Primary
& Early Years
Magazine 88

National Centre
for Excellence in the
Teaching of Mathematics

Welcome to another issue of our Primary Magazine, which has now been serving primary teachers for 88 issues with a varied collection of articles related to maths education and mathematics professional development - all of which are available in the Primary Magazine Archive.

Contents

In Digging Deeper each month we will explore an element of mathematics teaching linked to current developments and research: in this issue we look at ideas around learning multiplication facts.

Aspects of… provides a number of bitesize ideas related to a specific element of mathematics; this month: aspects of place value in the context of decimal numbers.

Seen and Heard provides a specific example of a child's response to mathematics in a classroom to stimulate thinking and provoke questions about how you would react to similar events in your own classroom. In this issue we consider number systems including Roman numerals.

If you have a photograph, or an account of a classroom conversation, that might stimulate similar thought, please email it to us. If we publish your suggestion, we'll put a £20 voucher in the post.

Next month we will have articles focused on journal-writing in mathematics, and marking. But first, as always, we have a News section, bringing news from the NCETM and beyond to keep you up to date with the fast-changing world of mathematics education.

News

The latest issue of Bespoke, the magazine of the Maths Hubs programme, has articles on Teacher Research Groups and one primary teacher's experience of teaching for mastery in a mixed-age class. This gives another perspective, following on from an article in Issue 86 of the Primary Magazine, which focused on the report from a Maths Hub Work Group into the same area. These can also be found in a new section of the mastery area of the NCETM website, Mastery case studies: aspects of teaching for mastery under the spotlight, which includes a case study, Meeting the needs of all without ability setting.

Have you seen the guidance from the NCETM, which came out earlier this term, on marking and assessment, and which encourages primary teachers to spend less time on marking their pupils' maths books? The paper also reflects expectations from Ofsted. Next month's Primary Magazine will include a look at 'Aspects of marking'.

Over the past year or so, there have been scores of events, led and co-led by the NCETM, where teachers have come to hear more about teaching for mastery, and compare notes with other teachers. A new short video reporting on one such event, organised jointly by the East Cheshire Association of Primary Heads (ECAPH) and the North West Two Maths Hub, is now available.

The DfE set up teacher review groups to make recommendations about reducing workload in three areas: marking, planning and data management. The three groups published their reports on 26 March 2016, and each contains recommendations relevant to subject leaders and teachers of primary mathematics.

The financial education charity pfeg is holding a conference to share the findings of a year-long project to assess the impact on engagement and attainment of primary pupils, as a result of an increased focus on financial education in 12 primary schools across London. The event, which is free to attend, is in London on 11 July. Speakers will include the lead teachers involved in the project. You can book your place online.

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Digging Deeper

Learning Multiplication Facts

The aims of the National Curriculum for Mathematics include a focus on fluency. There is a danger that fluency can be interpreted in a very narrow way, equated with speed and memorisation without an understanding of relationships, but this is not the intention, as the aim clearly states "…that pupils develop **conceptual understanding** and the ability to **recall and apply** knowledge…" (National Curriculum 2014).

Russell (2000) suggests that fluency "rests on a well-built mathematical foundation with three parts:

- an understanding of the meaning of the operations and their relationships to each other - for example, the inverse relationship between multiplication and division;
- the knowledge of a large repertoire of number relationships, including the addition and multiplication "facts" as well as other relationships, such as how 4×5 is related to 4×50;
- a thorough understanding of the base ten number system, how numbers are structured in this system, and how the place value system of numbers behaves in different operations - for example, that $24 + 10 = 34$ or $24 \times 10 = 240$."

This focus on understanding relationships is important to consider when thinking about the learning of multiplication (and related division) facts; pupils will benefit most if the learning of multiplication facts is embedded within learning **about** multiplication. This will involve exploring multiplication in relevant contexts, representing the situations with mathematical images which expose structure and relationships, describing these using mathematical language and connecting all of these to a symbolic representation.

There are various strategies and approaches that can be used; the important starting point in all cases is to identify what the children already know and understand and build on this. The following questions are intended as prompts to support thinking about how to approach the teaching of multiplication facts as part of the teaching of multiplicative reasoning:

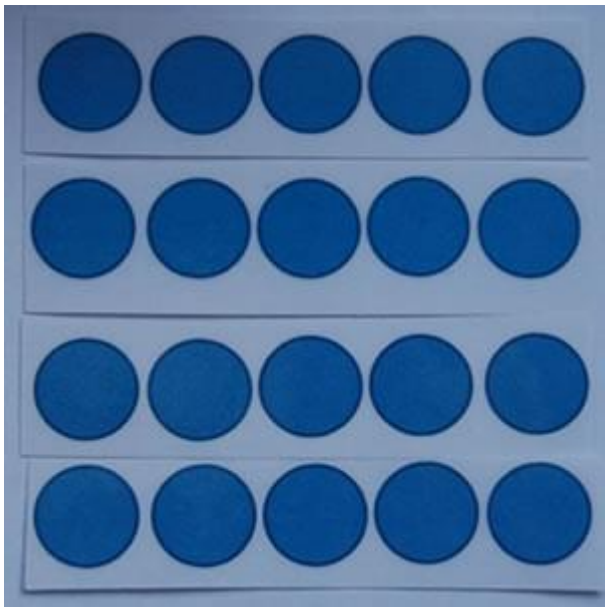
- Do the children understand what '$\times 1$' means and can they use this understanding to multiply any number by one? This can be modelled using a Cuisenaire rod (or the bar model) alongside the language of multiplication and the symbols of multiplication, which will allow for generalisation.

For example: "I have one of the yellow rods. I have the yellow rod one time. Yellow rod multiplied by one equals yellow rod. If the yellow rod is eight, I have eight one time. Eight multiplied by one equals eight. If the rod is… " matched to 'Yellow $\times 1$ = Yellow rod' and '$8 \times 1 = 8$'
Contexts for multiplying by one could be linked to single or unique events or situations. For example: linking $5 \times 1 = 5$ to 'in my purse all I have is one five pound note'; linking $80 \times 1 = 80$ to 'tea bags come in boxes of eighty and we have one box'; and linking $250 \times 1 = 250$ to 'bags of raisins weigh 250g and there is one bag.'

- Do the children understand what '× 0' means and can they use this understanding to multiply any number by 0?
- Do the children understand that doubling is the same as multiplying by two (× 2)? Explore doubling in different contexts and model using Cuisenaire rods (or the bar model).

For example: "If I double a purple rod I have two purple rods or I have a brown rod. If the purple rod is ten, then the brown rod is double this, it is twenty; 10 × 2 = 20". Cuisenaire rods and the bar model can be used to show how repeated addition and scaling are both part of multiplication.

- Do the children understand how repeated addition can be represented as a multiplication? This can be modelled by making a visual link between counting in steps the children are familiar with and the building up of an array using rows of dots (rather than individual counters). For example, building up the array below, row by row saying "There are five dots in a row. We have five dots one time, two times, three times, four times. We have five dots, four times, five multiplied by four (5 × 4). We have five, ten, fifteen, twenty dots. Five multiplied by four equals twenty." Contexts for building up arrays include planting vegetables in rows, setting out chairs in the hall and chocolates in boxes.



- Do the children understand the commutative property of multiplication? This can be modelled and explored with rectangular arrays, cut from squared paper, or in a context; it is important the children can explain how both multiplications are represented by the same array. For example, for the array below, discuss how this shows 5 × 4 - 'There are five children in each row and there are four rows' - and 4 × 5 - 'The children are lined up behind each other. There are four children in each line and there are five lines.'

- Which of the multiplication facts for twos, fives and tens do the children know/recall? From the points above, they should know that they know: 2×0, 2×1, 2×2, 5×0, 5×1, 5×2, 10×0, 10×1, 10×2. Support them to generalise about tens by focusing on the pattern of the calculation linked to an image (for example base ten or Numicon ten plates).
- Do the children know how to use what they know to work out other facts, including:
    - doubling and halving – connecting the five and ten times tables, connecting the two, four and eight times tables, connecting the three and six times tables, connecting multiplying by ten with multiplying by five, connecting multiplying by two with multiplying by four and eight and connecting multiplying by three with multiplying by six
    - adjusting (one more or one less) – connecting the ten times table with the nine times table (use the array to model) and other known multiplication facts with nearby facts; for example using 6×6 for 6×7.
- Special cases - are the children aware of different ways other children recall any of the facts including special cases, such as remembering 7×8 by thinking '5, 6, 7, 8…56 = 7×8'.

One teacher who has done some in-depth work in this area, as part of a Master's degree, is Katie Crozier, a primary mathematics teaching for mastery specialist teacher in the Cambridge Maths Hub. She says:

> *When asked about strategies used to recall times table facts I found two responses were common: a) I just know it; and b) I count up in ___s. With further questioning, I started to realise that many children didn't appear to have a strong visual representation of the structure of multiplication in its form of repeated addition. The times table facts were isolated points lined up in a sea of fog where the structure and connections lay hidden. I spent time devising a representation that clearly exposed the three parts to multiplication...*

Read more about Katie's research and her representation to support learning of multiplication facts, and you can also view the NCETM video on teaching multiplication.

Next time in Digging Deeper we'll explore journal-writing in mathematics.

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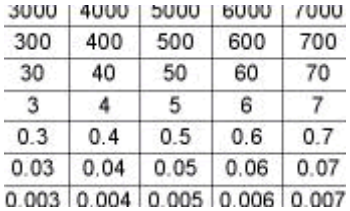

3000	4000	5000	6000	7000
300	400	500	600	700
30	40	50	60	70
3	4	5	6	7
0.3	0.4	0.5	0.6	0.7
0.03	0.04	0.05	0.06	0.07
0.003	0.004	0.005	0.006	0.007

Aspects of…

Place value in the context of decimal numbers

(Decimal numbers are the focus of this article, but many of the ideas are also relevant for whole numbers)

Children can find themselves manipulating and calculating with decimal numbers without a deep understanding of what the numbers represent. Exploring this in different ways will enable the children to make decisions when calculating with decimals, make sense of their answers and recognise when solutions are incorrect. Ways to explore the place value of decimal numbers include:

1 Where are decimals?

Explore decimals in a context which the children can directly experience, starting with one of something which will be divided up, so that they make sense of the relationship between the different parts of a decimal number. Measures are useful for this; for example, one kilogram of rice (shared between ten people) or one litre of water (used to water ten seedlings).

2 Decimal images

Use a mathematical image which can model the relationship between ones, tenths, hundredths etc. Different images will expose different bits of understanding:

- base ten can be used, with the large cube representing one. This allows children to see the relative size of ones, tenths, hundredths and thousandths and to understand, for example, that nine hundredths is smaller than one tenth. Using base ten in this way fits with the contexts suggested above; children should understand that one kilogram of rice is the equivalent to one thousand grams of rice, and so the block they are familiar with as representing one thousand also represents one in this context
- decimal place value counters can be used to create a visual image of the parts of a decimal number where the children already have an understanding of the relative size of the parts. This visual image will support aspects of place value explored below
- place value chart can be used to identify symbolically the different elements that make up a decimal number. Sometimes children can represent the number with a resource such as base ten or place value counters and can record a number represented with a resource, without fully understanding the value of the different parts of the number separately (see Seen and Heard in Issue 87).

3 Adding the parts

Write an addition to match how the number has been made using one of the resources above. This will show if the children understand the value of each part of the number; for example, 30 + 6 + 0.2 + 0.07 = 36.27.

4 From place value to subtraction

Identify numbers which can easily be subtracted from a decimal number. For example, explaining why it is easy to subtract 6, 0.2, 0.27, 36 etc. from 36.27.

5 Number facts combined with place value to add

Identify numbers which can easily be added to a decimal number. For example, explaining why it is easy to add 0.03, 4, 10, 0.8 etc. to 36.27.

6 A zero or not a zero, that is the question

Understand when zeros are necessary and when they are not necessary in a decimal number; children should be expected to not include unnecessary zeros when they are recording decimal numbers as solutions to problems etc. For example, 36.27 + 0.03 = 36.3 whilst 36.27 + 0.8 = 37.07.

7 Keeping decimals in order

Order a set of decimal numbers, explaining how they know. Use numbers which expose possible misconceptions about place value; for example, 24.03, 24.30, 24.3, 24.19 and 24.13. Where misconceptions occur, use the images above to model the numbers and explore the misconceptions.

8 Putting decimals in their place

Place decimal numbers on a number line marked with whole numbers, expecting the children to explain their decisions, and identify other decimal numbers lying between two decimals. For example, place 36.4 and 36.5 on a number line with 36 and 37 marked on it, explaining that 36.5 is halfway because 0.5 is half of one and 36.4 is smaller than this, and explain that numbers between 36.4 and 36.5 all start with 36.4 suggesting several numbers with different decimal places (36.457, 36.499999, 36.41 etc).

Seen and Heard

Seen and Heard *shines a light, via photographs and conversations from classrooms, on a specific example of the mathematics learning experience, the aim being to stimulate thought and questions about how you would react to similar events in your own classroom*

At the end of a maths lesson in Y5, the children were sharing how they had written the year of their birth in Roman numerals. One child said: "I have written MMVI because M is one thousand so MM is two thousand and VI is six so this is two thousand and six."

Another child in the class immediately said: "I have something I want to ask. Did you leave spaces where the zeros should be? I think you need to do this."

- Why did this child ask this question?
- What is their understanding of the base ten number system?
- What is their understanding of Roman numerals?
- What would you do next?
- How could you use this difference to support a deeper understanding of the base ten number system?
- What other differences between base ten and Roman numerals might you explore and how would you support understanding of these differences?
- How can you support children to ask questions of each other, as shown here?

With thanks to Sarah Moores from Northam Federation, Devon, for sharing this example with us.

If you have a thought-inducing picture, please send a copy (ideally, about 1-2Mb) to us at info@ncetm.org.uk with 'Primary Magazine: Seen and Heard feature' in the email subject line. Include a note of where and when it was taken, and any comments on it you may have. If your picture is published, we'll send you a £20 voucher.

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