Secondary Magazine Issue 70

National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 70 of the Secondary Magazine.

Contents

Contributors to this issue include: Margaret Brown, Rebecca Hanson, Mary Pardoe, Richard Perring, and Peter Ransom.

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From the editor

Welcome to this issue of the Secondary Magazine. We are delighted to have an interview with Professor Margaret Brown who, during the last 30 years or so, has greatly influenced national policy in mathematics education. Her work has resulted in much better understanding of many aspects of mathematics teaching and learning, impacting significantly on the curriculum and assessment, and on the development of teaching materials.

The National Centre online communities, forums and blogs provide opportunities for much discussion about the curriculum, assessment and teaching materials. The Secondary Forum is a good place to start communicating with each other for help, commenting or just sharing information. And many of the funded projects, research projects and regional networks of teachers are supported by dedicated communities, which may, or may not, be open to everyone. In this issue the Discussion about multiplication was inspired by several discussions in the online forums.

As usual we also have some ideas for the classroom that may help you develop your own teaching materials. Focus on change ringing might be a starting point for designing activities that link mathematics to other curriculum areas, and An idea for the classroom – 'the same' and 'different', suggests a kind of activity that can be used to encourage learning about any mathematical topic. Both 5 things to do and the Subject leadership diary contain links that may lead you to a variety of ideas for interesting activities.

At an inspirational meeting in July, Teaching mathematics for understanding, Professor Margaret Brown gave the keynote speech. Looking back over 30 years of changes in mathematics education, beginning with Cockcroft in 1982 and moving through to the new National Curriculum at Key Stages 3 and 4 in 2008 and the removal of Key Stage 3 Tests in 2009, Professor Brown reminded those present that "teachers find it difficult to change their practice, especially after teaching prescription and with high stakes tests – it needs time, talk, expertise and motivation".

Professor Brown urged mathematics educators to:

- forget grand schemes of national change – local innovations have at least as great an effect
- focus on releasing the creativity of teachers and others
- focus on what produces an effect on standards – connected teaching for understanding.

It's in the News!
Store wars

The fortnightly *It's in the News!* resources explore a range of mathematical themes in a topical context. The resource is not intended to be a set of instructions but a framework which you can personalise to fit your classroom and your learners.

Did you read about the price of rent on London's New Bond Street? It's the most expensive retail rental in Europe, with each square foot of space costing up to £950 per year! Little wonder then, that it's the very exclusive retailers such as Cartier and Chanel that fill the shops!

This resource uses the context of a retail space to give students the opportunity to develop work on length and area, as well as developing problem-solving skills. Students are challenged to create a space to contain 50m of clothes rails and 20m^2 of shelving, leaving space for access, but with the smallest footprint and, therefore, smallest annual rent.

This resource is not year group-specific and so will need to be read through and possibly adapted before use. The way in which you choose to use the resource will enable your learners to access some of the Key Processes from the Key Stage 3 Programme of Study.

Download this *It's in the News!* resource - in PowerPoint format

The Interview

Name: Margaret Brown

About you: I am married with three grown up sons and live in Southfields, near Wimbledon. I am now mainly retired but am still involved in research at King's College London working with Jeremy Hodgen and Dietmar Küchemann and a group of teachers on a project on Increasing Confidence and Competence in Algebra and Multiplicative Structures (ICCAMS). In my working life I have taught in primary and secondary schools and trained secondary mathematics teachers. While working at King's I have also directed many research and development projects in the teaching, learning and assessment of mathematics, spanning all ages from early years to adult numeracy. I have been a member of or chaired several government or other committees concerned with improvement of mathematics teaching, development of the mathematics curriculum and production of curriculum and assessment materials – most recently I was a member of ACME (Advisory Committee on Mathematics Education).

The most recent use of mathematics in your job was…
I use quite a lot of statistics in doing and reading educational research – most recently interpreting and presenting data from a large-scale attainment survey we carried out as part of ICCAMS.

Some mathematics that amazed you is…
Finite geometries – I really like the idea of exploring what happens to geometrical ideas, shapes and properties when you only have a finite number of points.

Why mathematics?
Partly because I realised I was quite good at it, and partly because, as a result of reading a lot of books about mathematics from the school library when I was in the sixth form, I became fascinated with mathematical ideas and how they were developed by people over time and were applied.

A significant mathematics-related incident in your life was…
I presume I can't count marrying a fellow mathematics student? One time I remember especially was at university when I spent several happy days playing with a new sort of number. Having realised that we got round the non-existence of roots of negative numbers by extending our number system by calling $\sqrt{-1}$ an imaginary number which we named i, and studying its properties, and the existence of non-recurring transcendental numbers we couldn't write down by calling the commonest ones e and π, it seemed a good idea to similarly deal with the impossibility of dividing by zero by representing 1/0 by a letter, say q, and finding out how it behaved. 'q' turned out to have some curious properties, and it soon got too difficult for me, but it was great fun while it lasted.

A mathematics joke that makes you laugh is...
Not really a joke, but I liked one Year 7 boy's explanation to me that there were infernally many decimals.

The best book you have ever read is…
I'm not sure I have an all-time favourite – and it's certainly many years since I read a novel – but the shortlist would probably include *Through the Looking Glass*, *The Collected Poems of T.S.Eliot*, *Barchester Towers* (Trollope), *Major Barbara* (Shaw's play), *The Lord of the Rings*, and *The Earthsea Trilogy* (Ursula Le Guin). Recently I've particularly enjoyed some mathematical biographies like The Strangest Man (Graham

Farmelo on Paul Dirac) and James Gleick's Chaos: the Amazing Science of the Unpredictable, a fascinating account of the development of Chaos theory.
Of course if you'd asked me about operas I'd simply have said Wagner's Ring Cycle – it blew my mind when I first saw it as a PGCE student in the 1960s and I still can't get enough of it.

Who inspired you?
First, one of my mathematics teachers at school, Enid Briggs. At university, the lectures of Christopher Zeeman on topology and algebra, and John Polkinghorne on quantum theory. In my years at Chelsea and King's, my ex-bosses Geoffrey Matthews and Paul Black.

If you weren't doing this job you would…
Spend a bit more time with my grandchild and on the hobbies I used to have like embroidery and playing the piano. I'd also do more walking and cycling in the countryside, and wild swimming. I'd even like to do a bit more maths teaching – I enjoyed regular team-teaching in Year 6 last year in the primary school where I'm chair of governors. But I'm happy to leave the writing of schemes of work, the detailed lesson plans, the SATs practice and the crowd control to the teacher experts!

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Focus on…change ringing

Lots Bell ringing is a context in which students can explore connected mathematical ideas such as the concepts of *order*, *change* and *system*. They can make and test hypotheses, and generalise. And there are opportunities for cross-curricular links with music and physics.

Change ringing is a team activity in which bells are rung one after another rhythmically according to patterns that the ringers memorise. It originated in England around the end of the sixteenth century as a way of ringing church bells that was compatible with the design and construction of sets of bells in church towers, and that fulfilled particular social functions.

This traditional activity has survived – as this school student explains (if you use this video in a lesson you can decide whether or not to show your particular students the last part – When it went wrong).

Change ringing is also done on hand bells, either by church bell-ringers when practising,

or in groups that are unrelated to church-bell ringing.

Change ringing bells are tuned to a normal *diatonic scale*. Pythagoras, it is believed, discovered that pleasant musical intervals are related by small integer ratios. He may have used a single-stringed instrument with a bridge that could be moved to divide the string into two parts in any proportion. Pythagoras found that plucking strings with lengths in the ratio 2:1 produced notes between which the

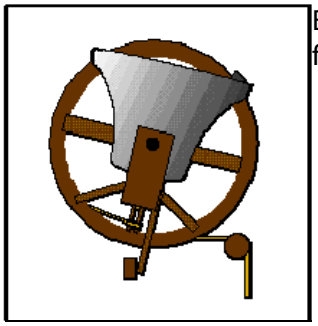
interval is what the ancient Greeks called *diapason*, and what we call an octave. The Greek word *dia* meant *between*, *through* or *across*.

This website was created *for* students *by* students (aged about 15) while they were working on a project. It provides more explanation about relationships that Pythagoras is thought to have investigated and established between musical pitches and tones. Students will also find information about the ancient Greek origins of the western musical scale at Midicode.

The C major scale is a *diatonic scale*.

In a change ringing room, the ringers stand in a circle, one behind each bell rope. Students can see in this one bell animation how a bell is rung 'full circle' by pulling on, and then releasing, the bell rope.

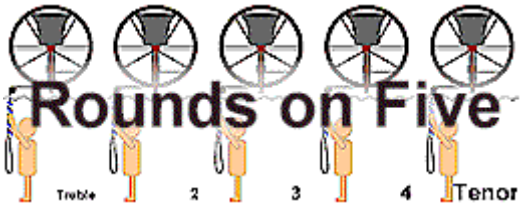
Animation by
Barney Stratford



Exactly how the clapper is made to strike the bell, and how the bell is prevented from over rotating is clarified in this swing bell animation.

Animation by
Chris Billinge

It is usual to start with ringing down the scale starting with the lightest bell – which rings the highest note. This sequence of rings is called a 'round', and is demonstrated in this five bell animation.



Animation by Ed Donnen. Animations used with permission of The Washington Ringing Society

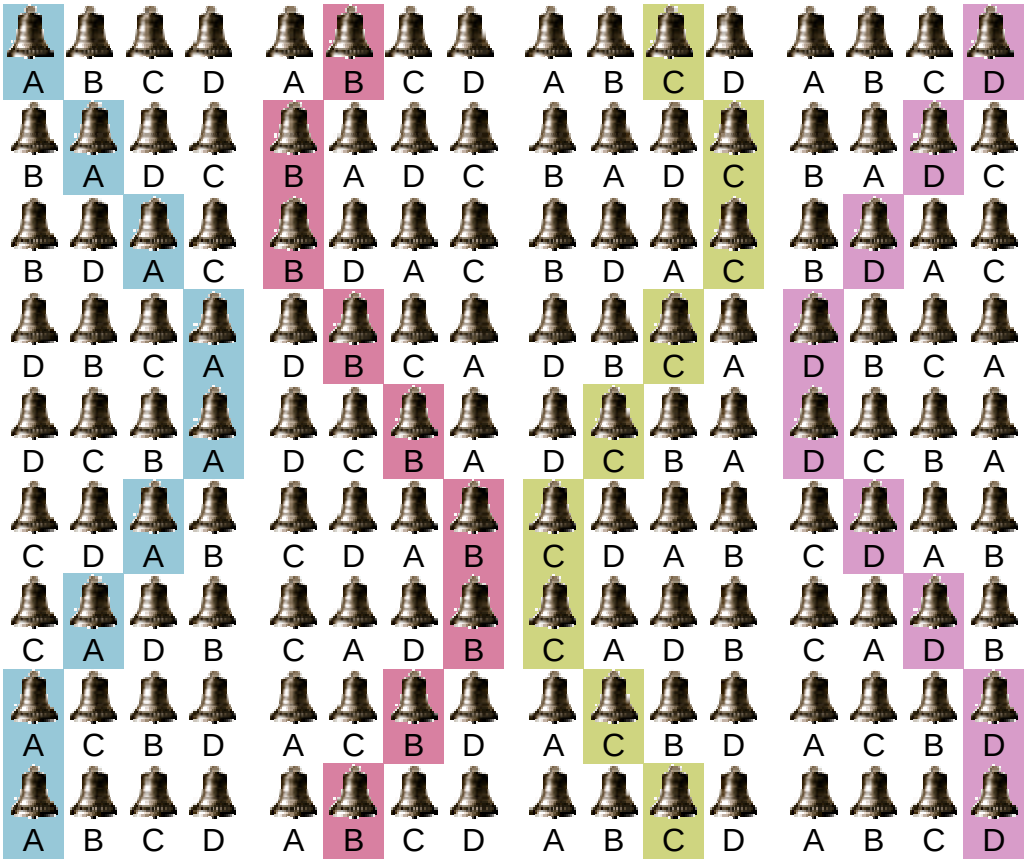
A round is usually repeated several times. Then the order in which the bells sound is changed from the order that produces the descending scale – so that all the bells in the set are again rung, but in a different order. The order is changed again, and all the bells are rung in this second order. Each different order in

which all the bells in the set are rung is called a 'row' or a 'change'. A 'method' is any sequence of 'rows' that satisfies the following conditions:

- the sequence begins and ends with a 'round'
- each bell sounds once in each row
- no bell may move more than one position from its place in one row to its place in the next row
- no row, apart from the first and last rows which are 'rounds', is repeated.

During ringing the ringers do not use any written prompts or music sheets. They memorise various methods, each of which has its own unique name, and they shift between methods when they are prompted to do so by 'calls' from their conductor. Methods are shown in diagrams that are collected in books, and on websites such as this introductory collection of methods for new bell ringers.

One well-known method is called 'Plain Hunt'. The changes in this method when it is rung on four bells are shown in these diagrams, in which each diagram highlights the 'route' of one particular bell:

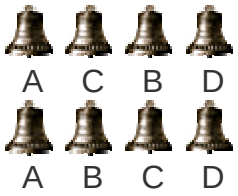


Students could be challenged to describe in words this system that is followed when the 'Plain Hunt' method is rung on four bells, before investigating the same method on other numbers of bells, or before exploring other methods.

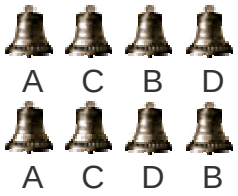
Students could also be challenged to work out the total number of possible orders of four bells (permutations of four objects).

It is possible to adapt the method shown above to create a method that includes every possible order. After seven changes, instead of following the system shown above by swapping the positions of the two

central bells and returning to the original round, another pair of bells are made to swap positions. For example, instead of the change from the 8th row to the 9th row being this,



it becomes this,



and then the original system restarts.

There are many traditional change ringing methods that can be explored on different numbers of bells. Or students can be challenged to devise their own methods for particular numbers of bells – perhaps with the added condition that every possible row is included. The patterns traced out through method diagrams by the 'routes' of particular bells are interesting!

You may be able to obtain a set of hand bells from your music department. Students can then ring, and listen to, methods that they devise or investigate – my students greatly enjoyed doing this, staying in during lunch hours to experiment and practice!

Students can try their hands at ringing 'Plain Hunt' on the change ringing simulator at LearnToRing.Com.

The Bells Applet is brilliant! It shows the number of each bell as it rings, so students can see how the order changes – start it, stop it, and they can see the sequence of rows!

There is much general information about change ringing at the Changeringing Wiki, which is an online community encyclopedia for change ringers.

Video and sound recordings of church bells and hand bells can be found at the Library of Bell Recordings.

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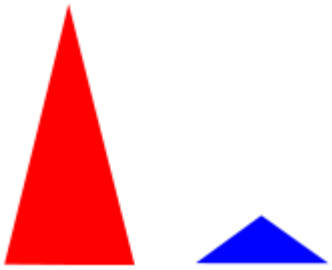
An idea for the classroom – 'the same' and 'different'

There are some kinds of mathematical task that ALL learners can access, and to which they can apply their natural thinking.

For example, if you show two examples of something and ask 'what is the same?' or 'what is different?' about them, all students are able to respond. If you choose the examples carefully, mathematical concepts can be developed naturally, using nothing more than the students' questions and natural thinking.

A simple starting point might be two isosceles triangles:



What is *the same* about the triangles? Students may notice that:

- in both triangles two sides are equal
- in both triangles the two angles that are between one of the equal sides and the third side are equal
- the 'bases' or 'third sides' are equal.

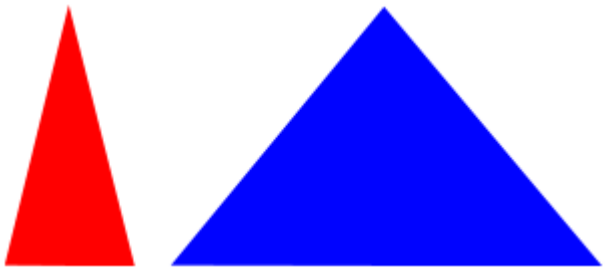
What is *different* about the triangles? Students may notice that:

- the angle opposite the 'third side' is smaller in the red triangle than in the blue triangle
- the equal angles in the red triangle are bigger than the equal angles in the blue triangle
- the red triangle is 'taller' than the blue triangle
- in the red triangle the equal sides are longer than the base, but in the blue triangle they are shorter.

Questions will arise naturally, such as:

- 'as the top corner of the red triangle moves down vertically towards the base what happens to the angles?'
- 'what happens to the length of the equal sides?'
- 'what are the smallest and largest possible sizes of the top angle/the base angles?'

A slightly different starting point will generate other questions.



- ‘as the base of the red triangle grows longer what happens to the angles?
- ‘what happens to the length of the equal sides?’
- ‘what are the smallest and largest possible sizes of the top angle/the base angles?’

Even with a situation involving just two triangles, the possibilities are endless!

These naturally arising questions are rich starting points for mathematical activity and, if they come from the students, they will carry commitment and motivation.

Discussion about multiplication

In a long forum discussion, How do you teach division?, contributors explored the idea that division can be visualised either as splitting/sharing/how many for one (partitioning) or as chunking/how many times it goes into (quotitioning). Algorithms are abstractions of either or both ideas and may be visualised in different ways by different users of them (although they may often not actually visualise what they are doing when using an algorithm).

My own focus on the split came from my observations regarding what happens when I demand that my students 'picture' what's going on.

I was interested in the idea that there are two fundamentally different routes through division. I then reflected that multiplication is the inverse of division. So if there are two fundamentally different routes through division, might there not correspondingly be two routes through multiplication?

But multiplication is commutative, which surely implies symmetry, and so suggests that this is not possible. But, after 10 months of reflection I am now convinced that commutativity is not symmetry, and that there are indeed two fundamentally different routes through multiplication. This difference disappears when you move into the abstract and is concealed by our use of numbers rather than real quantities, but it is very much there in concrete examples. The two routes are repeated addition and scaling.

You can see these ideas developing in another forum discussion, Teaching and learning fractions- a never ending journey!!!!!!.

Then I thought about how poorly we understand the idea of teaching multiplication through scaling. Exploring a section of the NCETM Self-evaluation Tools (Mathematics Content Knowledge > KS2 > Calculating > 2. Note: you will need to be logged in to see this) really brought home to me how confused we may be about this. I realised that scaling is something natural, something which is built into the beginnings of children's understanding of number, and which most current teaching methodologies suppress rather than nurture.

Now I have a perspective from which I can see why Cuisenaire rods and Montessori materials were so effective, and how Terezinha Nunes' street children and Celia Hoyles' student nurses have been able to develop mental strategies in ways that we do not understand. We indoctrinate children from the beginning of their education with the idea that the difference between 1 and 2 is the same as the difference between 8 and 9. But that's only true in the present context if we're focusing on repeated addition – isn't it? If we focus on scaling, the difference between 1 and 2 is much bigger than the difference between 8 and 9.

Young children have an incredible natural potential to calculate using the integer ratios – and we are not developing it. Montessori recognised this in her concrete methodology. Gattegno developed it through Cuisenaire rods which were discussed in the Primary Forum in The use of Cuisenaire in primary schools.

None of the forum discussions is closed – you are always welcome to join in, start a new discussion or post a comment.

5 things to do this fortnight

- The MathsJam is inspired by the biennial Gathering For Gardner. It will take place on the weekend of 13-14 November in the heart of Staffordshire. The gathering for Martin Gardner brings together people with an interest in magic, puzzles, mathematics, and the connection between them. Everyone at the Gathering is amazingly generous, sharing ideas and learning so much. While Colin Wright was at the British Congress for Maths Education (BCME 7) last April he was swapping puzzles and problems with David Bedford and others. David said, 'Why don't we have a Gathering in the UK?' MathsJam had been conceived.

- Have you read The Num8er My5teries: A Mathematical Odyssey through Everyday Life by Marcus du Sautoy, which was published in July? Now you can interact with it in an app! And you can watch recordings of Marcus' Royal Institution Num8er My5teries Christmas Lectures on the Royal Institution website. There will be an interview with Marcus du Sautoy in the next issue (71) of the Secondary Magazine.

- There is still time for students to enter their original projects in the 2010-11 National Science & Engineering Competition, either as individuals or as small teams. There are loads of prizes up for grabs! If your students missed last year's Big Bang, they can enter the competition now by completing an online entry form.

- Have you booked your place on the NCETM National CPD Conference, Professional Learning Networks: learning better through learning together? On 1 December you could join teachers from across the country to explore the benefits of networking and collaboration to enhance your mathematics teaching and learning. The conference, which will take place at the Megacentre in Sheffield, is free to attend and is open to teachers, lecturers and advisers from all sectors. To book your place, please email events@ncetm.org.uk.

- Tuesday 2 November 2010 would have been George Boole's 195th birthday if he had not died of a 'feverish cold' at the age of 49. But he lived long enough to invent Boolean Logic, which became the starting point for the development of all computer and microprocessor systems in the world today. The video, Forgotten Genius – George Boole outlines his life and achievements. And from the George Boole website you can reach a short docu-drama in which actors (one of whom plays George) try to convey aspects of the character and ideas of this forgotten genius from Lincolnshire.

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Subject Leadership Diary

The fatigue is starting to set in. After the initial buzz of returning to school, things have started to settle down so it is now time to reflect on the first few weeks and complete everyone's performance management.

I've had to deal with a few phone calls from concerned parents about the mathematics sets that their children are in this year. This mainly happens when students go into Key Stage 4 – where we move from two parallel bands to two non-parallel bands. However, once the situation is explained and parents realise that their child is in the best class for their mathematical education, things settle down. Quite a few students about whom we received a surprising lack of information started at the school this year. This makes it very difficult for us to decide which teaching group will be best for them – we don't want to have to test them when they arrive because starting in a new school is quite nerve-wracking for them anyway. If there is absolutely no information we start by allocating them to a middle set, and if we find that they are misplaced we move them as soon as possible.

All the teachers who have others to manage have taken part in the first of two twilight sessions on target setting. Over the past four years we have moved from just three rather woolly targets to four or more, depending on management responsibilities. Last year was our first year of doing performance management on Blue Sky Education - this allows targets to be agreed and set according to the school's improvement plan. It also allows the teacher and performance manager to add notes which build up into a portfolio of evidence that is used to determine whether the targets have been met. I have mixed feelings about this – in my opinion, performance management is a continuous process that should happen throughout the year, not just in two or three longer sessions (unless they are needed). Staff should feel that matters can be talked over at any point because surely it is better to discuss how things are going as often as possible in an informal way, rather than having only some more formal, longer, sessions. However, it allows the senior leadership team to see what stage everyone is at since the web-based programme allows that access. It also means that nobody should slip through the net!

I have an excellent colleague who does the book monitoring. He selects three names from each class in a specified year group (this term it is Year 9) and will ask for exercise books from those three students to be handed to him shortly. Because teachers do not know the names in advance, all books are kept marked reasonably well up to date. The books are checked to see that they have been marked according to our school standards. We check a different year each half term. This way we make sure that everyone is giving appropriate feedback to the students in their classes. The paper Inside the black box: raising standards through classroom assessments by Dylan Wiliam and Paul Black makes interesting reading.

I am interested in developing as much STEM (Science, Technology, Engineering and Mathematics) work as possible in the classroom. Not only are the activities interesting, but they will help enthuse my students to consider careers in this direction. Recently, I have found a wealth of material on Isambard Kingdom Brunel (IKB), and am developing that work in conjunction with Texas Instruments and the Brunel Museum at Rotherhithe. This is the site of the shaft of the Thames Tunnel, and there are activities planned for an evening event for teachers there on 21 October. Why not book a place?

I have also visited the Brunel special collection at Bristol University library where I handled Brunel's drawing instruments, and saw some of his calculation books. There's material there that can be used in

the classroom – it was interesting to find a calculation he worked on, crossed out and corrected, in his Clifton Suspension Bridge Calculations book.

Tuesday saw me burning the midnight oil – I'd had an invitation to submit a paper for the CERME 7 (Seventh Congress of the European Society for Research in Mathematics Education) conference in Poland in February and the deadline was the next day (my fault – I had intended doing it during the summer, but then forgot). So that meant pulling together a number of classroom episodes that had involved the history of mathematics. Anyway, it got done and sent off on the deadline date. Just have to wait now!