

Mastery Professional Development

Number, Addition and Subtraction



1.12 Subtraction as difference

Teacher guide | Year 2

Teaching point 1:

Difference compares the number of objects in one set with the number of objects in another set; or the difference between two measures.

Teaching point 2:

Difference is one of the structures of subtraction.

Teaching point 3:

Consecutive whole numbers have a difference of one; consecutive odd/even numbers have a difference of two.

Teaching point 4:

We can apply the structure of difference to compare data.

Overview of learning

In this segment children will:

- compare quantities using vocabulary of difference, for example, 'more', 'less', 'fewer', 'older', 'younger', etc.
- develop an understanding of difference as a numerical value used to compare two numbers
- relate difference to subtraction, using symbolic notation to represent difference stories as subtraction
- explore consecutive numbers and consecutive odd/even numbers by focusing on the difference between numbers.

This segment introduces a new structure for subtraction. It builds on segments *1.5 Additive structures: introduction to aggregation and partitioning* and *1.6 Additive structures: introduction to augmentation and reduction*, in which children were introduced to subtraction as partitioning and reduction. The 'difference' structure involves the comparison of two values – the difference is the 'gap' between the two values. Children will explore this relationship through a range of contexts, including measures and statistics. These contexts support children in using a wide variety of vocabulary for comparison, for example, 'more', 'less', 'fewer', 'older', 'younger', 'longer', 'shorter', 'heavier', 'lighter', etc.

The *structure* rather than the *solutions* should remain the focus throughout most of this teaching sequence, and the bar model and number line will be used to reveal and emphasise the structure; teachers should also plan using contexts for quantities within ten, to minimise focus on calculation.

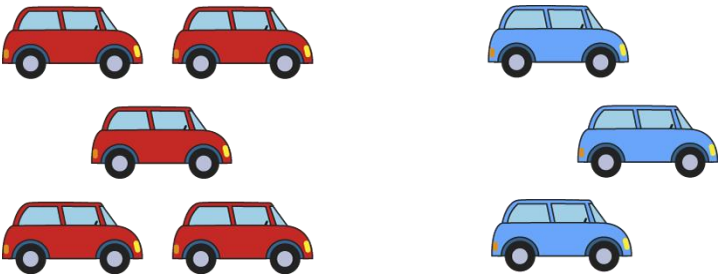
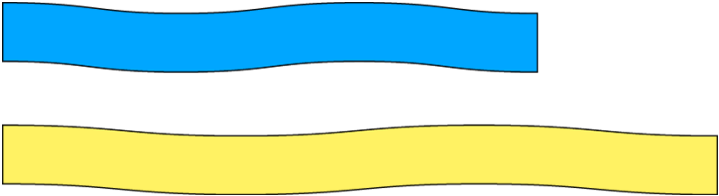
1.12 Subtraction as difference

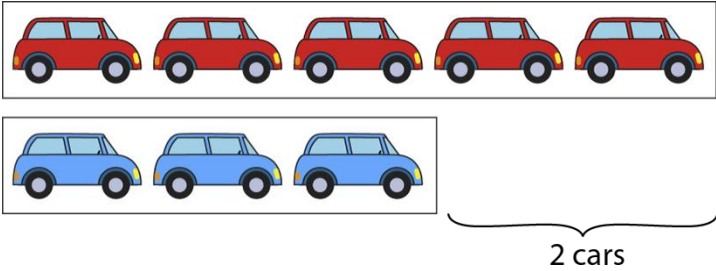
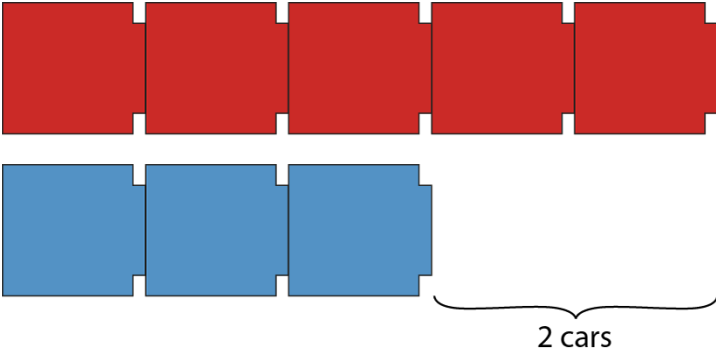
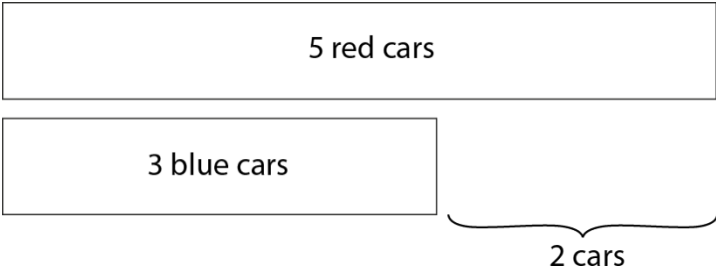
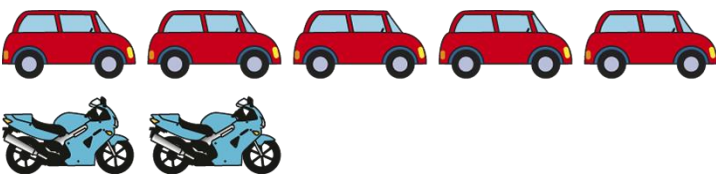
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

Difference compares the number of objects in one set with the number of objects in another set; or the difference in measure between two quantities.

Steps in learning

	Guidance	Representations
1:1	<p>Begin the teaching sequence by practising the language of comparison introduced in segment <i>1.1 Comparison of quantities and measures</i>. Present a range of practical contexts for children to compare, then build towards pictorial contexts. Ensure that children experience contexts involving both discrete objects which can be counted (for example, cars or sweets) and continuous measures (for example, length or age). Initially the comparison should be qualitative (for example, one ribbon is longer/shorter than the other) rather than quantitative (the yellow ribbon is one metre longer than the blue ribbon).</p> <p>Ask children to describe, in sentences, the relationship between the two values/measures. Children often more readily describe in terms of the larger value relative to the smaller value, so it is important that they practise describing each relationship in two ways, for example:</p> <ul style="list-style-type: none"> • There are five red cars and three blue cars. <ul style="list-style-type: none"> • <i>'There are more red cars than blue cars.'</i> • <i>'There are fewer blue cars than red cars.'</i> • The blue ribbon is three metres long and the yellow ribbon is four metres long. <ul style="list-style-type: none"> • <i>'The yellow ribbon is longer than the blue ribbon.'</i> 	<p>Discrete objects:</p>  <ul style="list-style-type: none"> • <i>'There are more red cars than blue cars.'</i> • <i>'There are fewer blue cars than red cars.'</i> <p>Continuous measures:</p>  <ul style="list-style-type: none"> • <i>'The yellow ribbon is longer than the blue ribbon.'</i> • <i>'The blue ribbon is shorter than the yellow ribbon.'</i>

	<ul style="list-style-type: none"> • <i>'The blue ribbon is shorter than the yellow ribbon.'</i> <p>Use a variety of examples to draw out a range of vocabulary, for example, 'bigger', 'smaller', 'older', 'younger', 'more', 'less', 'fewer', 'heavier', 'lighter'. Also remember to use 'less' for continuous variables and 'fewer' for discrete variables.</p>	
<p>1:2</p>	<p>Now move on from qualitative to quantitative comparisons, beginning to 'put a value' on the gap between the two quantities.</p> <p>Continue to encourage children to describe the contexts, in two ways, using full sentences, for example:</p> <ul style="list-style-type: none"> • <i>'There are two more red cars than blue cars.'</i> • <i>'There are two fewer blue cars than red cars.'</i> <p>Support this comparison by using bars to represent the two values. You can move towards this more abstract representation by first using concrete manipulatives such as multilink alongside the pictorial representation.</p>	  
<p>1:3</p>	<p>Once children can confidently compare values in a quantitative way, and in a range of contexts, link this comparison structure with the idea of difference.</p> <p>Introduce children to the use of the mathematical term 'difference' for describing the numerical gap between two values, for example:</p>	 <p><i>'The difference between the number of cars and the number of motorbikes is three.'</i></p>

- *'The difference between the number of cars and the number of motorbikes is three.'*
- *'The difference between the number of tractors and the number of cars is four.'*

It is important for children to see this relationship as the difference between the larger and the smaller amount, as well as the difference between the smaller and larger amount.

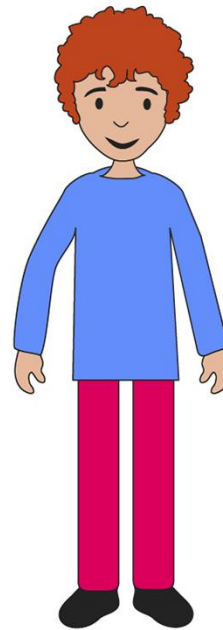
Note that the above sentence-forms no longer provide any indication as to which quantity is larger/smaller, more/less, etc. Use each 'difference' sentence in conjunction with the appropriate comparison sentence(s), for example:

- *'There are more cars than motorbikes; the difference between the number of cars and the number of motorbikes is three.'*
- *'There are fewer motorbikes than cars; the difference between the number of cars and the number of motorbikes is three.'*

Ensure that you also include 'continuous' contexts, for example, *'Ben is seven years older than Charlotte; Charlotte is seven years younger than Ben; the difference between their ages is seven years.'*



'The difference between the number of tractors and the number of cars is four.'



Ben is ten years old.



Charlotte is three years old.

'The difference between their ages is seven years.'

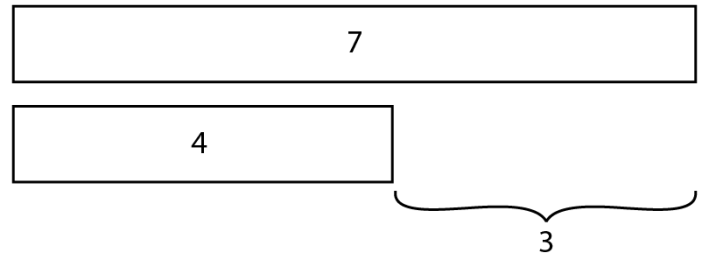
1:4

Once children are comfortable with the idea of mathematical difference in real-world contexts, move to the abstract – exploring difference simply as a way of describing the relationship between two numbers.

Again:

- focus on the 'gap' between numbers; the bar model and number line continue to be useful here

Bar model:

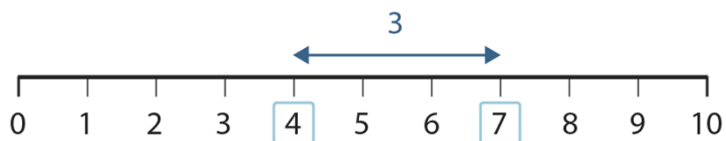


- ensure that the difference is described 'both ways' as before, for example:

- 'The difference between four and seven is three.'
- 'The difference between seven and four is three.'

As well as *presenting* difference using the bar model and number line, give children opportunities to show the difference between two numbers using these representations themselves.

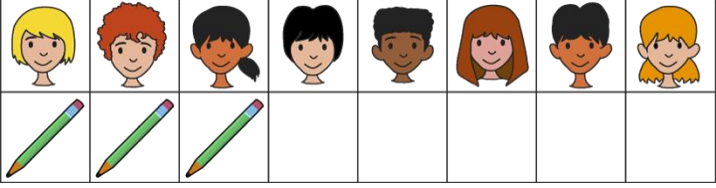
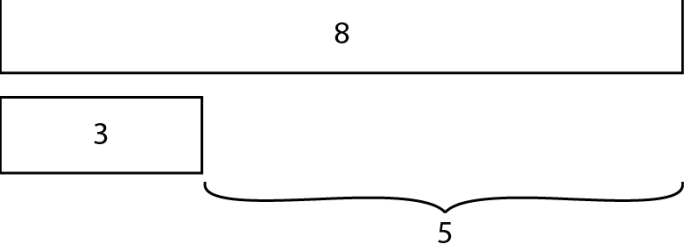
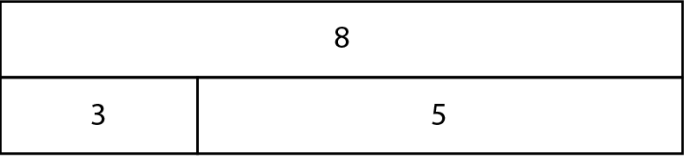
Number line:

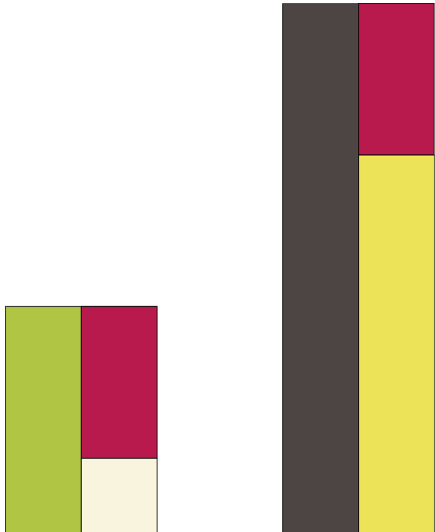
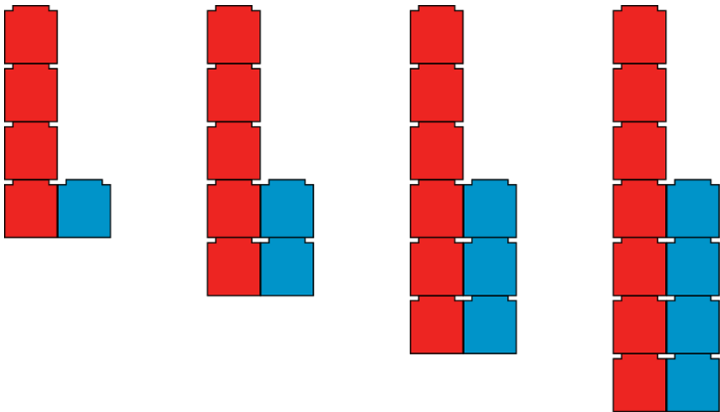


Teaching point 2:



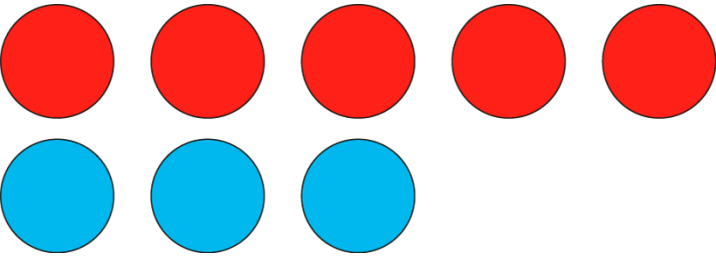
Difference is one of the structures of subtraction.

Steps in learning

	Guidance	Representations
2:1	<p>Now introduce children to difference as a form of subtraction. Use real-world contexts to show children that the difference between two numbers can be represented as subtraction. This is conceptually more complex than partitioning or reduction, since the difference is referring to an absence.</p> <p>Children will already have an understanding of subtraction as both partitioning (segment 1.5 <i>Additive structures: introduction to aggregation and partitioning</i>) and reduction or 'take away' (segment 1.6 <i>Additive structures: introduction to augmentation and reduction</i>). Build on the 'partial' bar model used in <i>Teaching point 1</i>, now using the usual form of this representation to link all three subtraction structures (partitioning, reduction and difference).</p> <p>Explicitly encourage children to describe what each number represents, in full sentences, for example:</p> <ul style="list-style-type: none"> • <i>'The 8 represents the number of children.'</i> • <i>'The 3 represents the number of pencils.'</i> • <i>'The 5 represents the difference; it is how many more pencils the teacher needs.'</i> <p>Link difference to the subtraction calculation, and encourage children to describe what the equation is showing, for example, the difference between eight and three can be written as $8 - 3 = 5$.</p>	<p>Real-world context: <i>'There are eight children and only three pencils. How many more pencils does the teacher need so each child has one pencil?'</i></p>   

	<p>Note that, until now, children have only explored situations involving the same kinds of object (for example, all cars, some of which are red and some of which are blue, or all vehicles, some of which are cars and some of which are buses). In the example provided here, we present a context with different kinds of object, involving an element of one-to-one correspondence. We are essentially partitioning the children in the problem into those who <i>do</i> and those who <i>do not</i> have pencils. Children will already be used to doing this in everyday life.</p>	
<p>2:2</p>	<p>Next, compare situations where the difference is the same. Begin with concrete manipulatives; for example, give children Cuisenaire® rods and ask them to find pairs of rods that have a length difference equal to that of the red rod. Then move onto explore other 'same difference' rods.</p> <p>At this stage, avoid giving a numerical value to the rods, and instead describe the differences in the 'unit' of a particular rod colour, for example, <i>'Yellow and green have a length difference of red.'</i></p>	
<p>2:3</p>	<p>Now extend the idea of 'same difference', using numerical values. Use multilink cubes for support:</p> <ul style="list-style-type: none"> • Start with four red cubes alongside one blue cube (the difference is three). • Add one cube of the corresponding colour to each set (five red cubes, two blue cubes). • Draw attention to the fact that the 'gap' has remained the same (the difference is three). 	

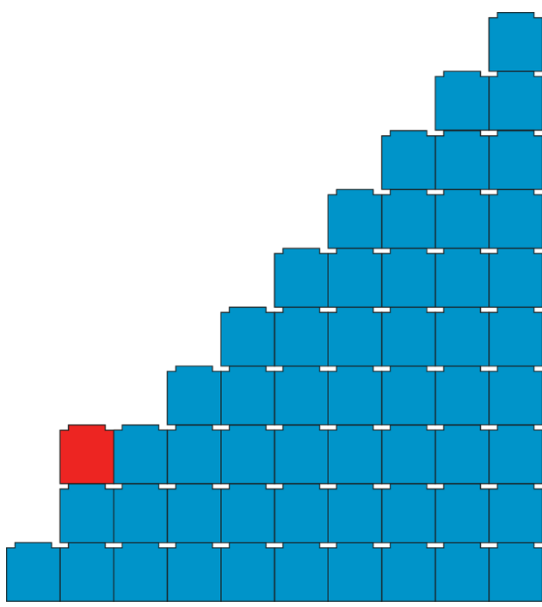
	<ul style="list-style-type: none"> Continue adding or subtracting an equal number of cubes from each set (the difference remains equal to three). <p>Introduce the symbolic notation alongside the concrete/pictorial representations, emphasising 'same differences', and ensuring variation in the format of the expressions:</p> $3 = 4 - 1$ $3 = 5 - 2$ $3 = 6 - 3$ <p>or</p> $3 = 4 - 1 = 5 - 2 = 6 - 3$ <p>or</p> $4 - 1 = 3$ $5 - 2 = 3$ $6 - 3 = 3$ <p>Encourage children to explore for themselves, by asking questions such as:</p> <ul style="list-style-type: none"> 'How many different pairs of numbers can you write that have a difference of three?' 'How would you write these using subtraction?' 	
2:4	<p>At this stage, draw children's attention to the fact that, for example, the difference between six and three is equivalent to the difference between three and six, but $6 - 3$ is not equal to $3 - 6$. Subtraction is not commutative.</p>	
2:5	<p>When children are secure in their understanding of difference as a representation of subtraction, explore subtraction stories which represent the range of structures that children have experienced so far, for example:</p> <ul style="list-style-type: none"> 'Five children are playing in the playground. Three are playing on the swing. The rest are playing on the 	

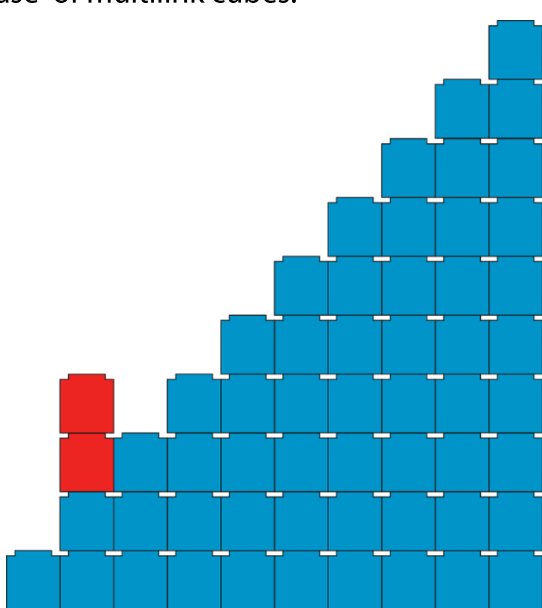
<p><i>roundabout. How many are playing on the round about?’ (partitioning)</i></p> <ul style="list-style-type: none"> • <i>‘There were five swans on a lake, then three flew away. How many swans are on the lake now?’ (reduction)</i> • <i>‘There are five ducks on the pond and three ducks on the lake. How many more ducks are on the pond?’ (difference)</i> <p>Note that in the partitioning and reduction examples we start with five and either remove three or split into three and two, but in the difference example both five and three are ‘there’ from the start.</p> <p>To promote depth, present children with some subtraction calculations and ask them to write their own difference stories to go with them.</p>	<p>Partitioning:</p>  <p>Reduction:</p>  <p>Difference:</p> 
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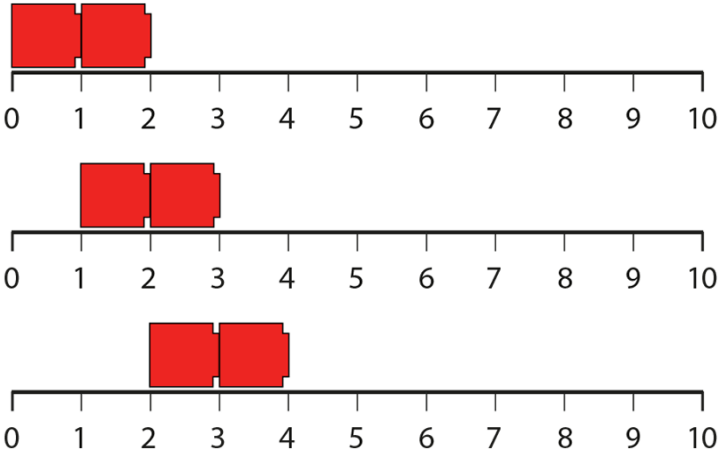
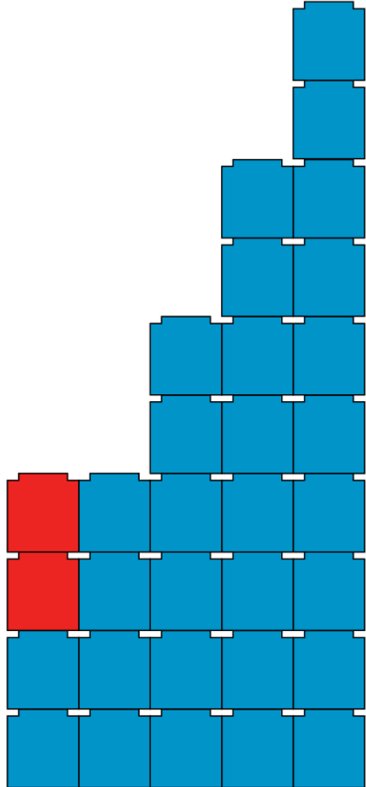
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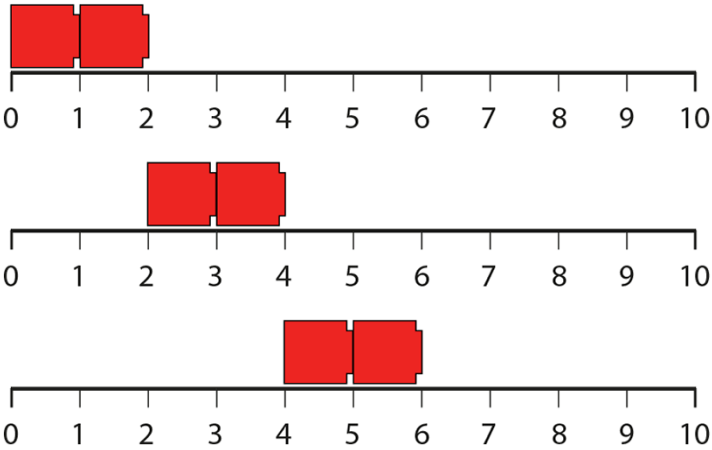
Consecutive whole numbers have a difference of one; consecutive odd/even numbers have a difference of two.

Steps in learning

	Guidance	Representations																																										
<p>3:1</p>	<p>Once children have mastered the idea of subtraction as difference, use 'difference of one' and 'difference of two' to support fluency in calculation – children need to become fluent in spotting these differences, using their knowledge of consecutive numbers and consecutive odd/even numbers.</p> <p>Begin by using children's understanding of difference to link consecutive numbers, using 'one more'/'one less' and 'difference of one'. You can use the following representations to support this:</p> <ul style="list-style-type: none"> • Use 'towers' of multilink from one to ten, plus an additional cube – children can 'walk' the extra cube up or down the 'staircase'; at each step emphasise the difference of one between the consecutive numbers. • Present children with a subtraction facts chart (up to ten) and ask them to find or colour in facts that show a difference of one (e.g., $4 - 3$). Ask 'What patterns do you notice?' <p>Work towards the generalised statement: 'Consecutive numbers always have a difference of one.'</p>	<p>'Staircase' of multilink cubes:</p>  <p>Subtraction facts – a difference of one 1:</p> <table border="1" data-bbox="758 1243 1484 1601"> <thead> <tr> <th></th> <th>0</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>...</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>$1 - 0$</td> <td>$1 - 1$</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <th>2</th> <td>$2 - 0$</td> <td>$2 - 1$</td> <td>$2 - 2$</td> <td></td> <td></td> <td></td> </tr> <tr> <th>3</th> <td>$3 - 0$</td> <td>$3 - 1$</td> <td>$3 - 2$</td> <td>$3 - 3$</td> <td></td> <td></td> </tr> <tr> <th>4</th> <td>$4 - 0$</td> <td>$4 - 1$</td> <td>$4 - 2$</td> <td>$4 - 3$</td> <td>$4 - 4$</td> <td></td> </tr> <tr> <th>⋮</th> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </tbody> </table>		0	1	2	3	4	...	1	$1 - 0$	$1 - 1$					2	$2 - 0$	$2 - 1$	$2 - 2$				3	$3 - 0$	$3 - 1$	$3 - 2$	$3 - 3$			4	$4 - 0$	$4 - 1$	$4 - 2$	$4 - 3$	$4 - 4$		⋮						
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<p>3:2</p>	<p>Present children with missing number calculations as shown opposite.</p>	<p>'Fill in the missing numbers.'</p> <p>$2 - 1 = \square$ $\square - 1 = 1$</p> <p>$3 - 2 = \square$ $\square - 2 = 1$</p> <p>$4 - 3 = \square$ $\square - 3 = 1$</p>																																										

		$5 - 4 = \square \quad \square - 4 = 1$ $6 - \square = 1 \quad 1 = \square - 5$ $7 - \square = 1 \quad 1 = \square - 6$ $8 - \square = 1 \quad 1 = \square - 7$ $9 - \square = 1 \quad 1 = \square - 8$ $10 - \square = 1 \quad 1 = \square - 9$ $17 = 18 - \square \quad \square = 18 - 1$ $17 = \square - 1$ $16 = 17 - \square \quad \square = 17 - 1$ $16 = \square - 1$ $15 = 16 - \square \quad \square = 16 - 1$ $15 = \square - 1$
<p>3:3</p>	<p>After exploring consecutive numbers, use the same strategies (step 3:1) to explore consecutive odd and consecutive even numbers. Here the children can 'walk' two joined multilink cubes up the same 'staircase' as before; at each step emphasise the difference of two between every second number, and record the subtraction calculations, for example:</p> <p>2 = 2 - 0 2 = 3 - 1 2 = 4 - 2 ...etc.</p>	<p>'Staircase' of multilink cubes:</p> 

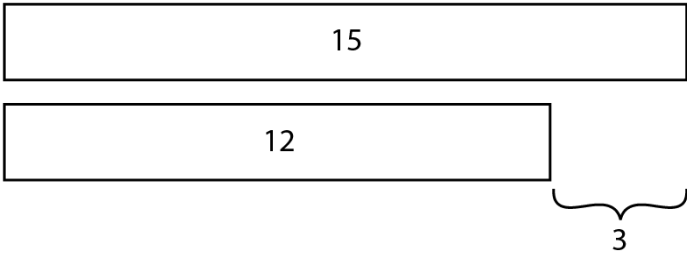





	<p>Examine the patterns of odd and even numbers, encouraging children to reason for themselves that the difference between consecutive odd numbers is two and the difference between consecutive even numbers is also two.</p>	<p>'Walking up' the number line:</p> 
<p>3:4</p>	<p>Now move on to walking the 'two' up and down an even, then an odd numbered staircase. Similarly, now walk the 'two' along the number line in steps of two, beginning at either the 0 or the 1 to explore even and odd numbers respectively.</p> <p>Again, record calculations, providing variation in the format of the expressions and emphasising the difference of 2:</p> <p>$5 = 7 - 2$ $3 = 5 - 2$ $1 = 3 - 2$... etc.</p> <p>Ensure that children are familiar with the following generalised statements by the end of this step:</p> <ul style="list-style-type: none"> • 'Consecutive odd numbers always have a difference of two.' • 'Consecutive even numbers always have a difference of two.' 	<p>Even 'staircase' of multilink cubes:</p>  <p>2 4 6 8 10</p>

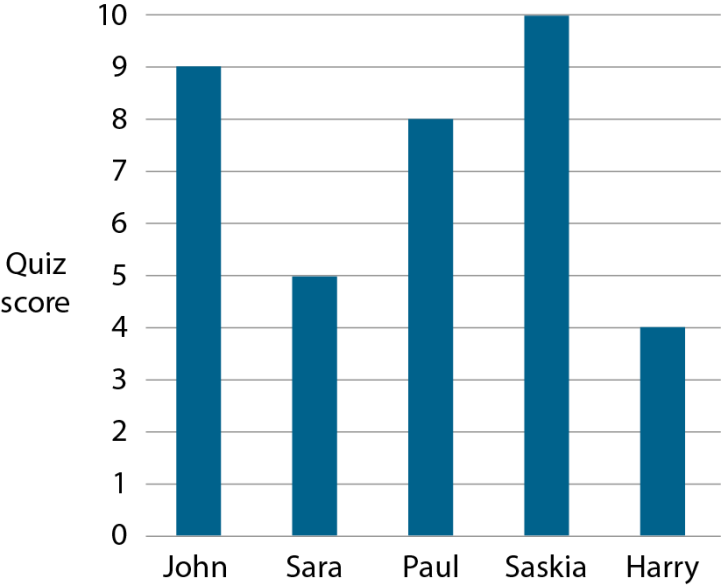





		<p>Even numbers – ‘walking up’ the number line in steps of two:</p> 								
<p>3:5</p>	<p>To promote depth of understanding, present children with questions such as those shown opposite.</p>	<p><i>‘Which expression is the odd one out?’</i></p> <p>6 – 4 13 – 11 9 – 6 2 – 0</p> <p><i>‘Which of these calculations represent a difference of two?’</i></p> <table style="width: 100%; border: none;"> <tr> <td style="width: 50%;">3 – 1</td> <td style="width: 50%;">4 – 2</td> </tr> <tr> <td>5 – 3</td> <td>6 – 3</td> </tr> <tr> <td>7 – 5</td> <td>8 – 6</td> </tr> <tr> <td>7 – 4</td> <td>10 – 6</td> </tr> </table>	3 – 1	4 – 2	5 – 3	6 – 3	7 – 5	8 – 6	7 – 4	10 – 6
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Teaching point 4:

We can apply the structure of difference to compare data.

Steps in learning

	Guidance	Representations
<p>4:1</p>	<p>Once children can confidently link the concept of difference with subtraction, move on to the application of this in a range of contexts, including graphical representations.</p> <p>When children encounter ‘How many more...?’ type questions such as the example shown here, they may initially ‘count on’ (for example, from 12 to 15), and record as addition (e.g. $12 + 3 = 15$). Ensure that they now also practise thinking in terms of difference, and record as subtraction (e.g. $15 - 12 = 3$). Note that it is just the <i>recording</i> of the calculation that is changing here and not the <i>strategy</i>; ‘counting on’ is a legitimate strategy for subtraction, at this stage, and children should be encouraged to use the strategy when it is an efficient choice.</p>	<p>‘John has twelve crayons and Sarah has fifteen crayons. How many more crayons does Sarah have than John?’</p>  <p>A bar chart with two horizontal bars. The top bar is labeled '15' and the bottom bar is labeled '12'. A curly bracket is drawn under the bottom bar, extending to the right edge of the top bar, with the number '3' written below it.</p>
<p>4:2</p>	<p>Working with statistics and graphical representations often involves comparison of values. Use pictograms to progress from purely pictorial to purely abstract graphical representations. With pictograms, children can easily see the difference in values, but encourage them to recognise that these differences can be recorded as subtraction.</p> <p>Make sure that you also present questions relating to the difference between a smaller and a larger amount, for example, ‘How many fewer sweets has Essie than Asif?’ This question is more challenging as Essie has six sweets and Asif has seven, but the calculation is written as $7 - 6$. The use of the term ‘difference’ to apply to</p>	<p>Asif </p> <p>Tom </p> <p>Sandra </p> <p>Essie </p> <p>Asif has two more  than Tom.</p> <p>This can also be recorded as the subtraction:</p> <p>$7 - 5 = 2$</p>

	<p>both 'larger compared to smaller' and 'smaller compared to larger' was introduced in step 1:3 above.</p> <p>To promote depth, give children a pictogram and ask them to design their own difference questions to go with it.</p>	
<p>4:3</p>	<p>Once children have mastered working with pictograms, introduce a more abstract representation, such as a bar chart.</p> <p>Ask children questions of the form:</p> <ul style="list-style-type: none"> • 'How many more...?' • 'How many fewer...?' • 'What is the difference between...?' <p>Encourage children to write their answers as subtraction calculations.</p>	 <p>Quiz score</p> <p>John Sara Paul Saskia Harry</p> <ul style="list-style-type: none"> • 'How many more did John score than Sara?' • 'How many fewer did Harry score than Saskia?' • 'What is the difference between Saskia's score and Paul's score?'
<p>4:4</p>	<p>To provide further challenge, provide pictograms in which the icons represent multiple objects. This highlights the importance of representing the problems as subtraction rather than just using the visual 'difference' in the number of objects.</p>	<p>'How many more socks does Tom have than Sandra?'</p> <p> represents 2 socks</p> <p>Asif </p> <p>Tom </p> <p>Sandra </p> <p>Essie </p>

		<p>Tom has two more  than Sandra; this represents four socks.</p> <p>This can also be recorded as the subtraction:</p> $10 - 6 = 4$
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