MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.16 Subtraction: two-digit and two-digit numbers

Teacher guide | Year 2

Teaching point 1:

Known strategies can be used to subtract a multiple of ten and a single-digit number from a two-digit number.

Teaching point 2:

A two-digit number can be subtracted from a two-digit number by partitioning the subtrahend into tens and ones.

Overview of learning

In this segment children will:

- apply known strategies from earlier segments to work towards subtraction of one two-digit number from another
- use a strategy in which only the subtrahend is partitioned to facilitate calculation.

From previous segments, children should be able to confidently:

- partition a two-digit number into tens and ones (segment *1.9 Composition of numbers: 20–100*)
- subtract a single-digit number from a two-digit number, including crossing a tens boundary (segment *1.13 Addition and subtraction: two-digit and single-digit numbers*)
- subtract a multiple of ten from a two-digit number (segment *1.14 Addition and subtraction: two-digit numbers and multiples of ten*).

In this segment children will combine these skills to subtract any two-digit number from another two-digit number.

In segment *1.15 Addition: two-digit and two-digit numbers*, children initially used a strategy of partitioning *both* addends to add two two-digit numbers. Due to their familiarity with that approach, children may 'over-apply' this and attempt to use a similar strategy for subtraction (e.g. $77 - 23 = (70 - 20) + (7 - 3)$); however, this strategy can be problematic when the ones digit of the subtrahend is larger than the ones digit of the minuend, since subtraction for the ones results in a negative number (e.g. $73 - 27 = (70 - 20) + (3 - 7)$), which children will not be familiar with; children may apply misconceptions regarding commutativity to reach an incorrect answer, for example:

$73 - 27 = (70 - 20) + (\mathbf{7 - 3}) = 50 + 4 = 54$ ✗

It is therefore important for children to learn to partition only the subtrahend (e.g. $73 - 27 = 73 - 20 - 7$).

Examples and questions should include cases of both subtracting the tens first (e.g. $73 - 20 - 7$) and subtracting the ones first (e.g. $73 - 7 - 20$) so children come to understand that this does not change the answer. Children should use known number facts to mentally subtract these decomposed parts, regardless of the order they are subtracted, rather than counting back, or counting manipulatives, to find the answer.

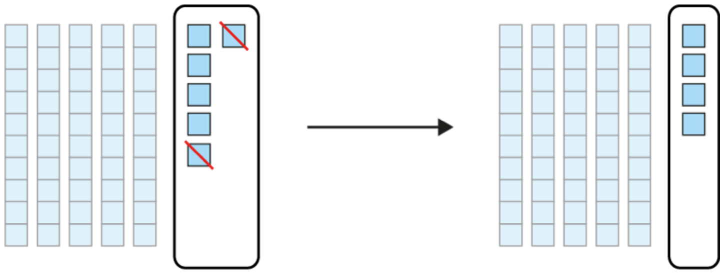
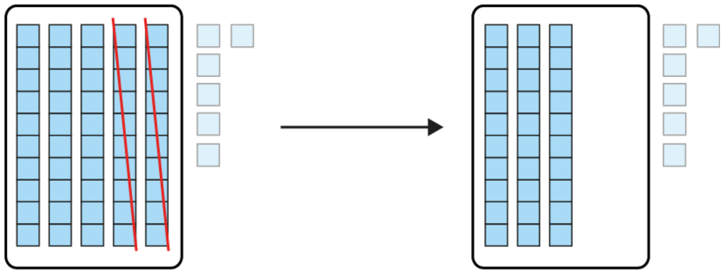
When choosing examples/questions it is also important to carefully consider the numbers used, to ensure the focus is appropriate for the given step. In each step, follow the guidance on whether the tens boundary should or should not be crossed during subtraction of the ones. Example practice problems have been suggested in steps *2:5* and *2:8*, addressing calculations which don't, and do, bridge a multiple of ten respectively. At the end of the segment it is recommended that children are also presented with mixed practice that includes calculations of both types.

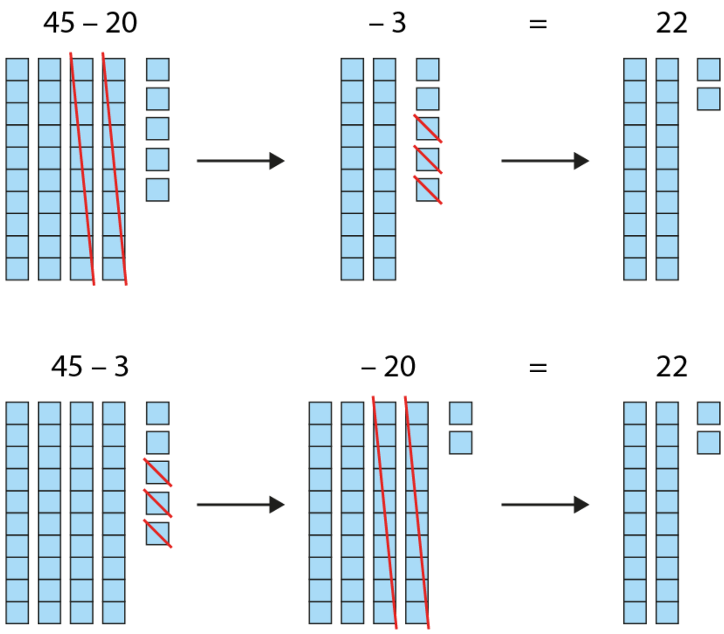
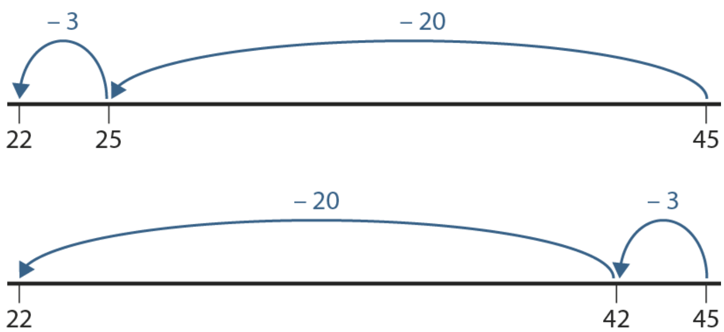
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

Known strategies can be used to subtract a multiple of ten and a single-digit number from a two-digit number.

Steps in learning

	Guidance	Representations
1:1	To begin this segment, ensure that children can confidently subtract: • a one-digit number from a two-digit number (see segment *1.13 Addition and subtraction: two-digit and single-digit numbers*) • a multiple of ten from a two-digit number (see segment *1.14 Addition and subtraction: two-digit numbers and multiples of ten*). As in previous segments, use a variety of representations to draw attention to which digit changes when subtracting ones or tens, including: • number lines • a hundred square • a Gattegno chart. As in segments *1.13* and *1.14*, emphasise the application of known facts, rather than 'counting back' or using the manipulatives/images to find the answers. To prepare for the next steps, also use Dienes. This will emphasise an important difference between representing two-digit addition and two-digit subtraction; for subtraction, only the starting number (the minuend) is made using physical resources, whereas both addends must be made for addition. During this review, include examples where the minuend is a multiple of ten, for example: 40 – 3 40 – 20	Subtracting a one-digit number from a two-digit number:  • *'I know that six minus two is equal to four...'* • *'…so fifty-six minus two is equal to fifty-four.'* 56 – 2 = 54 Subtracting a multiple of ten from a two-digit number:  • *'I know that five minus two is equal to three…'* • *'…so, five tens minus two tens is equal to three tens.'* • *'Five tens and six ones, minus two tens, is equal to three tens and six ones.'* 56 – 20 = 36

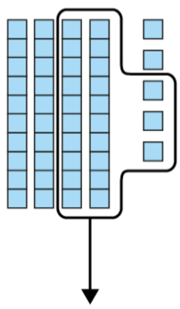
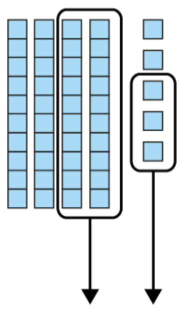
	Throughout this teaching point, use examples in which subtraction of the ones does *not* cross a tens boundary, so that in the subsequent steps the focus remains on the structure of subtracting the tens and ones separately.	
1:2	Once it is clear that children have mastered the concepts practised in step *1:1*, progress to the combination of subtraction of both tens and ones into one story, for example: *'At first I had forty-five biscuits. Then I sold twenty biscuits, and then I dropped three biscuits. How many are left?'* (Note that this is an example of a '***first**…, **then**…, **then**…, **now'*** structure, as explored in segment *1.11 Addition and subtraction: bridging 10*). Use Dienes, number lines and equations to represent the sequential steps of the story. Then change the order of the story to explore what is the same and what is different if the order of the subtrahends changes, i.e.: *'At first I had forty-five biscuits. Then I dropped three biscuits, and then I sold twenty biscuits. How many are left?'* Show the representations of the two stories, side-by-side, so children can see that both result in the same final amount (the difference).	*'At first I had forty-five biscuits. Then I sold twenty biscuits, and then I dropped three biscuits. How many are left?'* vs. *'At first I had forty-five biscuits. Then I dropped three biscuits, and then I sold twenty biscuits. How many are left?'* Comparing the Dienes representations:  Comparing the number line representations: 

<table>
<tr><td></td><td></td><td>Comparing the equations:
45 – 20 = 25   45 – 3 = 42
25 – 3 = 22   42 – 20 = 22</td></tr>
<tr><td>1:3</td><td>Provide practice with more stories, structured such that the tens and ones are subtracted separately, for example:
• 'Charlie counted thirty-six mini-beasts on a patch of grass; ten of them were grasshoppers, and four of them were ladybirds; the rest were ants. How many ants were there?'
(partitioning)
• 'There are eighty-four pens in a jar; two of them are red, and thirty of them are black; the rest are blue. How many blue pens are there?'
(partitioning)
• 'Joel had seventy-five pence. He spent five pence on a sweet, and then thirty pence on a pencil. How much money does he have now?'
(reduction)
• 'Pari had a thirty centimetre length of thread for making a bracelet. She cut off ten centimetres; the thread was still too long so she cut off another two centimetres. How long is the thread now?'
(reduction)
To deepen children's understanding, provide them with a set of number cards (with numbers chosen to avoid ones subtraction bridging the tens boundary), and ask them to create their own stories.
Throughout, children should make the starting number using Dienes, or sketch the Dienes; as usual, these should not be used as an aid to calculation, but to embed the place-value structure and support development of fluency.</td><td>'Create similar stories using these number cards.'<table><tr><th>Starting number</th><th>Multiple of ten</th><th>Single-digit number</th></tr><tr><td>88   76
67   59</td><td>10   20
30   40</td><td>6   4
3   2</td></tr></table></td></tr>
</table>

Teaching point 2:

A two-digit number can be subtracted from a two-digit number by partitioning the subtrahend into tens and ones.

Steps in learning

	Guidance	**Representations**
2:1	This teaching point is broken down into two stages – problems in which subtraction of the ones does *not* bridge a multiple of ten (steps *2:1– 2:5*) and problems in which the subtraction of the ones *does* bridge a multiple of ten (steps *2:6–2:8*). Begin by revisiting the story used in step *1:2*, showing the associated subtraction expression. Then show a numerically-related two-digit story alongside it, as in the example opposite. Compare the two problems, asking children: • *'What's the same?'* (Both stories start with the same number of items (45), and in both stories the same number of items (23) are subtracted; both stories give the same final answer (22).) • *'What's different?'* (In the first story the items are subtracted in two stages (20 and then 3); in the second story, all items (23) are subtracted 'at once'.)	*'At first I had forty-five biscuits. Then I sold twenty biscuits, and then I dropped three biscuits. How many are left?'* 45 – 20 – 3 vs. *'There were forty-five children in the playground. Then twenty-three of them went into the dinner hall. How many children are still in the playground?'* 45 – 23
2:2	Now examine the two-digit calculation more closely, using Dienes. Represent the 45, and show that taking away 23 all in one go leaves the same amount as taking away 20 and then taking away another 3. Note that we generally use the word 'subtract' to describe the operation, because it encompasses all three subtraction structures: reduction (taking away), partitioning and difference. Here, the use of the phrase	Dienes and abstract representations: 'taking away' 23  'taking away' 20 and 3 

<table>
<tr>
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<td>'take away' is acceptable – it is describing the specific action of removing a subset from a set.

Summarise with the explanatory sentence, jotting and equation, as shown opposite.

Model partitioning of the subtrahend for a range of calculations, using Dienes, and keeping to examples in which subtraction of the ones does not bridge a multiple of ten. Repeat the stem sentence:
• 'To subtract ___,…'
• '…we can subtract ___…'
• '…and then subtract ___.'

For now just look at the equivalent expressions rather than proceeding to find the answer to the calculations. Include examples where either the minuend or the difference is equal to a multiple of ten.

Provide children with incomplete equations so that they can practise expressing calculations with partitioning of the subtrahend. Include examples presented both with subtraction of the tens first, and with subtraction of the ones first.</td>
<td>'To subtract twenty-three, we can subtract twenty and then subtract three.'

45 – 23
/ \
20 3

45 – 23 = 45 – 20 – 3

Missing number problems:
'Fill in the missing numbers.'

67 – 21 = 67 – 20 – □
75 – 34 = 75 – □ – 4
55 – 27 = 55 – □ – □

84 – 12 = 84 – 2 – □
68 – 23 = 68 – □ – 20
47 – 25 = 47 – □ – □

80 – 13 = 80 – 10 – □
74 – 34 = 74 – □ – 4
63 – 23 = 63 – □ – □
50 – 24 = 50 – □ – □</td>
</tr>
<tr>
<td>2:3</td>
<td>Now begin to look at finding the answers to calculations, while continuing to emphasise the process of partitioning the subtrahend. Coins provide an excellent real-life context for this:</td>
<td></td>
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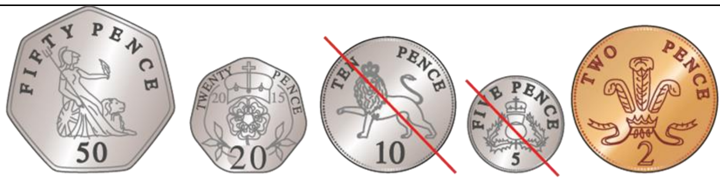
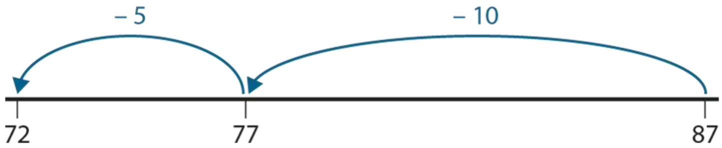
</table>

- Pose a question, such as *'I had eighty-seven pence. Then I spent fifteen pence. How much do I have now?'*
- Ask children to make 87 p using coins. Since this can be done in more than one way, some children might find this challenging; if this is the case, so as not to distract from the focus on subtraction, you may wish to tell children which coins to take (50 p, 20 p, 10 p, 5 p and 2 p).
- Then explore how to remove 15 p; it is not possible to remove a 15 p coin, but it is possible to remove coins to the *value* of 15 p. Discuss the most efficient way to do this, i.e. by removing a 10 p coin and a 5 p coin (note, if children made 87 p in the manner described above, then there will only be one way to remove 15 p).





You can represent the calculation on a number line, but must ensure that children use known facts to subtract the tens and the ones, rather than counting back in ones on the number line – the number line is used *only* to represent the two stages of the calculation. The removal of a 10 p and a 5 p coin, rather than a series of 1 p coins, will help to emphasise this.

As mentioned in the *Overview of learning*, children may be tempted to subtract the tens from the tens and the ones from the ones separately (partitioning both the subtrahend and minuend):

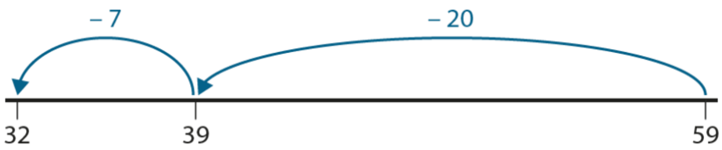
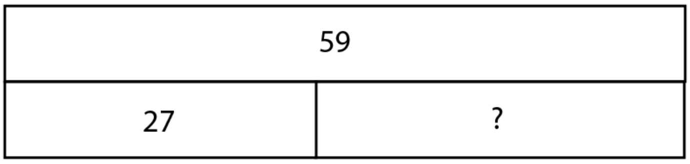
80 – 10 = 70

7 – 5 = 2

so

87 – 15 = 72

However, this approach will only be successful for calculations in which subtraction of the ones *doesn't* bridge a multiple of ten; children who persist with this approach will run into

	problems when they reach calculations in which subtraction of the ones *does* bridge a multiple of ten, since they have not yet learnt to subtract across zero. Look carefully at children's working to identify those who are partitioning both the subtrahend and the minuend, and address it (see also step *2:7* below).	
2:4	Now move on to problems represented as equations, continuing with examples in which subtraction of the ones does not bridge a multiple of ten, for example: 59 – 27 40 – 23 62 – 30 As before, encourage subtraction of the multiple(s) of ten, and then a single 'jump' for subtraction of the ones (as shown on the number line opposite), using known facts, rather than counting back in ones. Model problems using part–part–whole representations (bar-model or cherry diagram).	Number line:  Part–part–whole diagram: 

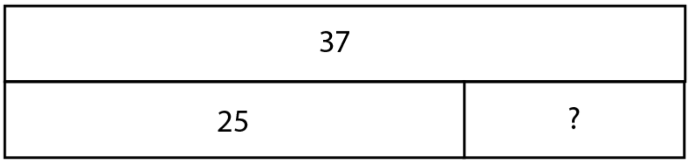
2:5	Provide children with varied practice, including:
	• missing number problems (equations, part–part–whole diagrams, number lines)
	• real-life contexts, for example:
	  • *'I saved up sixty-five pounds. Then I spent thirty-one pounds. How much money do I have left?'* (reduction)
	  • *'There are seventy-eight children at a party. Twenty-five children have already taken their party bags. How many children haven't taken their party bags?'* (partitioning)
		Missing number problems:
		'Fill in the missing numbers.'
		76 – 13 = □ □ = 84 – 21
		76 – 23 = □ □ = 84 – 41
		76 – 33 = □ □ = 84 – 61
		67 – □ = 43 53 – 21 = □
		67 – □ = 33 □ = 75 – 34
		67 – □ = 13 94 – 43 = □

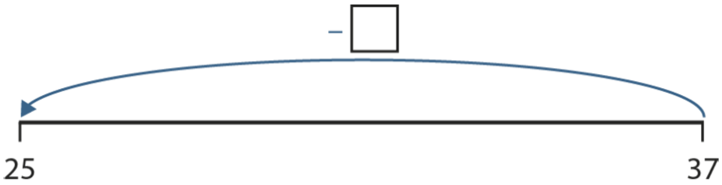
- *'I had fifty-nine metres of rope. Then I cut off and used twenty-seven metres of the rope. What length of rope do I have left now?'*
  (reduction)
- *'I planted forty-eight seeds. Twenty-three of them grew. How many did not grow?'*
  (partitioning)
- *'Zayn has eighty-five pounds. Yasmin has thirty-two pounds. How much more money does Zayn have than Yasmin?'*
  (difference)

Include questions where the minuend is a multiple of ten, for example: *'I had an eighty centimetre length of ribbon. Then I used twenty-three centimetres of the ribbon to wrap a present. What length of ribbon do I have left?'* Again, children should already be familiar with the constituent parts of the calculation:

- subtracting a multiple of ten from a multiple of ten (segment *1.8 Composition of numbers: multiples of 10 up to 100*)
- subtracting a single-digit number from a multiple of ten (segment *1.13 Addition and subtraction: two-digit and single-digit numbers*)

To provide challenge, use a dòng nǎo jīn problem such as the one shown oppoiste.





37 – □ = 25

Problems with the minuend equal to a multiple of ten:

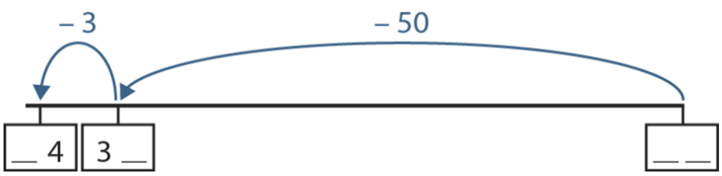
'Fill in the missing numbers.'

80 – 13 = □	□ = 70 – 22
80 – 23 = □	□ = 70 – 42
80 – 33 = □	□ = 70 – 62

60 – □ = 41	50 – 21 = □
60 – □ = 31	□ = 40 – 12
60 – □ = 11	90 – 43 = □

Dòng nǎo jīn:

'Fill in the missing numbers.'



2:6

Now progress to calculations in which subtraction of the ones *does* bridge a multiple of ten, for example:

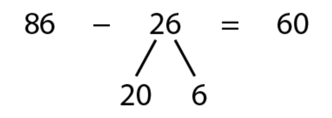
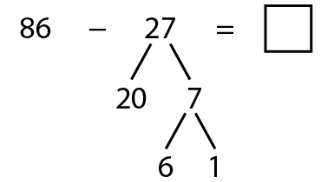
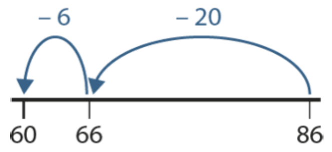
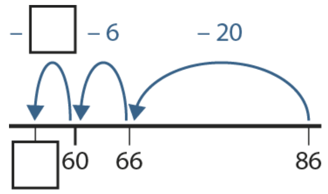
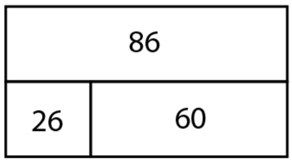
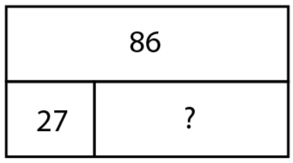
86 – 27

73 – 35

62 – 49

Begin with a calculation for which the difference is a multiple of ten (e.g. *86 – 26*). Find the difference, partitioning the subtrahend, as usual, and representing the calculation on a number line as shown opposite.

Then, keeping the minuend the same, increase the value of the subtrahend by one (i.e. *86 – 27*), relating this new calculation to the previous one. Refer to the number line for *86 – 26* and discuss how to amend it to show *86 – 27*. Note that the ones can be partitioned further to highlight the bridging through ten strategy that children should use to calculate the answer; this strategy builds on practice questions in step *2:5* where the minuend is equal to a multiple of ten.

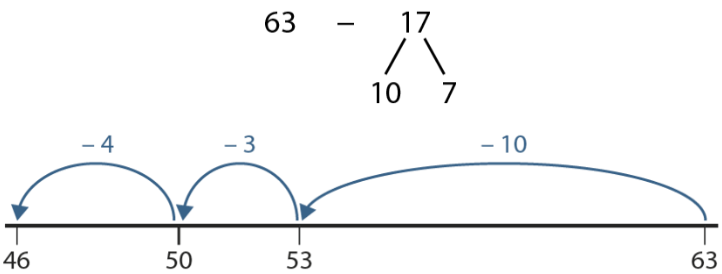
Difference is equal to a multiple of ten	**Bridging a multiple of ten**
	
	
	
86 – 26 = 60	86 – 27 = 59

2:7

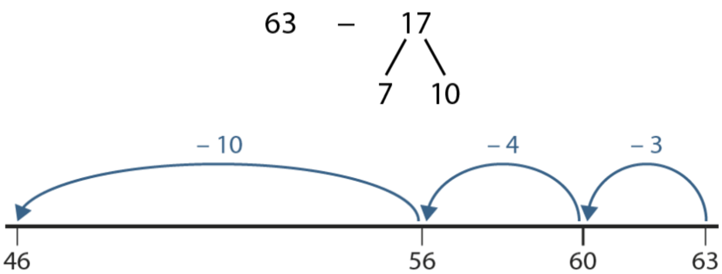
Model several more subtraction calculations for which subtraction of the ones bridges a multiple of ten, showing the three steps on a number line. Include examples in which you subtract the tens first, and in which you subtract the ones first.

Some children may still be tempted to subtract the tens from the tens and the ones from the ones separately (partitioning both the subtrahend and minuend). However, as discussed in step *2:3*, children will only be successful with this approach for calculations in which subtraction of the ones *doesn't* bridge a multiple of ten. For children who persist with this approach, it can be worthwhile to demonstrate *why* it is problematic when applies to

Bridging a multiple of ten – subtracting the tens first:



Bridging a multiple of ten – subtracting the ones first:



<table>
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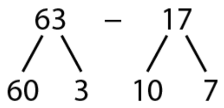
<td>calculations for which the ones digit of the subtrahend is larger than that of the minuend, as shown below, pointing out that they haven't yet learnt how to subtract one number from a smaller number.

60 – 10 = 50
3 – 7 = ?
7 – 3 = 4 (incorrect application of law of commutativity)
so
50 + 4 = 54 ✗
As shown here, on not being able to subtract the larger number (seven) from the smaller (three), children may incorrectly apply the law of commutativity, thus calculating, in this example, 67 – 13 instead of 63 – 17.
As children have not yet had sufficient practice with the correct strategy, you may need, in this case only, to count back to prove that this approach doesn't give the correct answer.</td>
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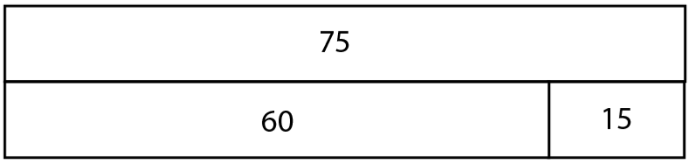
<tr>
<td>2:8</td>
<td>Children are likely to need a lot of practice to develop fluency in subtracting across a tens boundary in this way. Provide varied practice, including:
missing number problems (equations, part–part–whole diagrams, number lines)
real-life contexts, for example:
'I got sixty-two pounds for my birthday. Then I spent thirty-five pounds. How much money do I have left?'
(reduction)
'There are thirty-two children in the class. Seventeen children have already put their coats on. How</td>
<td>Missing number problems:

'Use the part–part–whole diagram to help you complete the following equations.'
75 – 15 = ☐
75 – 16 = ☐
75 – 17 = ☐</td>
</tr>
</table>

many children have not put their coats on?'
(partitioning)

- *'I had eighty-five centimetres of string. Then I cut off and used twenty-eight centimetres to play conkers. What length of string do I have left now?'*
  (reduction)
- *'Ginny has forty-six pictures in her colouring book. She has coloured in nineteen of them. How many has she not coloured in?'*
  (partitioning)
- *'Tara's sunflower is seventy-seven centimetres tall. Akesh's sunflower is fifty-eight centimetres tall. How much taller is Tara's sunflower than Akesh's?'*
  (difference)

Include questions which bridge ten (e.g. *92 – 84*). You can also include examples with two subtrahends, such as the one shown on the next page.

'Fill in the missing numbers.'

75 – 18 = □ □ = 92 – 24

75 – 28 = □ □ = 92 – 44

75 – 38 = □ □ = 92 – 64

75 – 48 = □ □ = 92 – 84

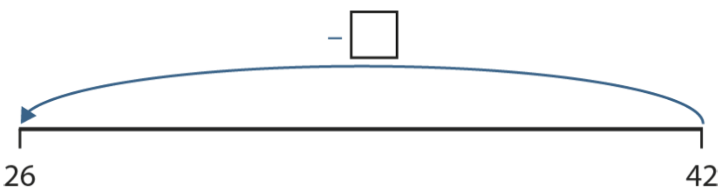
53 – □ = 38 57 – 29 = □

53 – □ = 28 □ = 73 – 64

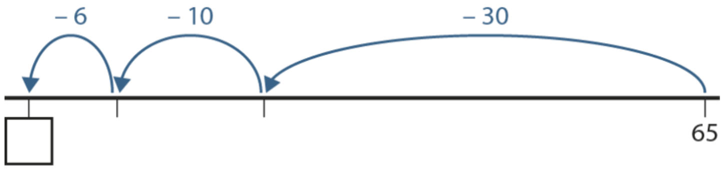
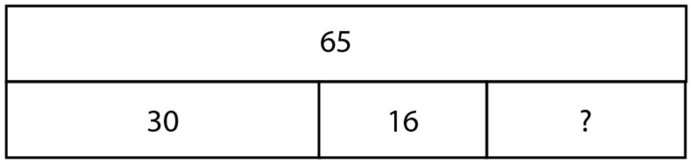
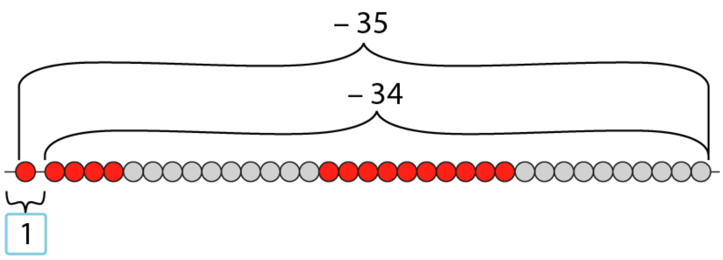
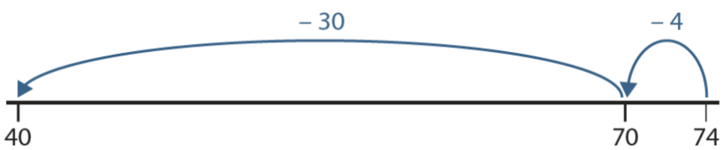
53 – □ = 18 95 – 47 = □

53 – □ = 8

42	
26	?



42 – □ = 26

		Problem with two subtrahends: *'Paul had sixty-five stickers. He gave thirty stickers to Andrew and sixteen stickers to Alicia. How many stickers does Paul have left?'*   65 – 30 – 16 = ☐
2:9	Once children have mastered subtracting one two-digit number from another, either crossing a tens boundary or not, explore the following special cases: • consecutive two-digit numbers / difference of one (e.g. *35 – 34*, *40 – 39*) • two-digit numbers with equal ones digits (e.g. *74 – 34*) For each, present an example calculation and ask children whether they know of an easier way to find the answer. • For calculations with consecutive two-digit numbers, children should be able to apply their understanding that consecutive numbers have a difference of one (segment *1.12 Subtraction as difference*), and their knowledge of the counting sequence for two-digit numbers. • For calculations in which the ones digit of the subtrahend is equal to the ones digit of the minuend, it may be easier / more efficient to subtract	Consecutive two-digit numbers:  Two-digit numbers with equal ones digits: 

	the ones first, and then the tens (*74 – 4 – 30 = 70 – 30*). The following generalised statement can be used: ***'For a subtraction calculation where both numbers have the same ones digit, the difference is a multiple of ten.'***	