Primary
& Early Years
Magazine 92

National Centre
for Excellence in the
Teaching of Mathematics

Welcome to another issue of our Primary Magazine, which has now been serving primary teachers for 92 issues with a varied collection of articles related to maths education and mathematics professional development - all of which are available in the Primary Magazine Archive.

## Contents

In Digging Deeper each month we will explore an element of mathematics teaching linked to current developments and research: in this issue we look at some of the things we can learn about teaching for mastery from the new suite of NCETM videos.

Aspects of… provides a number of bitesize ideas related to a specific element of mathematics; this month we have Aspects of ½.

Seen and Heard provides a specific example of a child's response to mathematics in a classroom to stimulate thinking and provoke questions about how you would react to similar events in your own classroom. In this issue a child's understanding of halving is explored.

First, as always, we have a News section, bringing news from the NCETM and beyond to keep you up to date with the fast-changing world of mathematics education.

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# News

With the ongoing growth of our materials related to teaching for mastery, we've simplified the collection by sorting it into different categories, including Case Studies, Lesson Videos, Resources, and more. Do have a look and let us know what you think!

Do you have views about last summer's SATs tests: the first under the new National Curriculum? Then MPs want to hear what you have to think. An inquiry by the Commons Education Committee is asking for written submissions by Friday 28 October. Find out more from parliament.uk.

Have you seen the collection of maths resources, newly assembled on the Foundation Years website? The site is the result of collaboration of a diverse group of professionals working in the Early Years sector.

Issues surrounding primary maths crop up most weeks in our Twitter-based CPD discussions under the **#mathscpdchat** hashtag. The record of every chat is posted on the mathscpdchat webpage. Recent weeks have covered the concept of variation in maths teaching, behaviour in maths lessons and how secondary schools can help new Y7s hit the ground running in maths lessons.

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# Digging Deeper

## Teaching for Mastery

The NCETM has identified five big ideas to develop mastery:

- Coherence
- Mathematical thinking
- Representation and structure
- Fluency
- Variation.

The new set of mastery videos provides an opportunity to explore and make sense of these ideas. In this article we consider the five big ideas in the context of the Y1 lesson, filmed in Brunel Field Primary School, Bristol and taught by Clare Christie who is the maths subject leader for the Ashley Down Schools Federation and co-Maths Hub Lead for Boolean Maths Hub.

### Coherence

> *"The planning and the structuring come through the careful choice of problems and activities …The interplay of dance between teacher intention and learner contribution is a complex weaving in and out, but each plays a role."* Askew 2009 [1]

The teacher has planned the lesson to enable the children to use and build on what they already know and understand in order to develop new understanding. This has involved thinking carefully about the steps along the lesson journey in order that the children can make connections and therefore makes sense of the mathematics explored. This lesson is part of a bigger journey that runs across a sequence of lessons and the teacher talks about where it sits in the sequence in the clip ***Maths subject leader Clare Christie talks about her Y1 lesson***.

The planning includes choosing the numbers, context and representations (see below) that will best support understanding. It also includes an awareness of what might cause the children to struggle. In the clip ***Moving pupils on to independent work*** the teacher explains that she identified the 'difficult points' of the lesson when planning. Rather than avoid these she instead planned so that the children would encounter them, and was then prepared to support the children as necessary. For example, the children had used the part-part-whole model for adding two numbers and they were then asked to use the same model to think about comparing two numbers; this was one of the difficult points. The teacher did not avoid using the model, because using it was important in order to support the children with understanding that the additive relationship between three numbers can be expressed as an addition or as a subtraction. Instead, she is ready to intervene when children are uncertain about how the model fits with the mathematics with which they are engaged.

### Mathematical thinking

> For ideas taught in a lesson to be understood deeply, it is important that they are not just 'received' passively, but worked on by the learner. They need to be thought about, reasoned with and discussed.

Throughout the lesson, 'doing' the maths is not enough; the teacher expects that the children will make sense of the mathematics at every step.

An important element of this is the expectation that the children represent their thinking through talk, not just in phrases or words but in full sentences for everyone to understand. Language can be challenging for children and a barrier for many. This is often due to limited experiences so it is important that in the mathematics classroom children are immersed in rich language experiences. This is much more than having vocabulary lists; it is about children structuring sentences that reflect their thinking and represent the mathematics they have encountered. Talking in full sentences can shift the expectation for the child from thinking that they are talking just for the teacher to understand to thinking that they are talking for everyone to understand, including themselves. Structuring sentences requires a greater depth of understanding of the mathematics being explored and often reveals where children are not clear or secure in their understanding.

In the Y1 lesson the teacher uses several strategies to support the talk, including:

- Praising full sentences. Towards the end of ***Warm up and review*** one child responds to the question 'What is the next odd number after twenty-nine and why?' with 'Thirty-one is the next odd number after twenty-nine because it's two more' and is told that it is a 'beautiful answer'.
- Providing sentence starters. In the clip ***Subtraction as take away*** the teacher provides the sentence starter 'The five counters…' for a child who has responded but not in a full sentence.
- Choral talking. At the end of ***Part-part-whole*** all the children repeat a sentence modelled by the teacher
- Modelling different sentences using the same language. In ***Subtraction as difference*** the teacher says 'So three is the difference. There is a difference of three'
- Modelling that the language order does not always match symbol order. In ***Subtraction as difference*** the teacher writes up 5 – 3 = 2 as she says 'The difference between five and three is two'.

**Representation and structure**

> In lessons, representations need to pull out the concepts being taught, and in particular key difficulty points. Representations expose mathematical structure.

During the lesson the teacher makes decisions about how to use representation to support understanding of the mathematics she wants the children to explore and learn about. Some points to notice are:

- The teacher chooses to use counters and the 'part-part-whole' model to physically represent the maths during the session. At every step the children are expected to make sense of the image and explain their understanding, often relating the image back to the context.
- At the start the children represent the five cars with five counters. The teacher notices and acknowledges there are different ways to show five with the counters, valuing them all, but she makes the decision to focus on one. This is deliberate as it is going to provide the structure that will support the learning for which she has planned.
- The teacher chooses one representation to focus on for taking away, keeping the five counters together. This allows the children to see the relationship between three, two and five. It connects taking away with a known fact linked to understanding the cardinal value of five; by keeping all five counters present and changing the image by turning some of the counters over, the fact

3 + 2 = 5 is physically displayed. Making the connection to addition encourages the children to use what they know and to think about the relationship between the three numbers rather than 'doing' a calculation. This is a key part of fluency.

**Fluency**

In considering the concept of fluency, it's important to remember that it (fluency) demands more of learners than memorisation of a single procedure or collection of facts. Attention to structure and relationships between mathematical facts can support their memorisation. Memorisation is important as it avoids cognitive overload and enables pupils to think about concepts.

The numbers chosen for the lesson are chosen to support developing fluency. In the clip ***Maths subject leader Clare Christie talks about her Y1 lesson*** the teacher explains how there is a focus in Y1 in the school on fluency with facts involving single digits. She then explains how this fluency with addition and subtraction facts with small numbers allows the children to generalise and recognise the same relationship with larger numbers. She gives the example of a child using that they know 2 + 3 = 5 to know that 200 + 300 = 500 and 2 000 000 + 3 000 000 = 5 000 000; generalising relationships between numbers is a key element of fluency.

This is seen in the lesson at the end of ***Moving pupils on to independent work*** when a child explains that 100 and 102 have a difference of two and that 602 and 604 have a difference of two; they are using what they know about single digit numbers to generate pairs of three digit numbers with a difference of two.

**Variation**

The lesson includes conceptual variation by exploring understanding of subtraction as both take away and difference. This is supported by representing the contexts in the same way, first with counters and then the part-part-whole model, so that the children can see how the two very different structures involve the same relationship between the three numbers and therefore can be represented with the same symbols: 5 – 3 = 2

The ***Moving pupils on to independent work*** clip shows that the practice also contains variation, with the questions purposefully varied each time in order to ensure that the children are thinking about the mathematics and not just following a pattern. The variation in the questions is as follows:

- Pictures set out horizontally, biggest number first and in line from the left hand side, which mirrors the experience with the counters during the lesson - find the difference
- Pictures in line from the right hand side - find the difference
- Pictures in line vertically with the smallest number first - find the difference
- Pictures randomly mixed together – find the difference
- Given a difference of two – find pairs of numbers that might have this difference.

[1] Askew 2009 Private Talk Public Conversation

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## Aspects of…

$\frac{1}{2}$

To support development of conceptual understanding pupils need to experience a concept in a variety of ways so that they can identify the essential structure, what is generalisable. Like all numbers, half can be a number with a specific place in the number system or an operator/multiplier, where the outcome is not necessarily one half. For example, the number five sits midway between four and six, but buying five packets of biscuits with ten biscuits in each packet results in thirty biscuits rather than five biscuits; similarly one half sits midway between zero and one but half of a packet of ten biscuits is five biscuits.

Developing understanding of one half can be supported as follows:

- **Vary the context for halving.** Find one half of different things in different contexts without using labelled measuring equipment. For example: a piece of paper, a set of small world animals, a length of string, a bag of rice. Ask 'What's the same about finding half and what's different?'

- **Count halves.** Use a context which is familiar to the children; for example apples from the school fruit. Start by counting 'one half, two halves, three halves, four halves…' and model the symbols $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, …alongside the count. Use this to immerse the children in the language and symbols. If appropriate this can lead to discussion of equivalence followed by counting 'one half, one, one and a half, two…'recording ½, 1, 1½, 2 …alongside.

- **Doubling and halving - part one.** Link halving to ÷ 2 and understand it as the inverse of × 2; the idea of 'doing and undoing'. For example, there are children on the school field and half of them go inside for lunch. Then children who have had lunch come out and the number on the field doubles. Use Cuisenaire rods or the bar model to represent the relationship:

  Numbers can be added to the context, for example at the start of the story there are ten children on the field, one hundred children, fifty children etc. with symbols recorded to reflect what happens: $10 \div 2 \times 2 = 10$, $100 \div 2 \times 2 = 100$, $50 \div 2 \times 2 = 50$ etc.

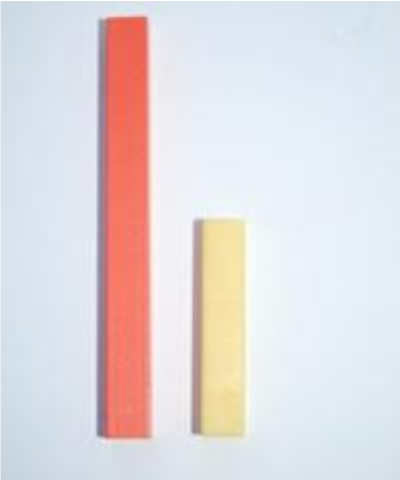
Explore this in the other direction, i.e. doubling and then halving. For example, one baking tray holds twelve cakes:

I needed double this number of cakes so made two trays of cakes:

I took them to school and half of the cakes were eaten, leaving me with twelve cakes:

$12 \times 2 \div 2 = 12$

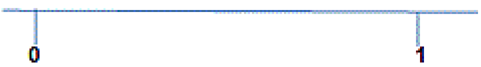
- **Doubling and halving - part two.** Understand half in the context of comparison, exploring the connection between halving and doubling as part of scaling. Use contexts which are familiar to the children and model with Cuisenaire rods or the bar model. For example: ‘My daughter is half my height. I am double my daughter’s height’:



Explore in different contexts, using different measures, for example the weight of a Cocker Spaniel compared to the weight of a Jack Russell, the ages of a child and their sibling, the time taken to travel to the swimming pool by car and by bicycle etc.

- **How big is a half?** Explore halves that are not equal, for example use bars of chocolate of different sizes and consider 'How big is half of a bar of chocolate? How can half be different sizes?'

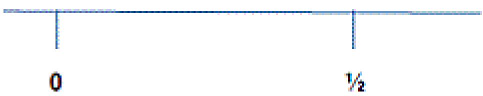
- **Half as a number.** Place the number ½ on different number lines, for example:




Ask: Why does half go in the middle of this line?


Ask: Why does half **not** go in the middle of this line?

Use a number line with half marked on it and invite the children to mark other numbers on the line and explain their thinking:





## Seen and Heard

Seen and Heard *shines a light, via photographs and conversations from classrooms, on a specific example of the mathematics learning experience, the aim being to stimulate thought and questions about how you would react to similar events in your own classroom*

A group of Y1 children are exploring halving. One child is given a ball of modelling clay to halve and the halves are shown below:

The child explained they had made two sausages. The teacher said 'I halved my ball of clay by making two sausages as well. Are you happy with what I did?'

The child agreed that the teacher had split the ball in half.

The same child was given a bag of counters and asked to show half of the counters. The child did this:

- What does this child seem to understanding about halving?
- What would you ask the child next?
- What understanding would you want them to develop?
- What other contexts would be useful to explore?