7 Using and applying numerical structure

Mastery Professional Development

Solutions to exemplified key ideas

Click the heading to move to that page. Please note that these materials are principally for professional development purposes; solutions are provided to support this aim. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

7.1.1.3 Use and interpret upper and lower bounds appropriately in calculations

Work confidently with different degrees of accuracy

Example 1:

All round to 82 000: 2 significant figures, nearest 1 000, nearest 100, 3 significant figures

All round to 81 972.36: 2 decimal places, nearest hundredth

Both round to 81 972: 5 significant figures, nearest integer

For information, the rounded solutions are:

2 significant figures	82 000	2 decimal places	81 972.36
	82 000	The nearest 100	82 000
The nearest tenth	81 972.4	The nearest hundredth	81 972.36
5 significant figures	81 972	5 decimal places	81 972.35640
The nearest million	0	The nearest integer	81 972
The nearest 100 000	100 000	3 significant figures	82 000

Example 2:

a) No b) No c) Yes d) No e) Yes f) No

Identify upper and lower bounds in both continuous and discrete contexts

Example 3:

a) 505 litres

b) 504 apples

Example 4:

Responses may vary but should demonstrate an understanding that Pip's calculation is inefficient and that an appropriate level of accuracy can be achieved with fewer decimal places. For example, *'It would depend on why Pip needed to know the maximum length, but* $\mathbf{2.005 + 2.005}$ *would be likely to tell her whether it would fit in a space or bridge a gap.'*

Recognise the effect of increasing or decreasing a value in different calculations

Example 5:

$d - [a]$	$[\,\mathbf{c}\,] \times \mathbf{d}$
$d + [\,c\,]$	$d \times [\,c\,]$
$[\,c\,] - d$	$d \div [\,a\,]$
$[\,\mathbf{c}\,] + \mathbf{d}$	$[\,\boldsymbol{c}\,] \div \boldsymbol{d}$

Example 6:

a) The top bar would remain the same.
The four bottom bars would increase in size and overhang the top bar.

b) The top bar would remain the same.
The four bottom bars would decrease in size and the top bar would overhang them more.

c) The top bar would remain the same.
The four bottom bars would increase slightly in size and only just overhang the top bar.

d) The top bar would remain the same.
The four bottom bars would slightly decrease in size.

Example 7:

a) Chris's rule will work.

b) Chris's rule will not work because you want the lowest value of y.

c) Chris's rule will not work because you want the lowest value of y.

d) Chris's rule will work.

e) Chris's rule will not work because you want the lowest value of x.

f) Chris's rule might not work, depending on the value of y.

Choose appropriately the upper or lower bound in a calculation

Example 8:

a) Minimum amount $= 995 \times 30 = 29\,850$ g

$29\,850 \times 60 = 7\,191\,000$ g per hour $= 1\,791$ kg per hour

b) $600 \div 1.005 = 597.01$ (to 2 d.p.)

597 bags

c) $200 \times 0.995 = 199$ kg of flour

Example 9:

Responses may vary but should demonstrate an understanding of which combination of upper and lower bounds will produce the maximum and minimum values. Below are some examples to consider.

a) What is the least amount of oil contained in one tanker and one household tank combined?

b) What is the maximum amount of oil in three tankers and a tank?

c) What is the smallest number of household oil tanks that could be filled by an oil tanker lorry?

7.1.2.1 Understand the mathematical structures that underpin multiplication and division of numbers represented in standard form

Recognise that standard form represents a number as a product of two factors

Example 1:

a) 10^2

b) 10^2

c) 10^{-2}

Example 2:

a) Responses may vary but should demonstrate an understanding that they share a common factor of 2 because both are even; and powers of 10 up to 10^5 as well as associated factors.

b) Responses may vary but should demonstrate an understanding that they share a common factor of powers of 10 up to 10^5 as well as associated factors.

Example 3:

$\mathbf{6.1 \times 10^7}$

Apply the associative law to reorder multiplications and divisions of numbers represented in standard form to allow for efficient evaluation

Example 4:

Responses may vary but should demonstrate an understanding that both are correct, and that Elinor's method is easier to work out because $25 \times 4 = 100$.

Example 5:

Responses may vary but should demonstrate an understanding that both are correct, and that Sameen's method is more efficient in this case.

Example 6:

Neither is greater because they are equivalent.

Understand and write equivalent factorisations for numbers represented in standard form

Example 7:

a) $\mathbf{10}$

b) $\mathbf{10}$

c) $\mathbf{10^2 = 100}$

d) $\mathbf{10^4}$

e) $\mathbf{10^6}$

f) $\mathbf{10^1}$

Example 8:

a) $\mathbf{6 \times 10^{11}}$

b) $\mathbf{1.5 \times 10^{12}}$

c) $\mathbf{9 \times 10^{10}}$

d) $\mathbf{9 \times 10^{12}}$

Responses may vary but should demonstrate an understanding that Valbona might not have left the answer in standard form with an initial number between 1 and 10 because $\mathbf{5 \times 3 = 15}$, a two-digit number.

7.1.2.2 Understand the mathematical structures that underpin addition and subtraction of numbers represented in standard form

Understand that numbers of the same magnitude can be easily added and subtracted *Example 1:* a) 6 b) 21 c) 6 d) 3
Example 2: Responses may vary but should demonstrate an understanding that Surinder's method calculates each multiplication calculation separately, then adds them together. Catriona's method is using the distributive law. Students should recognise the efficiency of Catriona's method in this case.
Recognise that numbers in standard form can be rewritten to allow for ease of addition and subtraction *Example 3:* a) 31 b) 3 100 000 c) 0.071 d) 0.501 e) 0.000 000 68
Example 4: Responses may vary but should demonstrate an understanding that Jane makes both powers of 10 equal, then converts to standard form at the end. Nicola converts both numbers to ordinary numbers, calculates, then converts to standard form at the end. Students should justify why Jane's method is more efficient.

7.1.3.1 Understand the infinite nature of recurring decimals

Understand that decimals and fractions are both representations of the same value

Example 1:

a) Responses may vary but should demonstrate an awareness of appropriate levels of accuracy as well as the equivalence of the expressions used.

b) Responses may vary but they might discuss the fact that Benji is focused on whether 10 is divisible by 3, not whether 10 can be divided by 3.

Recognise that a fraction represents a division and the outcome of that division can be represented by an equivalent decimal

Example 2:

$\mathbf{7 \div 16}$

Word problem responses may vary but should demonstrate an understanding that 7 is being shared equally between 16.

Understand that the formal written method for division can be used to write a fraction as a terminating decimal

Example 3:

a) 'Twelve divided by four,' or equivalent.
'Thirteen divided by four,' or equivalent.
'One divided by four,' or equivalent.

b) $\mathbf{12 \div 4 = 3}$
$\mathbf{13 \div 4 =}$ Responses may vary but should demonstrate an understanding that the answer will be more than three but less than four.
$\mathbf{1 \div 4 = \frac{1}{4} = 0.25}$

c) $\mathbf{3, 3.25, 0.25}$

Example 4:

a) $\mathbf{62.5, 0.625, 1.6.}$

b) $\mathbf{\frac{8}{5} = 1.6,\ \frac{5}{8} = 0.625}$

Understand that the formal written method for division can be used to write a fraction as a recurring decimal

Example 5:

a) Responses may vary but should demonstrate an understanding that none of the students has reached a complete decimal answer, which would involve the use of the recurring symbol (dot), or that the magnitude of the remainder each has shown is not clear.

b) Responses may vary but should demonstrate an understanding that:
Akram showed 'r4' for 4 remaining $\mathbf{\frac{1}{10000}}$s, so the remainder is 0.0004
Brenton showed 'r4' for 4 remaining $\mathbf{\frac{1}{100}}$s, so the remainder is 0.04
Cameron showed 'r4' for 4 remaining $\mathbf{\frac{1}{10000000}}$s, so the remainder is 0.0000004

7.2.1.2 Use the structures underpinning multiplication and division of numbers written in index notation to understand negative and zero indices

Appreciate that divisions can be written as fractions and vice versa

Example 1:

Responses may vary but could include number line, bar model, fraction, decimal, written word, shaded part of a shape, selected part of a set of items, etc.

Example 2:

Sometimes true, for the reasons discussed in the guidance notes.

Understand that simplifying fractions involves taking out a factor of 1 using the properties $\frac{ab}{ac} = \frac{a \times b}{a \times c}$ and $\frac{a}{a} = 1$

Example 3:

Responses may vary but should demonstrate an understanding that it is only when the highest common factor is extracted and simplified to 1 that the fraction is fully simplified.

Example solutions could include:

$$\frac{24}{48} = \frac{24 \times 1}{24 \times 2} = \frac{1}{2}$$

$$\frac{24}{48} = \frac{12 \times 2}{12 \times 4} = \frac{2 \times 1}{2 \times 2}$$

$$\frac{24}{48} = \frac{8 \times 3}{8 \times 6} = \frac{3 \times 1}{3 \times 2}$$

Example 4:

Responses may vary but should demonstrate an understanding that $\frac{3}{3} = \frac{7}{7} = 1$ and so neither is greater.

Connect the understanding of simplifying fractions with simplifying expressions with indices

Example 5:

Solutions given are examples and not exhaustive.

a)

$$\frac{48}{12} = \frac{12}{3} \times \frac{4}{4} \qquad p = 12, \quad q = 3, \quad r = 4$$

$$\frac{48}{12} = \frac{24}{6} \times \frac{2}{2} \qquad p = 24, \quad q = 6, \quad r = 2$$

Responses may vary but should demonstrate an understanding that r should be a factor of 12 and that the product of p **and** $r = 48$ and the product of q **and** $r = 12$.

b)

$$\frac{8 \times 8 \times 8 \times 8 \times 8 \times 8}{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8} = \frac{8}{8^3} \times \frac{8^5}{8^5} \qquad p = 8, \quad q = 8^3, \quad r = 8^5$$

Responses may vary but should demonstrate an understanding that $r = 1, 2, 4, 8, 8^2, 8^3, 8^4, 8^5, 8^6$ and that the product p **and** $r = 8^6$ and the product of q **and** $r = 8^8$.

c)

$\frac{5^3}{5^7} = \frac{5^2}{5^6} \times \frac{5}{5}$ $p = 5^2$, $q = 5^6$, $r = 5$

Responses may vary but should demonstrate an understanding that $c = 1, 5, 5^2, 5^3$ and that the product of $p\ and\ r = 5^3$ and the produce of $q\ \ and\ r = 5^7$.

Appreciate that a^n is a number in its own right, which can be operated on

Example 6:

$13^{11} \div 13^6 = 13^5$

$13^{11} \div 13^7 = 13^4$

$13^{11} \div 13^8 = 13^3$

$13^{11} \div 13^9 = 13^2$

$13^{11} \div 13^{10} = 13^1$

$13^{11} \div 13^{11} = 13^0$

$13^{11} \div 13^{12} = 13^{-1}$

$13^{11} \div 13^{13} = 13^{-2}$

Use knowledge of simplifying fractions to appreciate why $a^0 = 1$

Example 7:

Responses may vary but should demonstrate an understanding that Mike is wrong because he is confusing division with subtraction, and that a number divided by itself is 1.

Students might make use of related facts such as:

$\frac{2}{2} = 1$ $\frac{2 \times 2}{2 \times 2} = 1$

Example 8:

Responses may vary for the last two rows in the table but should demonstrate an understanding that they simplify to 3^3or 3^0.

The general rule may vary but should demonstrate an understanding that $a^m \div a^m = a^0 = 1$.

$3^5 \div 3^2$	$\frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$	$3 \times 3 \times 3 \times \frac{3 \times 3}{3 \times 3}$	3^3
$3^4 \div 3^1$	$\frac{3 \times 3 \times 3 \times 3}{3}$	$3 \times 3 \times 3 \times \frac{3}{3}$	3^3
$3^7 \div 3^5$	$\frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3}$	$3 \times 3 \times \frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3}$	3^2
$3^4 \div 3^2$	$\frac{3 \times 3 \times 3 \times 3}{3 \times 3}$	$3 \times 3 \times \frac{3 \times 3}{3 \times 3}$	3^2
$3^4 \div 3^4$	$\frac{3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3}$	$\frac{3}{3}$	3^0
$3^5 \div 3^5$	$\frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3}$	$\frac{3}{3}$	3^0

Use knowledge of simplifying fractions to appreciate why $a^{-b} = \frac{1}{a^b}$

Example 9:

Responses may vary for the last two rows in the table but should demonstrate an understanding that they simplify to 3^{-2} and 3^{-4}.

$3^2 \div 3^5$	$\frac{3\times3}{3\times3\times3\times3\times3}$	$\frac{1}{3\times3\times3}\times\frac{3\times3}{3\times3}$	3^{-3}
$3^5 \div 3^8$	$\frac{3\times3\times3\times3\times3}{3\times3\times3\times3\times3\times3\times3}$	$\frac{1}{3\times3}\times\frac{3\times3\times3\times3\times3}{3\times3\times3\times3\times3}$	3^{-3}
$3^1 \div 3^4$	$\frac{3}{3\times3\times3\times3}$	$\frac{1}{3\times3\times3}\times\frac{3}{3}$	3^{-3}
$3^2 \div 3^4$	$\frac{3\times3}{3\times3\times3\times3}$	$\frac{1}{3\times3}\times\frac{3\times3}{3\times3}$	3^{-2}
$3^6 \div 3^8$	$\frac{3\times3\times3\times3\times3\times3}{3\times3\times3\times3\times3\times3\times3\times3}$	$\frac{1}{3\times3}\times\frac{3\times3\times3\times3\times3\times3}{3\times3\times3\times3\times3\times3}$	3^{-2}
$3^2 \div 3^6$	$\frac{3\times3}{3\times3\times3\times3\times3\times3}$	$\frac{1}{3\times3\times3\times3}\times\frac{3\times3}{3\times3}$	3^{-4}

Example 10:

$$3^8 \div 3^{10} = \frac{3\times3\times3\times3\times3\times3\times3\times3}{3\times3\times3\times3\times3\times3\times3\times3\times3\times3}$$
$$= \frac{3\times3\times3\times3\times3\times3\times3\times3}{3\times3\times3\times3\times3\times3\times3\times3}\times\frac{1}{3\times3}$$
$$= 3^{-2}$$

Example 11:

Responses may vary but should demonstrate an understanding that:

Freddie’s value for n must be positive.

Gurpreet’s value for n must be negative.

Maya’s value for n must be zero.

Example 12:

Responses for the alternative may vary but should demonstrate an understanding that there can be multiple ways of naming the same number.

Jane	$\mathbf{100}$	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{\frac{1}{10}}$	$\mathbf{\frac{1}{100}}$	$\mathbf{\frac{1}{1000}}$
Pete	$\mathbf{10^2}$	$\mathbf{10^1}$	$\mathbf{10^0}$	$\mathbf{\frac{1}{10^1}}$	$\mathbf{\frac{1}{10^2}}$	$\mathbf{\frac{1}{10^3}}$
Sachin	$\mathbf{10^2}$	$\mathbf{10^1}$	$\mathbf{10^0}$	$\mathbf{10^{-1}}$	$\mathbf{10^{-2}}$	$\mathbf{10^{-3}}$
Alternative	$\mathbf{100}$	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{0.1}$	$\mathbf{0.01}$	$\mathbf{0.001}$

Reason and problem solve using the laws of indices

Example 13:

Responses may vary but should demonstrate an understanding that x is a non-integer value between 0 and 1.

Example 14:

a) 3	b) −4	c) −2	d) −4	e) 4	f) −2

7.2.1.4 Use the structures that involved when working with numbers written in index notation to understand fractional indices

Appreciate that some numbers can be written as repeated multiplication in a number of different ways

Example 1:

a) 3^6 b) 3^8 c) 3^{10} d) 9^{10} e) Not possible since $3^{19} = 9^{9.5}$

Example 2:

a) 4^6 b) 2^{12} c) 8^4 d) Not possible e) 64^2

Example 3:

Responses may vary but should demonstrate an understanding of equivalence of the calculations and their results.

Understand that $a^{\frac{1}{n}}$ is equivalent to $\sqrt[n]{a}$

Example 4:

a) 9 9 25 25 8 8

b) Responses may vary but should demonstrate an understanding that $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 5:

$a = 3$ $b = 2$ $c = 6$

Example 6:

a)

Set A	Set B
$(2^2)^{\frac{1}{2}}$	$(2^3)^{\frac{1}{3}}$
$(3^2)^{\frac{1}{2}}$	$(2^4)^{\frac{1}{4}}$
$(4^2)^{\frac{1}{2}}$	$(2^5)^{\frac{1}{5}}$
$(5^2)^{\frac{1}{2}}$	$(2^6)^{\frac{1}{6}}$

b) For Set A, the values can be simplified to 2,3,4 and 5 respectively. For Set B, the answers can all be simplified to 2.

Appreciate that $\sqrt[n]{a^m} = a^{\frac{m}{n}}$ is a consequence of the primary principle $(a^m)^n = a^{mn}$ *Example 7:* $a = 6$ $b = 4$ $c = \frac{2}{3}$ $d = \frac{1}{4}$ $8^{\frac{2}{3}} = 8^{2\times\frac{1}{3}} = (8^2)^{\frac{1}{3}} = \sqrt[3]{8^2}$
Example 8:   Responses may vary but should demonstrate an understanding that $(a^m)^n = a^{mn}$ and $a^{\frac{1}{3}} = \sqrt[3]{a}$, perhaps by generalising from the following: $64^{\frac{1}{3}} \times 64^{\frac{1}{3}} \times 64^{\frac{1}{3}} = 64^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 64 = \sqrt[3]{64} \times \sqrt[3]{64} \times \sqrt[3]{64} \therefore (64)^{\frac{1}{3}} = \sqrt[3]{64}$
Understand that applying the laws of indices is commutative   *Example 9:*   Responses may vary but should reference the commutative nature of laws of indices when explaining the equivalence of both approaches, as indicated in the guidance notes.
Appreciate the case when $(a^m)^n = a^1$ *Example 10:*   All are equivalent to 8.   Responses may vary but should demonstrate an understanding that a number can be written in root or fractional index notation.
Example 11:   Responses may vary but should demonstrate an understanding that m and n have a product of 1.
Solve problems involving fractional indices   *Example 12:* $a = 6$ $b = 4$ $c = 1$ $d = \frac{1}{5} = 0.2$

7.2.2.7 Understand the structure that underpins multiplication of surds, for example $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

Understand and use the equivalence of $\sqrt{a}\sqrt{b}$ and $\sqrt{ab}$ (as well as $\frac{\sqrt{a}}{\sqrt{b}}$ and $\sqrt{\frac{a}{b}}$)

Example 1:

a) 9	b) 25	c) 5
d) 2	e) 5	f) 2
g) 10	h) 10	i) 10

Example 2:

$\sqrt{10} + \sqrt{8} \neq \sqrt{18}$

Responses may vary but should demonstrate an understanding that surds cannot be added in this way.

Example 3:

a) Both answers are 2.

b) That the answers are the same.

c) Responses may vary but should demonstrate an understanding that this will always be true; square rooting the individual values and then dividing the results is equivalent to dividing the values and then finding the square root.

Use factorisation in order to rewrite expressions of the form $\sqrt{a}$ into expressions of the form $b\sqrt{c}$

Example 4:

a) 4	b) 16	c) 25	d) 36	e) 9×5	f) 100×5

Example 5:

Responses may vary but should demonstrate an understanding that the 'best' start will be one which has found a square number as a factor of 48, so Austin or Ella.

Example 6:

$\sqrt{4} \times \sqrt{9} = 6$

$\sqrt{20} - \sqrt{2} = 2\sqrt{5} - \sqrt{2}$

Example 7:

7	28	63	700	$7a^2$

Example 8:

$\sqrt{45} = 3\sqrt{5}$ and $\sqrt{500} = 10\sqrt{5}$

Appreciate that, when a number of expressions are all multiples of the same surd, then they can be added (or subtracted) *Example 9:* a) $4a$ b) $-4a$ c) $6a^2 - 4a^3$ d) $4\sqrt{5}$ e) $-\sqrt{5}$ f) $7\sqrt{5} - 4\sqrt{3}$
Example 10: Responses may vary but should demonstrate an understanding that all are mathematically possible, but only some can be simplified. a) $3\sqrt{5}$ b) $4 + 2\sqrt{2}$ c) $\sqrt{5} - 5\sqrt{2}$ d) 10 e) $\sqrt{5} + \sqrt{29}$
Example 11: a) $9\sqrt{3}$ b) $2\sqrt{3} + 2\sqrt{5} + 4\sqrt{2}$ c) $13\sqrt{5} + 5\sqrt{3} + \sqrt{10} + \sqrt{30}$
Confidently manipulate expressions with surds *Example 12:* a) Area of rectangle $= 18\sqrt{2} + 9\ units^2$ Perimeter of rectangle $= 8\sqrt{3} + 4\sqrt{6}\ units$ b) Area of triangle $= \frac{1}{2}\sqrt{6} \times \sqrt{6} sin60° = \frac{3\sqrt{3}}{2}\ units^2$ Perimeter of triangle $= 3\sqrt{6}\ units$

7.2.2.8 Understand and use the technique of rationalising the denominator to transform a fraction to an equivalent fraction

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Understand the difference between rational and irrational numbers

Example 1:

a) $\frac{4}{11}$, $\frac{4}{10}$, $\mathbf{0.\dot{3}}$, $\mathbf{0.1425}$, $\frac{22}{7}$, $\sqrt{4}$

b) $\frac{x^2}{x^2+3}, \frac{x}{13}, \frac{\sqrt{3}}{2}, \frac{a}{b}, \frac{a}{\sqrt{b}}$

The last one depends on the value of $\boldsymbol{b}$.

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Appreciate that when an irrational number is multiplied by an appropriate term, the resulting number can be rational.

Example 2:

$\boldsymbol{a} = \sqrt{3}$ $\boldsymbol{b} = \sqrt{2}$ $\boldsymbol{c} = \sqrt{2}$ $\boldsymbol{d} = 2$ $\boldsymbol{e} = \sqrt{8}$

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Example 3:

a) $2 + \sqrt{6}$ b) $3 + \sqrt{6}$ c) $5 + 2\sqrt{6}$ d) 1

e) Part d

f) Responses may vary but should demonstrate an awareness that this is a result of the question involving a difference of two squares.

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Appreciate that fractions involving surds in the denominator can be expressed as equivalent fractions where the denominator is rational

Example 4:

$\boldsymbol{p} = \sqrt{3}$

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Example 5:

a) $\frac{\sqrt{2}}{2+\sqrt{6}}$	b) $\frac{\sqrt{3}}{3+\sqrt{6}}$	c) $\frac{\sqrt{3}+\sqrt{2}}{5+2\sqrt{6}}$	d) $\sqrt{3} - \sqrt{2}$

e) Part d) has a rational denominator.

f) All have the same value as they have been multiplied by $\frac{p}{p} = 1$.

g) Now part c has a rational denominator.

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Appreciate that, when fractions have integer denominators, they are easier to calculate with

Example 6:

Responses may vary but should demonstrate an understanding that finding a common denominator with integer denominators is more efficient than with irrational denominators.

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7.2.3.3 Appreciate what constitutes the proof of a statement and what is required to disprove it

Understand that generalisations can be made that hold true for any value

Example 1:

Responses may vary but should demonstrate an understanding that the properties and definitions of triangles can be built upon to generalise for any triangle, using the properties of parallel lines.

Example 2:

Responses may vary but should demonstrate an understanding that there are no other consecutive primes because 2 is the only even prime number, and any set of two consecutive integers always contain one even and one odd number.

Example 3:

a) The standard test for identifying multiples of 3 is to find the digit sum, if the digit sum is a multiple of 3, then the original number is also a multiple of 3.

   Proving that the rule works can be achieved in different ways. One example is:

   Consider a number in which the digits are $\boldsymbol{a, b}$ **and** $\boldsymbol{c}$ and is written "$\boldsymbol{abc}$" (note that in this case, this *does not* represent $\boldsymbol{a} \times \boldsymbol{b} \times \boldsymbol{c}$).

   Using place value, the actual number can be represented as $\mathbf{100}\boldsymbol{a} + \mathbf{10}\boldsymbol{b} + \boldsymbol{c}$

   $\mathbf{100}\boldsymbol{a} + \mathbf{10}\boldsymbol{b} + \boldsymbol{c} = \mathbf{99}\boldsymbol{a} + \mathbf{9}\boldsymbol{b} + (\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$

   $\mathbf{99}\boldsymbol{a} + \mathbf{9}\boldsymbol{b} + (\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}) = \mathbf{3}(\mathbf{33}\boldsymbol{a} + \mathbf{3}\boldsymbol{b}) + (\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$

   $\mathbf{3}(\mathbf{33}\boldsymbol{a} + \mathbf{3}\boldsymbol{b})$ is a multiple of 3, therefore, if $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}$ is also a multiple of 3, then "$\boldsymbol{abc}$" is a multiple of 3.

b) Responses may vary but should demonstrate an understanding that if a number is a multiple of both 2 and 3, then it will be a multiple of 6.

Appreciate the difference between demonstration and proof

Example 4:

Responses may vary but should demonstrate an understanding that being able to represent a generalisation of two odd numbers (as Priya does) is more convincing than choosing specific numbers.

Example 5:

Responses may vary but should demonstrate an understanding that:
Mr Perry's class demonstrate the result, but since they cannot give every example it does not constitute a proof.
Similarly, Ms Hawke is demonstrating rather than proving.
Ms Akhtar is generalising using algebra, and this constitutes a proof.

Understand that counter examples can be used to disprove a conjecture

Example 6:

Responses may vary but should demonstrate an understanding that Rowan has selected certain numbers to try, which does not constitute a proof but shows that for certain cases this is true. One counter example may be used, for example 24.

Example 7:

Responses may vary but should disagree and demonstrate an understanding of forming factors from prime factors. Looking at 6! will show that a new prime factor is not uncovered.

Use algebra to express numbers with specific properties

Example 8:

a) Bashaar

b) Responses may vary but should demonstrate an understanding that Bashaar now definitely has an odd number. There is still no further information for Abi or Cathy.

Example 9:

a) $n+1$ b) $n-1$ c) $2n$ d) $4n$ e) $2n+1$

f) $2n-1$ g) $7n$ h) $14n$ i) $5n-3$ j) $10n-3$

Responses may vary, but should demonstrate an understanding of the structure of the number system.

Example 10:

Always odd	Could be odd or even	Always even
$8n-1$	n $n+1$ $n+2$ $7n$ $7n+1$ $7(n+1)$ $8n+n$	$2n$ $8n$ $8n+2$ $8(n-1)$

Interpret an algebraic statement to draw a conclusion

Example 11:

a)

i. $10(x-4)$

ii. $3(x+1),\ 3x+1,\ x+5$

iii. $3(x+1)$ and $6x+3$

iv. $10(x-4),\ x+5,\ 5x+4$

v. $3x+1$

vi. $10(x-4)$

b) Responses may vary but should demonstrate an understanding of multiples and divisibility.

Example 12: a) Expressions may vary but could include $\boldsymbol{n, n+1, n+2}$ or $\boldsymbol{n-1, n, n+1}$. b) Either $(\boldsymbol{n})(\boldsymbol{n+2}) = \boldsymbol{n^2+2n}$ or $(\boldsymbol{n-1})(\boldsymbol{n+1}) = \boldsymbol{n^2-1}$. c) Either $\boldsymbol{n^2+2n}$ is not equivalent to $\boldsymbol{4(n+1)}$ or $\boldsymbol{n^2-1}$ is not equivalent to $\boldsymbol{4(n)}$.
Example 13: a) Responses may vary but should demonstrate an understanding that we cannot be certain that $\boldsymbol{n+1}$ or $\boldsymbol{n+3}$ were odd numbers. b) Responses may vary but should demonstrate an understanding that by using $\boldsymbol{2n+1}$ and $\boldsymbol{2n+3}$ we can be certain these are odd.