and at the end of the project:

- **Teachers written and oral feedback at workshops**
  In particular collected feedback relating to impact on thinking and practice of:
  - units, lessons and workshop activities
  - completed **lesson studies**[1]
  - *all workshops were attended by the project lead for observation and to support evaluation*

- **Observation of:**
  - workshop discussions
  - pupils' work including videoed interviews of pupils justifying solutions

- **Interviews in schools:** project teachers, pupils, subject leaders and head teachers

- **Post project professional development booklets**
  - each teacher completed a booklet at the end of the project where they reflected on the impact on their thinking and practice of key aspects of the project. Engagement with the booklet was started in the final workshop where sessions were linked to sections in the booklet. It was completed by the teacher after the workshop and returned for project evaluation purposes. The booklet contained qualitative and quantitative data, the latter informing the data quoted in section 8 of this report. A spreadsheet collating comments and analysing judgement data from the booklets is available as an appendix.

Integration of project evaluation into teacher professional development

A key part of the professional development design was the opportunity during workshops and school based tasks for teachers to reflect, discuss and record the impact of activities and materials on their practice and thinking. The formats used by teachers to do this also provided a key source of evaluation for the project. Some of the main formats used for doing this are given in the appendix of this report.

Contemporary relevance of the project

Key approaches in the project resonate with wider developments current in maths education.

- The project focuses on significant content aspects of the new curriculum. The teaching and learning approaches used reflect the key themes of the new curriculum and the associated teacher PD needed to support their implementation.

- The project brings together university research expertise into how pupils learn these aspects of the curriculum with professional development leaders and classroom practioners. The project uses classroom evidence and observation of pupils' responses to the learning materials as the main focus for the PD.

- The project content and model offer a suitable work group for Maths Hubs and there is significant interest from hubs in using this model.

[1] ***See page 7 for a description of the lesson study model used for the project and page 27 for feedback on its use.***

Section 2: Project structure and delivery

- The project was organised centrally by NCETM with delivery focused around three regional centres – Nottingham, Manchester and Southampton. These were 'pathfinder' hubs based around a lead school appointed by NCETM and given responsibility for providing the regional training venue and administration related to the project schools allocated to their region.

- Project schools were recruited centrally to the project via application following advertising through various outlets but mainly through the NCETM website. The application details specified the release of two teachers from the maths department to attend all 6 days of workshops provided at the local hub venue and support the gap tasks which included the teaching of the project lessons and completion of lesson studies.

- Within a relatively short period of time over 100 schools had submitted applications from which 30 intervention schools and 30 control schools were selected by the external evaluator (Sheffield Hallam University) in line with the requirements of the Randomised Control Trial process. The intervention schools were allocated to one of the three regions (hubs) according to geographical location, each hub receiving ten schools from which the nominated project teachers would attend all the workshops provided at the hub venue.

- Each school had been asked as part of the application process to nominate two teachers from the maths department involved with teaching KS3 classes to be the project teachers, attend all the workshops together and complete the gap tasks. Schools were asked to select where possible a more experienced KS3 teacher alongside a less experienced or non-specialist maths teacher as the project teachers.

- A separate process was then undertaken by the external evaluator (Sheffield Hallam University) to select pupils from the intervention classes (and classes in the control schools) as part of the RCT test. Details of this are given separately in the SHU external evaluation report

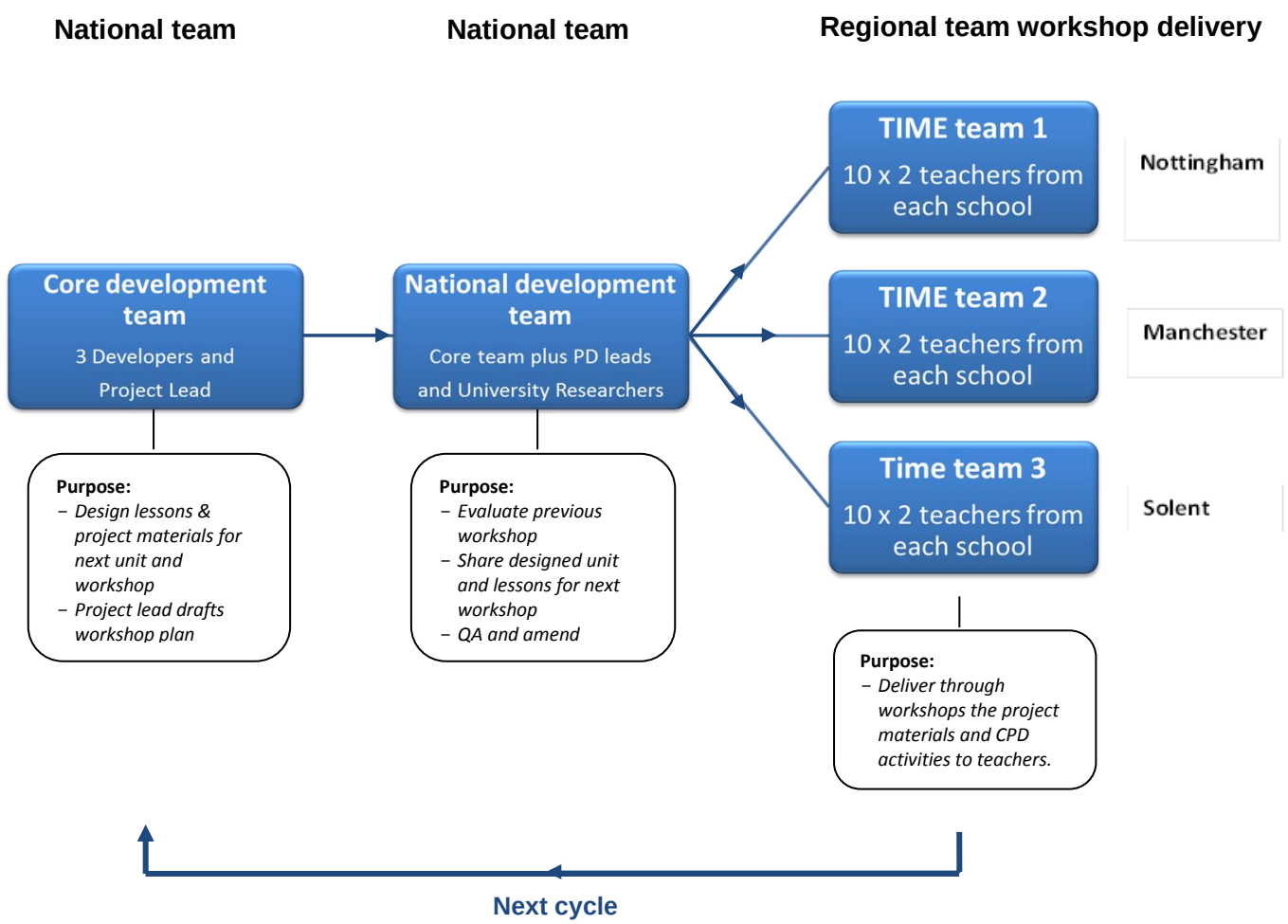
- Each hub now had twenty teachers from their ten allocated schools attending the workshops spread over the school year from October to June. For each Hub two 'professional development leads' were appointed by the Hub (NCETM criteria – experienced maths teacher and CPD provider) to work alongside a 'researcher' (appointed by NCETM) from a local HEI to deliver the workshops for their Hub. The aim of the HEI 'researcher' was to bring research knowledge rigour to the workshop discussions and expertise to the lesson study. They were also able to bring a certain degree of evaluation expertise. Together the leads, researcher and the project teachers for each hub were known as a **TIME team** (Teachers Improving Maths Education).

- The workshops were designed centrally (with some local flexibility) around materials specifically designed by a separate team of researchers (the **Core Development Team – CDT**) selected for their knowledge and experience of the latest research into effective teaching and learning in this area of the curriculum.

- The professional development leads and researchers for the three hubs met as a team prior to each workshop to be introduced to the next workshop and support its design (the **National Development Team – NDT**)

See next page for diagrammatic representation

Sequence of team meetings leading to each TIME team workshop



Summary of delivery and design:

Delivery:

- The project was arranged around 3 pathfinder Maths Hubs
  - **Manchester, Portsmouth, Nottingham**
- Each Hub ran a **TIME team** consisting of
  - **10 intervention schools in each team - *2 teachers from each school***
- Each TIME team was run by
  - **2 professional development leads** and **1 university researcher**
- Each TIME team completed 5 cycles of workshop and in school gap task
  - **5 TIME team workshops (6 days) over three terms**
  - **Gap tasks between workshops informing feedback at next workshop**

Design:

- A team of core designers met to design materials for each workshop
  - **(Core Development Team)**
- Prior to each workshop the hub professional development leads met to assess the previous workshop and be introduced to materials designed for the next workshop
  - **(National Development Team)**

Time teams and workshops

The **TIME team** model aimed to develop teachers' (and departments') subject specific and pedagogical knowledge together with the associated classroom practice in this particular area of the curriculum. **Evaluation** tasks were built into the workshops also forming a key opportunity for teachers to reflect on their professional development.

Developing subject specific knowledge

This was primarily done through careful discussion of the mathematics associated with the project designed lessons as they were worked through with teachers in the workshop prior to teaching. Sometimes this includes the use of additional activities designed to highlight or extend an aspect of subject knowledge for teachers.

Developing subject specific pedagogical knowledge

Pedagogical discussions take place as teachers work through the lessons during the workshops, however the principal source of PD here is the careful focus on the impact of the project activities on pupils learning; this is achieved through the lesson studies undertaken as part of the gap tasks and discussed at the workshops. A particular aim of the **lesson study process** adapted for the purposes of the project is to allow pedagogical discussions resulting from lessons to be moved between the teachers, department and workshop, hence engaging a wider CPD audience and contribution.

The structure of the lesson study process used for the project

- For each workshop the TIME team researcher chooses one of the project lessons together with a suitable research question to be the focus of a common lesson study to be undertaken by all the teachers in the TIME team.
- Back in school, the project teachers undertake the lesson study engaging the rest of their department as part of the gap task. A format and guidance is supplied to support this and record key aspects of the department post lesson discussions on the research question.
- The outcomes of the department lesson and its subsequent discussions are discussed all together at the next workshop under the guidance of the HEI researcher in order to further refine and deepen understanding.
- Teachers reflect and record key pedagogical insights they feel they have gained.

TIME team leadership

The TIME team workshops were led by the two hub professional development leads supported by an appointed HEI researcher/co-lead.

Role of the HEI co- lead

The HEI involvement was to support the rigour and effectiveness of this process through:

- Supporting the PD leads in planning the workshops and 'in school'gap tasks (*within the agreed project structure, guidance and aims*)
- Suggesting the common classroom task for the focus of the lesson study and the research question
- Leading these sessions in the workshops and leading the feedback from the common lesson study at the subsequent workshop in order to focus on maximising the PD

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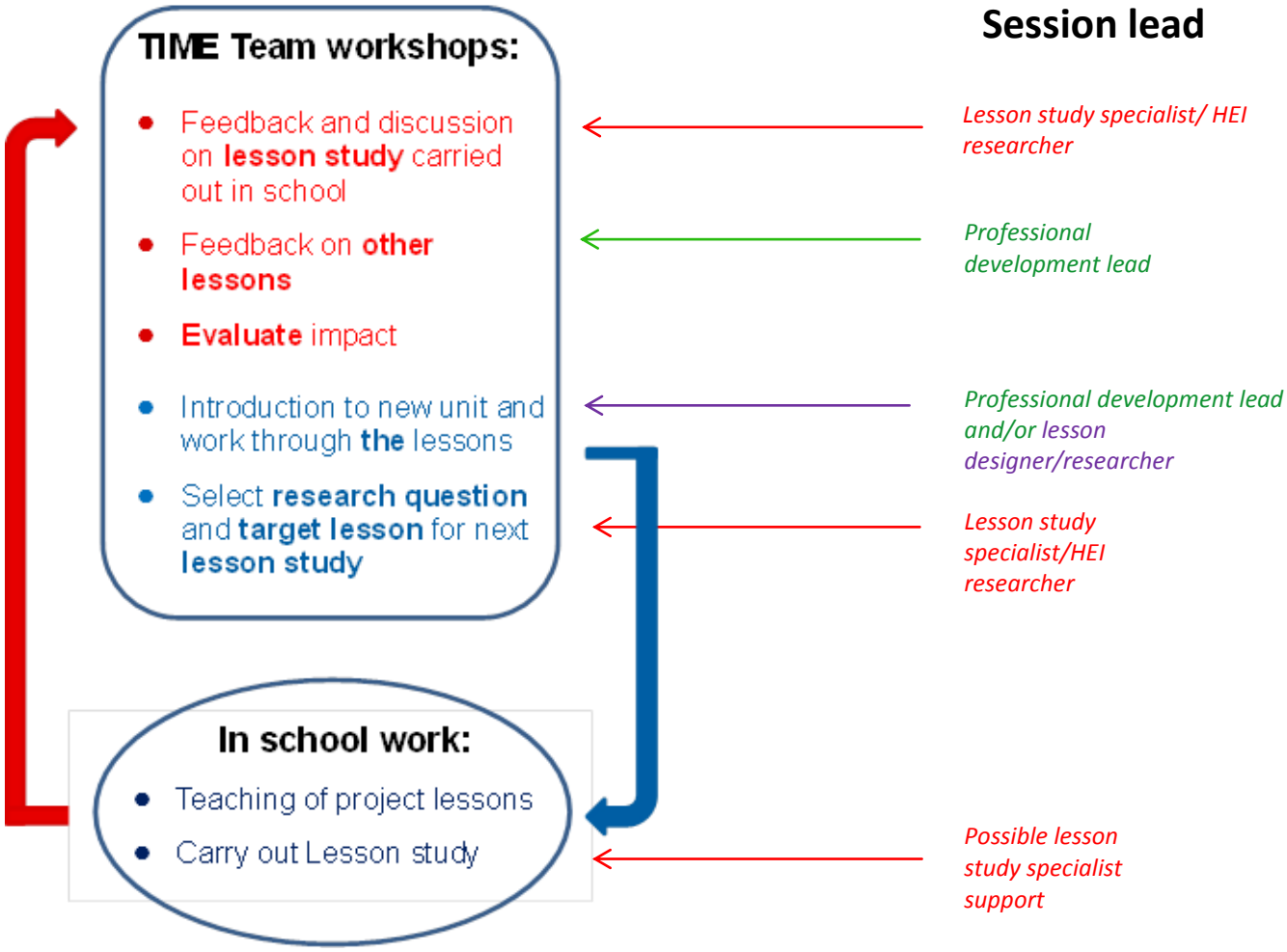
Basic Structure for the project TIME team workshops and Gap task



See next table for sequence and content of workshops and gap tasks

TIME team workshop and gap task cycle

Date	Event	Focus	School Gap task
Sept –Oct 2013	CDT and NDT meetings	*Designing & planning*	
Oct 2013	**Workshop 1** 2 days	**Introduction** **Unit 0** - Formative assessment task	Complete Unit 0 assessment tasks
Nov 2013	CDT and NDT meetings	*Feedback on workshop and planning*	
Dec 2013	**Workshop 2**	**Feedback on gap task** **Unit 1** - Reasoning and making sense of fractions – **3 sets of 2 lessons**	Teach lessons & do lesson study
Dec 2013	CDT and NDT meetings	*Feedback on workshop and planning*	
Feb 2014	**Workshop 3**	**Feedback on gap task** **Unit 2** - Understanding and identifying proportional contexts – **3 sets of 2 lessons**	Teach lessons & do lesson study
Feb 2014	CDT and NDT meetings	*Feedback on workshop and planning*	
March 2014	**Workshop 4**	**Feedback on gap task** **Unit 3** - Application to a range of proportional problems – **3 sets of 2 lessons**	Teach lessons & do lesson study
April 2013	CDT and NDT meetings	*Feedback on workshop and planning*	
June 2014	**Workshop 5**	**Feedback on gap task** **Evaluation**	**Pupils take PIM test** June 4th as part of SHU RCT
July 2014	CDT and NDT meetings	*Feedback on workshop and evaluation*	Complete evaluation booklets

See Section 3 for workshop link to subject specific professional development

Activities	Materials produced
5 workshops (6 days) Unit 0 assessment task and questions A **teaching unit** introduced at each workshop	• **Teaching units** ○ *One assessment unit* (*4 core questions*) ○ *3 teaching units* (*between 3 and 6 lessons per unit*)
In school gap tasks: • carry out **unit 0 assessment** tasks • **Teach lessons** • **Lesson study** and other activities	• **Lesson study booklet** to support and record the in school lesson study process • **Common anticipated issues sheet**
Access **on line community**	Forum topics, discussion, access to materials

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Section 3: Project features

Project approach:

Teaching focus:

To support pupils in making sense of problems and the mathematics involved by challenging them to:

- **Represent problems using appropriate visual images**
  - Bar, double number line, ratio tables
- **Justify and explain their approaches and solutions**

Professional development focus:

To develop teacher subject knowledge, pedagogy and practice by:

- **Using specifically designed lessons and reflecting on them**
  - based on research into learning
- **Listening carefully to pupil responses to inform further teaching**
  - lesson study* and assessment interviews (see appendix for a definition of the type of lesson study used in the project)

Project dimensions *Key features of the CPD*

- **Making connections in mathematics and deepening the understanding of multiplicative reasoning**
- **Using representations to make sense of problems**
- **Paying close attention to pupils explaining and justifying answers, dealing with misconceptions and moving thinking on**
- **Working with colleagues in school and at workshops**

Section 4: Project subject knowledge component

Workshop teaching focus	Workshop Subject knowledge focus
Workshop 1 *(2 days)* Project introduction The mathematics of Multiplicative reasoning Unit 0 assessment tasks	**A. Recognising areas of the curriculum connected by proportionality** • recognising connections where the underlying mathematics is multiplicative • Considering common approaches to solving proportional problems *(unitary and scale factor)* I. Identifying the meaning of the operations involved II. Showing their equivalence mathematically. **B. Considering the mathematics of multiplicative reasoning** *(this was optional, see appendix1)* • Transforming one number into another – via two operations - multiplication and division – reducing this to a single operation of either multiplication or division. • Moving towards a broad definition of the term *multiplicative reasoning* used for the project • Ratios and fractions • Definition of a proportion **C. Considering different interpretations of multiplication and division** • Division as sharing (partitive) or grouping (quotative) and multiplication as repeated addition or scale factor enlargement • Interpreting the meaning of calculations from a given context **D. Generating equivalent expressions involving fractions**
Workshop 2 **Teaching unit 1:** Deepening understanding of fractions	• Identifying the whole in solving problems involving fractions • Applications of the distributive law for fractions • Geometric images to support the visualisation of infinite power series • Consideration of a fraction as an operator
Workshop 3 **Teaching unit 2:** Understanding and identifying proportional contexts	• Partition and its relation to area partition -*partition of brie segment in appendix* • Fractions in relation to ratios • The effect of proportional changes and their inverses • Definition of a reciprocal as an inverse operator • Equivalence of division to multiplication by reciprocal, hence their role in relation to division of fractions • Exploring the properties of different relationships including directly proportional relationships and linear relationships • Exploring the representation of functions through mapping diagrams and double number lines
Workshop 4 **Teaching unit 3:** Application to a wider range of contexts	• Making sense of procedures involving calculating with fractions • Interpreting the meaning of multipliers and chains of multipliers in fraction and decimal form and simplifying such expressions to show equivalence • Models for multiplication and division including decimals and fractions
Workshop 5 **Plenary workshop**	• Necessary and sufficient conditions for proportionality: • Exploring a proportional problem across representations • Combining intermediate steps (operations) to a single operation • Proving the distributive law for proportional functions

Section 5: Main teacher findings and recommendations

Section 5a - Main teacher findings:

This section details those aspects of the evaluation we have quantitative data for, drawn from an analysis of the final teacher evaluation booklets. This section draws together and summarises key points from all the evaluation sources described in section 2 in order to give a reflection of what might be expected as outcomes from engagement in the project. Consideration is also given to lessons that might be learned to improve the effectiveness of the project if it were to be the basis of a scalable product. This then leads in the final part of this section to a consideration of possible follow up actions for NCETM.

Impact on CPD - *Teachers reported:*

1. The **main change** the project made to their **thinking and practice** was a greater focus on **securing understanding** before moving pupils on to other topics.
    a. Through the processes of the project teachers found that many of their pupils' **procedural applications** *(even applications resulting in correct answers)* were often backed by **limited understanding** of the mathematics involved and the problem being solved. **Hence learning was not secure**.
    b. Teachers, including experienced teachers, reported surprise at the extent of pupils' **misconceptions** and felt it was important to allow these to be drawn out and addressed in their teaching.

2. Developing a recognition that **multiplicative reasoning connected** many areas of the mathematics curriculum and that making this explicit in their teaching and schemes of work could greatly improve **pupils' conceptual understanding**.
    a. Comments reflected a belief that an awareness of how different areas of the curriculum were underpinned by the same mathematical idea of proportionality would support the development of pupils' understanding. They also believed that exploring this through the project had supported their own subject knowledge.
    b. In particular, a focus on developing pupils' understanding of the mathematics behind proportionality could improve their **procedural fluency** across the full range of the curriculum, making the apparent **size of the new curriculum more manageable**.

3. The use of particular **visual models** supported pupils' ability to represent and make sense of proportional problems and, together with the focus on **explaining and justifying** approaches and solutions, helped pupils reveal **misconceptions**, **deepen understanding** and **see connections.**
    a. Pupils' use of visual representations facilitated their explanations and gave opportunities for teachers to challenge and develop thinking through carefully posed questions and tasks.
    b. Views were expressed that these approaches provided opportunities to go deeper into the mathematics, hence offering **appropriate challenge to all pupils within the same topic**.
    c. The use of the bar model particularly supported pupils' modelling of contextual problems. The double number line and ratio table supported discussion and understanding of many properties of proportional problems.
    d. The use of a particular model for different problems supported pupils in seeing and explaining connections.
    e. Teachers claimed they are adopting many of these approaches across all their teaching.
    f. During workshops and in written feedback, teachers emphasised the importance of:
        - approaches that avoided the use of visual representations to simply mimic procedure, rather than develop understanding.
        - avoiding questioning that led pupils' thinking, rather than supporting them in making meaningful sense of the problem or the underlying mathematics.

- starting these approaches in primary education and building them into schemes of work to develop continuity in learning across key stages.

4. Teaching was more effective when teachers were able to **work together** to plan the lessons. Teachers reported they benefitted when planning focused on **anticipating pupil responses** to the **carefully planned lessons** followed by discussion of **actual responses** in order to consider how to **move learning on** for subsequent lessons.

   a. This was greatly facilitated where time was provided for teachers to work together in school.

   b. Where **lesson study** was the focus of the gap task, this contributed significantly to the quality of the CPD. The assessment activities (*in particular videoed interviews of key selected pupils*) were considered a useful CPD resource for department use.

   c. Where there was not time for a full lesson study, some teachers felt the use of the '**common anticipated issues' sheet** *(a copy is included in the appendix)* had great potential to support PD and improve the teaching of the lesson.

   d. There was potential for the subject leader to integrate participation in the project as part of a whole department improvement plan.

   e. The **research background** and particularly the **lesson commentary** accompanying the project lessons supported teacher's effective use of the lessons and enhanced their **PD**.

   f. The use of teachers' time to focus on judging the significance of the impact on pupils' learning and how to move this on, rather than on designing lessons and activities, allowed greater access to CPD for all - (*often teachers felt they did not have time to come up with lesson ideas and key learning activities, but felt they could successfully adapt lessons in the light of pupils' responses*).

5. The format of the **TIME team model** (*workshops and gap teaching tasks, including a version of lesson study with feedback supported by HEI expertise*) played a crucial role in the effectiveness of the **PD.**

   a. Teachers found the balance of activities in the model worked particularly well for them and in most cases, for their departments, though the timeline often proved challenging.

   b. Workshop sessions discussing the project lessons following their teaching were important for teachers to *make sense* of what they had observed or experienced

   c. **Observation of workshops (*including by external evaluators*) indicated that significant professional development occurred during the guided feedback sessions on the lesson study gap task** in those hubs where the lesson study had taken place**.** (*A version of lesson study was adapted for use as part of the TIME team model with guidance provided for teachers and their departments*)*.

      During a workshop the HEI co-lead would choose one of the newly introduced lessons and an appropriate research question to be the focus for the lesson study gap task. Teachers would then follow the guidance and carry out the lesson study with their department, recording the key points from the department post lesson discussion on the research question. These key points were then brought to the subsequent workshop feedback session where the HEI co-lead used their expertise and knowledge of the lesson study process to carefully guide the discussion to allow teachers to further unpick the issues. Observation suggested that in these sessions in particular, teachers appeared (or stated as such) to be gaining significant deeper insights into the teaching and learning. **A reason for this** may be this second opportunity (under the skilled guidance of the expert lead) for teachers to reflect on their insights alongside others who have undergone the same process.

      This particular scenario would not normally be available through a typical lesson study cycle undertaken with just the department and hence may be a particular advantageous feature of the TIME team lesson study model.

d. **Having a pair of teachers from the same department** was considered important to the PD as they could work together in school on the gap tasks, especially the lesson study, and gain full joint benefit from workshop discussions.

e. **As the workshop programme progressed, the impact on teachers' PD became more significant**, reflecting greater familiarity with the format, the other teachers, the tasks and the nature of discussion and emerging thinking. This emphasises the importance of the same two teachers attending all workshops.

f. **The use of at least one teacher/consultant with experience in leading professional development, together with another person with lesson study expertise** (*preferably a link with local university*) to lead the workshops, was considered important to their effectiveness. This opportunity was also considered both by the leads and the HEI's to be of benefit to their respective professional development.

g. Carefully planned **evaluation** sessions, following key activities that gave teachers time to reflect and record changes they felt had taken place in their subject and pedagogical knowledge and classroom practice were an important feature of the ongoing PD.

h. **National Development Team meetings,** involving the professional development leads and university researchers who designed the lessons, took place prior to each workshop to support the workshop leaders through discussing the project lessons and key messages. Participants claimed these opportunities also **provided an important format for the research ideas to be explored in depth by academics and practitioners**.

6. A key issue for the national development team was determining the level of demand placed on project teachers and departments that reflected the commitment they had entered into but was also realistic in terms of workload. Some aspects of these judgements were interpreted differently between hubs and led to some variation in the expectation of what was to be carried out by teachers.

a. In one hub there was less emphasis on the carrying out of further lesson studies after the initial study. Feedback focused on general reflections and discussion of how the lesson went. The formal evaluation evidence was not sufficiently targeted to identify the impact on teachers PD but observations of the feedback sessions suggest the PD was less rich.

b. There was variation between participating schools with regard to the wider involvement of the maths department in sharing and discussing the project materials and participation in any lesson study. This was due to a decision not to pursue this aspect of the project based on workload concerns fed back by workshop leaders.

Impact on Pupils - *Teachers reported:*

- The practical contexts engaged pupils' interest.
- The use of the bar model was particularly popular with pupils.
- Pupils began to use diagrams or pictures to support their explanations more often and this led to improved understanding '*pupils began to realise that understanding was more than just getting it right*'.
- Pupils had begun to work more independently representing problems in a variety of ways '*once pupils had become confident at illustrating their thinking the lessons ran far more smoothly*'.
- Pupils displayed greater resilience when attempting problems.
- Pupils had greater confidence in communicating their thinking, comparing their own approaches with others and justifying and explaining their work.
- Pupils who were able to answer correctly found the requirement to go deeper challenging, but this was seen as productive: '*as lessons in the project progressed, some pupils began to see this as the thing to do*'.
- **Teachers felt that pupils were demonstrating improvement in their understanding, reasoning and problem solving, but longer term exposure would be needed to impact more fully on pupils' performance in exams and that this was unlikely to happen in the curriculum time of this project.**

Section 5b - Subject knowledge findings:

Given the scope and timeline for this particular pilot, the evaluation processes of the project focussed mainly on teachers pedagogical gains. However observations were made on the engagement and impact of those tasks where the principal purpose was to deepen subject knowledge of the mathematics itself. These subject knowledge activities involved teachers working together doing mathematics either from the lessons or from activities designed for the teachers, to give deeper insight into the mathematics related to the lesson.
The outcomes of these observations are summarised below; the subsequent recommendations include a recommendation to incorporate impact measures for teachers subject knowledge more directly.

Subject knowledge outcomes

These mainly reflect workshop observations but also feedback from evaluations relating to the subject knowledge activities.

- In discussion many teachers on the course viewed the professional development in terms of improving teaching knowledge and practice rather than acquisition of knowledge of the subject itself which can be a sensitive issue. Often subject knowledge acquisition is seen mainly as a requirement for non-specialist teachers and there are specific courses which experienced or specialist teachers would not go on. However observation of project workshops indicated that even very experienced specialists and mathematics graduates did gain from considering the mathematics itself.
- In particular the specialist and experienced mathematics teachers were able to probe the mathematics in the subject knowledge activities more deeply with the resulting discussion furthering their own insight as well as that of the non-specialist teachers who they were happy to work with.
- Important to this was the recruitment of a suitable balance of specialist, non-specialist and experienced teachers. Discussion with teachers suggested they and their departments were attracted to participate by the project focus on *developing practice based on research backed pedagogy* (in this key area of mathematics and the new curriculum). The specialist and more experienced teachers suggested they would not be attracted to courses where subject knowledge was seen as the main focus rather than a component. However their presence greatly contributed to the effectiveness for all the teachers of both the pedagogical and subject knowledge elements of the project. This suggests the significance of integrating these two elements of professional development as a factor in attracting wider engagement from teachers and departments as a whole, rather than courses focussing on individual subject knowledge development.
- A key feature of the course design which supported the **professional development of pedagogy and subject knowledge in parallel** was the integration of the subject knowledge specific activities into the sessions exploring the lessons. This allowed teachers to see the relevance of the specific activities which were designed to deepen subject knowledge of the actual mathematics related to the lesson. The resulting discussions often highlighted important pedagogical issues supporting the discussion of the teaching.
- Workshop observations suggested participating teachers had a range of mathematical knowledge including views on what subject knowledge is and what was relevant to the teaching of mathematics. This varied from those who considered themselves confident mathematicians to those (*including some that would consider themselves specialists*) who admitted to being confident in procedural knowledge and application rather than deeper conceptual understanding (*e.g. some teachers were able to apply a rule such as dividing two fractions but were unsure of the reasoning behind the rule*). A few teachers questioned the need for such understanding for pupils or even themselves (as it was not necessary for exam mathematics therefore an unnecessary aspect of the course, *e.g. questioning the need to know why the rule for dividing fractions works as long as you can apply it*). It wasn't clear to what extent these views may have changed by the end of the course for those particular teachers.

Areas observed to be particularly relevant to the subject knowledge needs of the project teachers

- The range of topics in the curriculum underpinned by the same mathematics.
- The different interpretations and associated models for multiplication and division (*this appeared to be an issue with all including graduate mathematicians*). In particular:

  - Division as sharing or grouping and multiplication as repeated addition or scale factor enlargement
  - The technical language associated with the meaning and interpretation of multiplication and division (*partitive and quotative, divisor, quotient, multiplicand etc.*)
  - How the different interpretations, particularly of division, relate to different solution strategies e.g. unitary or multiplier
  - The more formal properties (*e.g. distributive law*) of multiplication and division allowing different solution strategies to be linked and their related calculation expressions shown to be equivalent (*including combining intermediate steps into a single step*)
- Consideration of fractions as operators
- Interpretations of the reciprocal and its role in fraction arithmetic
- Greater understanding of reasoning behind particular arithmetic procedures involving fractions *e.g. dividing fractions.*
- A greater understanding of key multiplicative relationships and their manipulation in the solving of proportional problems, including finding multipliers, their inverses in different forms (*fraction, decimal and percentage change*) and their effect as operators.
- A deeper understanding of the properties of proportional relationships and their difference from other relationships such as inverse or inverse square and particularly linear relationships that are not directly proportional. This includes exploring how their properties appear in different representations including graphical and double number lines.
- How considering a problem from different mathematical perspectives or representations can be a powerful tool for improving teacher subject knowledge but also a potential pedagogic approach.

Section 5c - Recommended actions for NCETM arising from the project

A. Developing the project for wider use

The project was enthusiastically engaged with by the core teachers and their departments and the evidence suggests that this was of benefit to participating teacher's professional development. The SHU report also judged the project to be a cost effective form of professional development. Evidence of its suitability for wider uptake include the number of schools and Maths Hubs who are already doing this following either their participation in the project or hearing about it from schools who were involved.

Since the completion of the project:

- two project schools have used the materials and their experience of participation in the project as a basis of providing INSET to other local schools including primary schools.
- four Maths Hubs are currently running or planning to run the project as a Work Group

NCETM are preparing and refining the materials and workshops through the production of a user guide to support Maths hubs in offering the project as a work group.

B. TIME team model and building Maths Hub CPD capacity

1. The TIME team model has demonstrated capacity to bring together effective partnerships from the mathematics community to improve mathematics education. In this project the model has successfully brought together university teacher CPD expertise, research into how pupils learn mathematics and teachers' classroom practice to focus on improving teaching. This model of working has strong potential for application to other areas of the curriculum, in particular for use as a **Maths Hub Work Group** activity. Key messages learnt from the project with regard to TIME teams and workshop design should be shared and wider uptake and development of the model should be encouraged.
2. Ways to apply the model to other **key areas of the curriculum** (*geometric, algebraic reasoning, etc.*) through pilot projects with particular consideration to the use of a TIME team model should be sought.
3. The TIME team requires involvement of members with specialist PD expertise. The development of this expertise should be a priority. The TIME team itself provides a context in which this expertise can be developed, particularly where an experienced PD lead can *mentor* a less experienced lead. Consideration should be given to how this could be facilitated as part of a strategy to **build the PD capacity of Maths Hubs**. (*Such partnership may contribute to an accreditation process for a less experienced PD lead*).
4. The CPD expertise developed by the PD leads involved in delivering the MR project through the TIME team model provides a **powerful resource to support the scaling of this CPD expertise** in the manner described above. Actions to involve this group in such work should be explored.
5. A key factor in the quality of the PD was the lesson feedback sessions, in particular **the lesson study feedback** focusing on the common lesson and research question decided at the previous workshop. Further work should be carried out into the nature of these **emerging collective lesson study feedbacks** and how to maximise their PD value in the workshop context.
6. The TIME team model should be promoted with **education research funding organisations** in order for them to consider possible implications for increasing the impact and reach of their funded work.
7. The opportunities for small groups of researchers and classroom practitioners to meet (*as in National Development Team meetings*) to discuss research designed lessons, **provided an important format for the research ideas to be explored in depth by academics and practitioners**. Opportunities for such meetings should be encouraged and facilitated through the Maths Hubs programme.

C. Departmental CPD and subject leadership.

1. Evidence suggests the CPD activities (*in particular - lesson study, assessment interviews, anticipated issues sheets, etc.*) undertaken by project teachers in their schools played a key role in the effectiveness of the professional development. Actions should be taken to further explore and refine these aspects as a **possible model for department developmental work**, which may exist independently of participation in a TIME team.
2. The sharing and embedding of effective practice gained through participation in such projects are at the heart of good **subject leadership**. In this respect, the outcomes of the project offer possible follow up models to improve and build on subject leadership in mathematics and should be further explored by NCETM.

D. Multiplicative reasoning

1. The power of multiplicative reasoning approaches to link together areas of the curriculum (*within mathematics and in other subject areas*), and its potential to **transform teaching and learning** should be widely promoted as an example of how a focus on understanding underlying mathematical relationships can improve both conceptual understanding and procedural fluency.
2. The KS3 Multiplicative Reasoning project should be promoted nationally, particularly to Maths Hubs. **Further development should be undertaken** to address more applications of multiplicative reasoning within the KS3 curriculum and across all key stages **- especially in the primary phase and at the transition between phases**.
3. The approaches that underpin the KS3 Multiplicative reasoning project directly support current wider priorities of mathematics departments nationally and should be promoted as such.
   In particular:
   a. Implementation of the three core principles of the **new curriculum** (fluency, reasoning and problem solving).
   b. Approaches to developing depth rather than 'coverage' to **challenge all pupils**.
   c. Collaborative CPD and lesson design, focused on the learning responses of pupils.
   d. The use of **visual representations** and **accessible concrete and practical contexts** to deepen learning and problem solving.

E. Recommendations relating to the subject knowledge component

1. Consider courses designed to develop pedagogy and subject knowledge in parallel.
2. Give careful consideration to attracting a mix of specialist and non-specialist teachers to work together at workshops (ideally a specialist and non-specialist from the same department).
3. Design subject knowledge activities focussed on teachers working together doing mathematics. In particular deepening knowledge of the mathematics related to the teaching focus or lesson materials and allowing related pedagogical issues to be discussed.
4. Incorporate relevant (*teacher focussed*) subject knowledge questions into the lesson commentary.
5. Design collaborative gap tasks to involve the whole department (*these could be available online and completed online for course leaders to view and feedback on*).
6. Give references for further study related to the subject knowledge.
7. Consider giving a subject knowledge overview at the start of the project in order to support coherence and opportunities for teachers to reflect on their progress.
8. Consider an appropriate way of measuring the impact of the subject knowledge activities on teachers as a whole and individually. This might include before and after surveys/assessments. Such surveys should also include opportunities to determine changes in beliefs, attitudes and confidence as well as taking experience and specialism into account.

F. General suggestions

1. **Lesson study** should be promoted as a universal PD tool to support the work of the Maths Hubs. Attempts should be made to support this through the establishment of partnerships and links with centres of lesson study expertise.
2. The use of **before and after assessment interviews,** as described in the project, could be explored for its potential to assess pupils' **progress in conceptual understanding** and related CPD for teachers.
3. The **commentary** accompanying the project lessons was significant for teachers' PD in supporting their subject knowledge and pedagogy. This approach should be encouraged in the design of materials for teachers and **textbooks**. Such commentaries should reflect key PD points e.g. highlighting the learning significance of particular tasks or stages of the lesson, the importance of particular questions for drawing out misconceptions and anticipation of pupil responses with suggestions of how to move learning on.
4. Further support for approaches to deepening understanding, rather than accelerating coverage, is required.

G. Development of an NCETM multiplicative reasoning national support structure

In order to achieve the above recommendations the NCETM should consider the establishment of support structures to coordinate developments within the Maths Hubs and in the wider mathematics community relating to multiplicative reasoning and the development and uptake of the TIME team model.

This might take the form of a dedicated ***microsite*** within the **NCETM website** with both open and limited access sections. In addition to facilitating work groups relating to MR, it could ensure:

- appropriate access to NCETM project materials linked to a **quality assurance process**
- links to key relevant **operational partners**
- **training** opportunities for PD leads
- **production of a user guide incorporating all the elements needed to run the project as a hub work group**
- evaluation support and **sharing of emerging practice, resources* and new developments**
- a **wider debate on the teaching and learning of multiplicative reasoning across the mathematics community (nationally and internationally)**

Such a structure may become a blueprint for supporting other common areas of curriculum or teaching and learning collaboration.

****a number of resources developing the ideas and approaches of the project, including ICT, were suggested or created by participants. Such sharing could stimulate engagement and further creativity.***

Section 6: Lessons learnt from the project

As the project progressed, feedback from workshops was reflected on at National Development Team meetings and adjustments were made to approaches for subsequent workshops. Following the completion of the project careful consideration has been given to aspects of the project that did not go as planned or feedback that indicated areas for improvement. These are detailed below along with suggested improvements. Many of these will be incorporated into the project user guide, which will support work group leads in running the project as a hub work group.

Initial workshop and subsequent sequencing of workshops

A work group aimed at replicating the project at secondary level should use the full six TIME team days, although the format may change slightly. The project started with day 1 and day 2 as consecutive, in order to support the introduction and setting up the project. In one of the post project workgroups currently being run, day 2 and day 3 are consecutive, and in another they are starting with a twilight session. This follows project workshop evaluations indicating that certain parts of the initial (two day workshop) spent too much time on describing the project.

Suggested improvement:

- A more focused and concise introduction to the project in the initial workshop.
- This could be supported by the production of a project guide available to participating teachers and schools prior to starting the workshops.
- Such a guide might include a more detailed description of Multiplicative Reasoning and the project together with some useful surveys on teacher beliefs and values to be completed prior to attending the first workshop.
- A more effective model might be to consider the initial workshop as one day but with day 2 and 3 as consecutive to facilitate the lesson study suggestions below. Details and options on workshop sequencing will be given in the user guide.

Workshop sessions on introducing the new lessons and materials:

It was difficult to give adequate time to all the lessons in the first two units during the workshop sessions. Teachers commented that they found it more productive to spend time in workshops focussing in detail on one or two lessons rather than them all. Where this took place teachers reported that those particular lessons were the ones that they felt were more successful when taught in school.

Suggested improvement:

- Workshops to make available all the lessons produced for each unit but identify the three lead lessons to focus on in the workshop and expect them to be taught back in school.
- More time to be spent on the lesson identified in advance for the lesson study in order to prepare to carry out the lesson study back in school.

Lesson developer input:

The lessons were designed by a team who have particular experience and expertise in research into the learning of these areas of mathematics. Where a lesson developer visited a workshop they were able to give an input on the research background and purpose of their lessons. Although the lessons themselves included a section detailing this, the additional input of the lesson developer was greatly valued by teachers.
However this would not be a scalable aspect in a wider role out.

Suggested improvement:

- Consider commissioning developers to do small video clips regarding the lessons that could be used at workshops and in preparing the workshop leaders.

The lesson study

Following the carrying out of the lesson study in school, the subsequent workshop session focussing on feedback on the lesson proved to be an important source of PD. This justified the inclusion of a lesson study expert to guide these sessions. Their role in agreeing and designing the lesson study element of the work group will be important when setting up the work group leadership and design. They will play a key role in the decisions below.

Suggested improvement:

- Where possible arrange an actual lesson study to take place during an early workshop so that the lesson study process can be introduced more effectively. This might involve project teachers observing a lesson (*a chosen project lesson introduced previously –see suggestions on* ***workshop sequencing***).
  - If this was not feasible then consider the use of a suitable videoed lesson.
- Continue the practice of agreeing a single common lesson and research question to be carried out in school prior to the next workshop to facilitate communal feedback.
- The lesson study commitment is clear in details given to schools prior to participation.
- The role of the subject leader and department in participating in the lesson study should be made clear.

Professional Development materials

The lessons were detailed and substantial and as such became referred to as professional development booklets. When this was highlighted teachers accepted the size and detail of the lessons and found them useful.

Suggested improvement:

- The documents containing the lessons should be titled and referred to as professional development documents based around a lesson in order to clarify their purpose and use. The lesson should be easily accessed on its own but the accompanying PD commentary should also be clear. Adjustments have been made in their presentation to reflect this.
- They need to be emphasised as forming a coherent professional development package rather than a coherent or complete scheme of work for pupils.

Evaluation

1. Evidence gathering templates and approaches were amended and refined as the project progressed. The completion of these at various stages of the project and on different aspects was integrated into the project design and became an important part of the professional development itself. However in some cases teachers felt there seemed to be too much form filling.

   Suggested improvement:

   - Consider bringing together the evaluation templates into a single booklet that teachers have in advance of the project. This might detail aims and how the evaluation will take place and contribute to the professional development. Completion of relevant aspects of the booklet would be built into the workshop sessions and school activities and be retained at the end of the project by the teacher as a useful record of their professional development.

2. An important aspect of the professional development is the impact on teachers' attitudes and beliefs with respect to the mathematics and its teaching and learning. The project lacked a way of explicitly measuring this.

   Suggested improvement:

   - Commission a survey to use as a before and after measure and allow teachers to reflect on these changes and the implications as part of their PD.

Whole department participation

As highlighted above, early decisions on workload focussed attention away from this aspect of the project. However the impact from greater participation could potentially be of benefit in improving the teaching and learning of the whole department. Subsequent revision to the project following feedback (in particular the number of lessons focussed on) suggests that the workload issue can be addressed.

Suggested improvement:

- Develop this aspect of the project and incorporate whole department involvement into the user guide.

Section 7: Wider response to the project

A. Project Teachers:

- The evaluation of all the evidence sources indicates an overwhelmingly positive reaction to the benefits of participation in the project.
- Many teachers are keen to continue to be involved with developments
  - Some plan to lead developments in their department's work and are discussing approaches with other teachers and the department as a whole
  - a number of teachers are particularly motivated and keen to lead work based on the project in their locality – *partner schools and primary schools.* Many have put forward plans to run similar development work, either in their own school or as part of a local Maths Hub.
- The project has developed a group of **highly positive and active teachers** (*implementers and influencers of others*) who can be powerful agents for change
  - **Non specialist teachers** involved in the project have **responded very positively** to the effect of the project on their subject knowledge, pedagogy and classroom practice.
- In some cases the project teachers brought along a trainee teacher, who also responded very positively.

B. Subject leaders:

Completion of subject leader section of the teacher evaluation booklet or personal interview indicated:

- Subject leaders were very positive about their participation in the project, believing their teachers have benefitted and that this is benefitting pupils.
- Those interviewed stated that participation in the project had not caused any undue management or logistical problems.
- Many have substantial plans to further embed the work of the project next year into department practice or schemes of work.
- Some have indicated they are looking in particular at integrating Lesson Study and, in some cases the assessment interviews, as part of regular department CPD.
- Some want to work collaboratively with other schools on further development of lessons and schemes of work in this area.
- Some want to lead work in other schools using the project materials, particularly with their primary feeder schools.

C. Head teachers

- Those interviewed were very positive. They saw this as having the potential for galvanising improvement in the mathematics department.
- Others were excited about the opportunity to show leadership in running related courses with local schools, in some cases as part of Maths Hub work.
- **A key factor in this positive reaction was the enthusiasm shown by their subject leaders, which was generated by the enthusiasm of the teachers involved in the project.**

D. Project developers and deliverers

- Those involved in developing and delivering the project have been brought together from university mathematics education research, professional development and school improvement practitioners and classroom mathematics teachers. Over the period of the project there has been increasing enthusiasm for the potential of the relationships being formed with schools.
- Involvement in the project has provided a strong developmental experience in subject knowledge and in refining, managing and delivering this model of CPD.
- Many of the professional development leads plan to repeat the programme, organised through a local Maths Hub. Local university expertise and participation is being continued.

- This has also resulted in a number of separate but related projects being initiated by universities, demonstrating the power of the project to stimulate further collaborative work between schools and universities.

E. Reaction by the wider mathematics community

- Presentations of the project, its approaches, content and PD methodology have been enthusiastically received in many different maths education settings – education shows, conferences (BCME, MEI, SSAT, NEC, CPD provider networks, etc.)
- There is a real appetite for the transformative potential for this area of mathematics and the PD delivery model.

Section 8: Detailed feedback relating to project dimensions

This section relates in more detail views expressed by teachers in the evaluation booklets completed at the end of the project. It also draws on an analysis of the data generated in the booklet when they made judgements against each of the project dimensions which are given as headings below. In each case they used the following scale to judge the impact on their thinking and practice: *1 = Very significant; 2 = significant; 3 = some; 4 = no change – if not give reasons*. Teachers were asked to make accompanying supporting statements with examples and evidence to back up their judgements. Time was given in the final workshop for teachers to reflect on these questions prior to completion after the workshop. A copy of the evaluation booklet is included as an appendix together with an anonymised spreadsheet collating and analysing all the responses.

Using representations to make sense of problems

Using representations to make sense of problems		
Impact on thinking and practice	**Thinking**	**Practice**
Significant or very significant	**90%**	**86%**

The project focussed on the use of the bar, double number line and ratio table to model problems

Teachers reported:

- The use of visual models greatly supported pupils to represent problems, construct possible solutions and make sense of the mathematics involved (in particular the nature and roles of fractions, ratios and percentages).
- Teacher's comments reflected these points
  - **The bar model** was particularly successful in allowing pupils to understand and develop solutions for themselves and had widespread applications across many topics. It also supported greatly pupils' understanding of fractions and ratio. It was felt this was due to the *bar being a simple starting point for pupils to think about these sorts of problems and build confidence.*
  - **The double number line** particularly supported ideas of conversion, across and between calculations and in comparing properties of proportional and non-proportional situations.
  - **The ratio table** allowed pupils to model many different approaches and consider how the solutions are connected. Many teachers indicated that they use the ratio table across curriculum topics and it has been particularly useful to support pupils at KS4.
- The use of contexts that were realistic and accessible to pupils (particularly practical situations) greatly improved their engagement.
- As models and mathematics were developed around a problem, it was felt important to link the mathematics back to the model and original context and vice versa (e.g. where would this part of the problem be in the model?)

  "*The need for a good model to allow pupils to approach problems is extremely important. This will allow pupils to see that many topics that are usually taught as distinct are actually connected and can be approached using the same model.*"
- An important strategy teachers used was to **'stay with the context longer'** to support some pupils to make progress. Included in this was constructing stories around the information in a problem in order to support pupils to make sense of the problem and the meaning of the information given. A key prompt was to 'draw a picture' when pupils felt that the quantity and complexity of the information presented was a barrier to engagement with the problem.
- Models need to be **carefully developed** over time with pupils in order to support understanding and for pupils to develop the ability to use them independently.
- Models such as the ratio table were more effective when there was a clear understanding and familiarity with more 'concrete' models (e.g. the bar model).
- A particular danger in the way a model may be used in teaching is that pupils just see this **as another procedure, rather than as a way to develop their understanding.**

- Teachers noted that the year 7 pupils were often much more receptive to these approaches than older pupils who appeared less flexible in their approaches. Teachers suggested that it is important for these approaches **to start at year 7 and progress through KS3 and 4.**
- Over time, pupils became better at choosing an appropriate model to represent a problem effectively.

Paying close attention to pupil responses

Impact on thinking and practice	Thinking	Practice
Significant or very significant	**90%**	**77%**

Many teachers reported that the approaches in the project focusing on pupils explaining and justifying their solutions had an important impact on learning and was changing their classroom practice.

Teachers reported:

- Many pupils struggled early in the project to begin to explain their answers or approaches, even when they had found the correct answer to the problem.
- The use of a visual model to represent the problem greatly supported pupils being able to explain and justify their approach and why they thought they were right.
- A key question was "*How do you know you are right*?"
- Paired and grouped work facilitated this approach.
- This approach revealed key insights into pupils' understanding and often revealed misconceptions.
- Teachers felt it was important not to step in and '*show them*' but allow the misconception to be drawn out and challenged through appropriate teacher and peer questioning. This they believed led to deeper and more secure understanding.
- However, many teachers felt they were not always confident they could pose the type of questions that would challenge and move a pupil's thinking on effectively, and that this was an important CPD area for teachers and departments.
- Many teachers felt the lesson study, the common issues sheet and other project assessment activities greatly supported their professional development in this respect.
- Many teachers also felt the '***Anticipated issues sheet***' *(see appendix)* provided a very useful way of recording any particular pupil responses or issues arising from the teaching of the lesson and how they may be responded to. Attached to the lesson plan, these issues can then inform subsequent teaching with further comments accumulated as the lesson is taught by different teachers across the year. **This was considered a simple but potentially rich resource for department CPD.**
- Teachers noted that as the project lessons progressed pupils became more confident in their ability to explain and justify solutions and the pupils themselves began to recognise this as an important mathematical activity.
- These approaches made the use of different activities for pupils at different levels of progress (i.e. differentiation by task) less of an issue as the focus was on deepening understanding rather than increased or accelerated coverage.

Making connections and deepening understanding

Impact on thinking and practice	Thinking	Practice
Significant or very significant	**96%**	**86%**

- Teachers felt the opportunity for pupils to use the same model for different problems and topics would support them to see and understand how these areas were connected by the underlying idea of proportionality.
- Many teachers felt these links provided opportunities to go deeper rather than move groups of pupils on to new topics at different rates.

- Some teachers found the teacher activities involving exploring multiple representations of the same problem– **exploring the same problem from different mathematical viewpoints** (*e.g. graphical, enlargement, algebraic etc.*) **gave insight into their own subject knowledge** and had potential for pupil use.
- Teachers mentioned the use of questions such as, "*How do you know you are right*?" *and* "*Where would this part of the solution be in these different models*?" were useful.
- Many teachers recognised how multiplicative reasoning linked across topics and curriculum strands, but had not previously made these links explicit in their teaching.
- Teachers felt this had greatly influenced their thinking on how these areas of mathematics should be approached, believing that continuity in approaches and development of ideas should be aligned across key stages including primary.
- Many stated they would be involved in working with department colleagues to share work from the project, including developing or adapting schemes of work. Some were keen to link with their primary feeder schools for some of this work.

Working with colleagues

Impact on thinking and practice	Thinking	Practice
Significant or very significant	**100%**	**95**

- Working with colleagues was considered by teachers to be the most important feature of the workshops.
  In particular:
  - working through one or two of the new lessons in detail and sharing each other's reflections built confidence and supported a more common vision of the related subject knowledge and pedagogy.
  - teachers reported the lessons studied in detail during the workshops were the lessons that they felt went best in school. These were often the lessons chosen for lesson study.

Section 9: Detailed feedback relating to lesson study and formative assessment activities

Lesson study in school

What is referred to as lesson study in the project involves core teachers from each school investigating a research question through the careful observation of pupils learning during the teaching of a selected lesson. A common research question and target lesson was taught back in school following a lesson study format involving at least the project teachers. One teacher taught the lesson while the other observed and noted responses against the research question, after discussion the lesson was adapted and retaught. Feedback and analysis on the research question took place in school but was also be done collectively at the subsequent workshop under the guidance of the **university lesson study expert** who played the role of the *'knowledgeable other*'. Hence what was discussed at the workshop related to a common lesson plan that had been taught to different classes.

The expectation was that teachers in their school pairs would carry out one *lesson study* per teaching unit and report back on this at the subsequent TIME workshop. Teachers were supported by an accompanying booklet which outlines possible stages to record and aids completion. The stages are: Planning, teaching, discussion, re-teaching, final discussion. ***A copy of the booklet is included in the appendix.***

Where lesson studies took place in school teachers reported:

Planning stage:

- The paired discussions helped teachers to better understand the purpose of the lesson, insight into the development of the mathematics, teaching approaches and where the key learning points might be; listening to others during the workshop also supported this and stimulated thinking about wider issues concerning pedagogy and subject knowledge.
- Teachers began to think more deeply about pupils' learning and what obstacles there might be.
- Teachers began to anticipate misconceptions and consider the responses that pupils might give and their own possible responses to these, in particular considering where these points might occur in the lesson.
- Teachers felt they became much better at anticipating misconceptions as the project progressed and they had more experience of the lesson study.
- Some teachers reported that the process made them see the lessons differently in terms of what the key learning would be. The feedback sessions at the workshops were important in this respect and enhanced their understanding of the lesson study process.

First teaching of the lesson:

- The planning and the specific focus helped the observing teacher target the relevant pupil responses more effectively.
- Many teachers were surprised by the extent of the pupils' misconceptions.
- Some teachers felt it was a "*revelation to look at how pupils were learning, rather than what they were learning*".
- Many reported that they enjoyed the process and felt this was very different from the 'pressure' of a more formal observation. It felt more of a joint endeavour (though not team teaching) and was seen as helpful rather than judgemental.

Post teaching discussion:

- The observer notes helped to identify the nature of pupils' responses and, through discussion, helped to unpick the pupils' thought processes.
- The key point of the discussion was to consider how aspects of the lesson might be changed in the light of the responses observed. This might include adjusting tasks or questions and/or how to expose a misconception more deliberately and challenge responses.

- The discussions helped to identify and draw out the key learning moments in the lesson and their significance. This often helped to develop teachers' own understanding of how pupils learnt topics addressed in the lesson.
- The discussion enabled key moments that hindered or accelerated progress to be recognised.
- A key part of the discussion was on how some of the misconceptions could be dealt with or how thinking might be developed. In particular, what further questions might be posed.
- The discussion helped teachers identify what was effective in the lesson.
- Different views emerged, which stimulated deeper discussion.
- "*Things were picked up on that I hadn't noticed*".

Teachers noted:

- Sometimes unexpected learning issues might arise, also students grasped concepts quicker or slower than expected.
- Some things took longer than expected from the planning, important to judge how to deal with this so pupils did not become 'bored'.
- Once pupils had become confident at illustrating their thinking the lessons ran far more smoothly.
- The pupils realised what the underlying skills needed for this learning were.
- It would be good to see the follow up lesson given the learning issues that arose and the discussion on how we might deal with them. Often this was not possible through a further observation, though individual teachers felt they were much more aware of the significance of the pupils' responses.

Second teaching of lesson:

Often teachers mentioned time restraints on this being able to take place

Where it did take place teachers reported:

- The observation was easier due to the familiarity with the lesson and awareness of issues and what to focus on.
- This was particularly powerful to see if changes or adaptations to the lesson impacted on pupils' learning and on addressing issues arising from the initial teaching.

  "*The first lesson identified key learning issues and the second lesson enabled us to judge how effective our adaptations were*."
- Seeing the lesson a second time had a big impact giving far greater insight into the mathematics, pedagogy and learning issues for pupils.
- Sometimes subtle changes had big impacts, for example asking pupils to explain their method, or to suggest using a different representation to solve the problem, or "*how would you prove it to somebody else?*"

Impact on thinking and practice

As a result of the lesson studies teachers reported:

- that the process was "Incredibly useful"
- that they began to think about lessons differently, focusing more on the learning rather than covering the content
- that they anticipated misconceptions and where the key learning moments might be and how to respond.
- *"I was more aware of responding to learning than rushing on".*
- *"I give more thinking time"*
- *"I am talking less and asking 'why?' more".*
- 'I have changed the way I encourage pupils to respond to each other rather than just the teacher'

Unit 0 - Assessment activities *(see appendix)*

Impact on thinking and practice	Thinking	Practice
Significant or very significant	83%	65%

1. **The Unit 0 questions and related tasks played a significant role at the start and end of the project in establishing the focus on conceptual understanding and reasoning for both pupils and teachers.**
   a. **Unit 0 class task** The use of the questions as a class task to focus pupils on presenting a justification for their method or answer, rather than just getting a correct answer, had several advantages:
      I. It revealed a wide range of misconceptions, many of which were unexpected by teachers.
      II. It challenged pupils who had the correct answer, many of whom were unable to offer a convincing justification or any reasoning at all.
      III. Many pupils were unable to make sense of the problem outside of applying a procedure.
      IV. It revealed a lack of understanding about many common mathematical ideas (e.g. fractions, ratio, etc.) and how these related to the problem.
      V. It showed where pupils had little vocabulary to support their explanations and reasoning.
      VI. The approach helped to give pupils and teachers an idea about conceptual understanding as opposed to procedural application alone, and how deep conceptual understanding might be demonstrated.
   b. **Return to Unit 0 task**
      I. Where this took place teachers described pupils being more confident in articulating their reasoning and using a greater variety of images to support their justifications.
      II. While only a few teachers were able, in the time available, to carry out re-tests of Unit 0 questions, they thought this would be a valuable measure for teachers and pupils in relation to progress in understanding.
   c. **Pupil interviews** allowed teachers to think more deeply about the teaching and learning issues connected with this area of mathematics, in particular:
      I. Teachers felt they improved their ability to plan and respond to pupils through the use of 'probing questions' to assess and develop understanding.
      II. Teachers learnt how not to lead pupils in the direction of their thinking or answers but allow them to wrestle and make their own meaning and sense.
      III. Discussion of pupils' work from this point of view was developmental for teachers.
   d. **Video interviews** Videos are able to provide a department CPD and teaching and learning resource.
      I. Teachers found reviewing a video interview often allowed them to notice or pick up on issues they had missed in the actual interview and continued to further develop pedagogical knowledge particularly in relation to conceptual understanding and the use of probing questions.
      (Not all project teachers were able to arrange time for this to take place on more than one occasion; nevertheless they recognised its usefulness.)
      II. Use of video provides a useful resource for the department as part of an ongoing assessment and CPD programme.
      III. The video has potential as a classroom resource to exemplify and generate discussion

2. **Overall feedback on the use of assessment questions and pupil interviews**
      I. **There was a strong feeling that the use of pupil interviews on key assessment questions provided not only assessment of key pupils' understanding but also professional development for teachers in recognising conceptual understanding in pupils discussion; and these should be embedded as part of secondary school maths department practice.**
      Views expressed by teachers included:
      II. All new units or topics should be started with an assessment pre task to inform teaching, and returned to at the end of the unit to demonstrate progress. This would allow pupils to show what they know without being prompted by the teaching of the topic.
      III. Unit 0 style tasks could form the basis of periodic department assessment.

IV. Selective interviews of targeted groups on priority parts of the curriculum can provide assessment while also being a significant CPD resource and as such should be embedded as part of department practice.
V. It was recognised that this was time consuming and that it therefore needs to be carefully planned and selectively used.
VI. Vocabulary related to justification and reasoning should be a part of teaching across the curriculum and at all Key Stages to support pupils to represent and solve problems.

Section 10: Detailed feedback relating to other project aspects

Lesson format

The lessons were designed in a format under a standard set of headings to enable teachers to plan and teach the lesson but also to support their professional development. The supporting commentary linked to each stage of the lesson by the designer was crucial to this. However this often made the lesson booklets substantial documents. Hence the lesson booklets are referred to as professional development documents rather than just lessons, reflecting their role in the project. ***An example of a lesson is included as an appendix.***

Lesson headings:

- ❑ **Lesson summary** - *the aim/purpose of the lesson*
- ❑ **Lesson preparation** - *resources and materials needed for the lesson*
- ❑ **The lesson** - *usually set out on a single page divided into sections*
- ❑ **Lesson commentary** - *supporting commentary by designers linked to each section of lesson detailing purpose, possible pupil and teacher response and professional development points*
- ❑ **Adapting the lesson** - *notes on how the work might be pitched to meet the needs of different groups of pupils*
- ❑ **Suggestions for lesson study focus** - *possible 'research questions' suggested by the developer that could be used as a focus in any 'lesson study'*
- ❑ **Research background to the lesson** - *research background that supported the design of the lesson.*
- ❑ **Resources and hand-outs** - *attached at the back of the lesson*

The lesson commentary

Many teachers felt the lesson commentary was very useful. It allowed them to think carefully about the purpose of different parts and give better insight into the lesson. In particular predicting unexpected pupil responses and identifying the key learning moments in the lesson. This made it particularly useful in supporting discussion as part of the planning for a lesson study.
Many teachers felt the commentary supported their deeper understanding of MR and its teaching. However some teachers felt it was too wordy and gave too much detail. Not all recognised its role in supporting the PD

The research background

- Many teachers felt knowing and understanding the main research themes that underpinned the design of the project was interesting and insightful.
- Many felt it gave coherence to the lessons in the project and "*…allowed greater pedagogical knowledge and insight to be developed*".
- Some teachers mentioned it affected their wider teaching beliefs and approaches thus affecting how they taught other topics.
- Not all teachers felt they had (or were able to prioritise) the time for reading the individual lesson research sections.
- Many felt the **attendance to a workshop of a designer** and to hear them talk about the research background was very influential on their thinking.

Adapting to the needs of different pupils

- Each lesson was produced for KS3 pupils but not for any particular year group or level of challenge. The focus on understanding and the scope and depth of the lesson allowed its potential use throughout KS3 and teachers were encouraged to adapt accordingly. The approach focusing on going deeper for those who made rapid progress meant this could be done.
- Teachers reported that pupils who could get the correct answer benefited from trying to explain why they were right or demonstrating this through different representations and seeing connections. Many such pupils found this challenging but productive.
- Many teachers were using the lessons or variations on the ideas in their KS4 teaching and reporting rapid progress.

Workshops

The workshops were highly rated by the project teachers

General workshop format:

- Feedback on previously taught lessons
- Structured feedback on lesson study lesson
- Introduction to new lessons
- Planning

Working with colleagues was considered by teachers to be the most important feature of the workshops

In particular:

Lesson feedback sessions

- The impact of the lesson study and feedback sessions varied between different Hubs.
- Where lesson feedback sessions focused on a brief descriptive structure they tended not to be as rich in terms of CPD as those that gave time to unpicking key issues that arose from the learning. Here teachers often reported that these discussions had helped them make sense of what they had observed or experienced, enabling links and changes in their thinking to be made.
- This needed to be skilfully led in order to prevent the discussion 'dragging'; to keep it focussed and productive.

Lesson study feedback sessions

- Not all Hubs embedded the lesson study as a requirement of the gap tasks. However, the best CPD discussions observed at subsequent workshops, and reflected in evaluations, took place where the lesson study was the main focus of the feedback.
- The success of these sessions depended on the dynamics and relationships in the group and the ability of the session leader to manage the discussion, pose key questions in response to what was being said and to develop the discussion. Keeping a focus on the research question and having an understanding of the principles of lesson study were important but in many cases discussion became more general. However, this was not necessarily to the detriment of the quality of the CPD. **More work here on maximising this process would be interesting and beneficial.**

Introduction to new lessons

- Where there was an opportunity for a lesson to be presented at a workshop by one of the developers, this often greatly enhanced the teachers' engagement with the ideas and research elements.