

Multiplicative Reasoning

This document is part of a set that forms the subject knowledge content audit for Key Stage 1 and Key Stage 2 maths. Each document contains: audit questions with tick boxes that you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications; explanations; and further support links. At the end of each document, there is space to type notes to capture your learning and implications for practice. The document can then be saved for your records.

Question 5

How confident are you that you understand and can support children to recognise distributive law in multiplication?

1

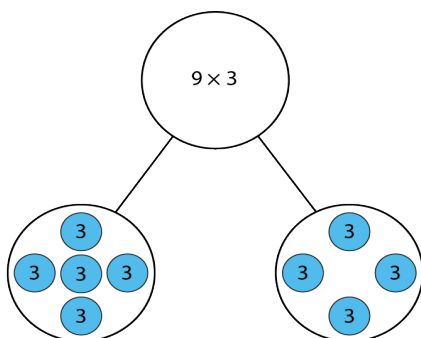
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3

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How would you respond ...?

a. How can this diagram support children to understand the distributive law?



b. How many different strategies, using the distributive law, can you find for the calculation?

You may use representations or equations to demonstrate your strategies.

Danior spends four hours each week practising the piano. How many hours of practice does he do in 16 weeks?

Responses

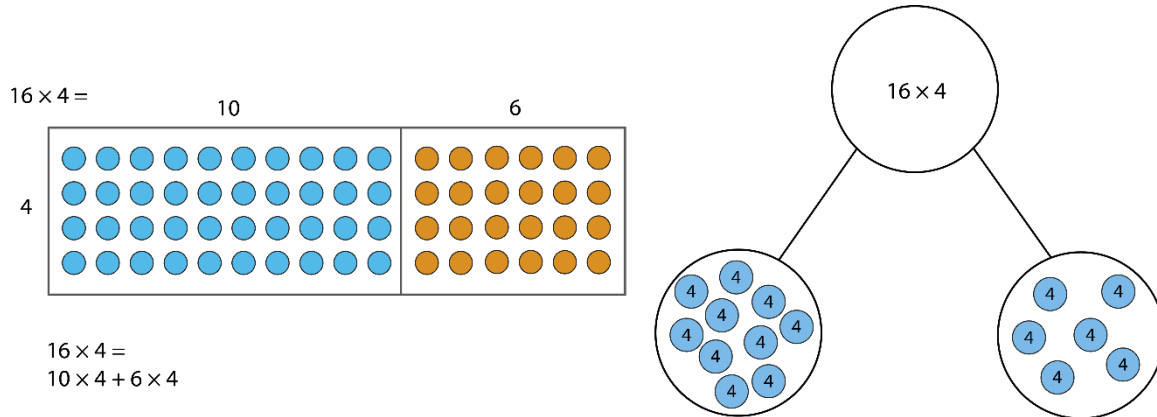
Note your responses to the questions here before you engage with the rest of this section:

Did you notice that...?

a. $9 = 5 + 4$
 $3 \times 9 = 3 \times 5 + 3 \times 4$
 $= 15 + 12$
 $= 27$

'Nine is equal to five plus four, so three times nine is equal to three times five plus three times four.'

b. Distributive law could be applied to the calculation 16×4 in the following ways:



This is not an exhaustive list but some suggested strategies. You may have used a different strategy.

Applying distributive law

This section will explore the distributive structures of multiplication. It will firstly focus on distribution as the partitioning of factors to derive related known facts, before considering a range of strategies for deriving multiplication facts beyond those known.

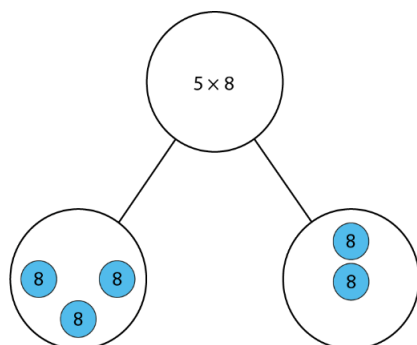
Children have already used the distributive law informally, in the following contexts:

For each times table learnt, children have noticed and applied the fact that adjacent multiples of a number have a difference equal to that number (e.g. $5 \times 2 = 4 \times 2 + 2$ and $4 \times 2 = 5 \times 2 - 2$).

For some times tables (six, seven and nine), children have learnt how to use arrays to derive 'new' multiplication facts from known facts (for example, relating 7×7 to 5×7 and 2×7).

Building on the first point, children can formalise and extend this by understanding how known multiplication facts can be used to derive other products by adding or subtracting with one group or several groups: in other words, by partitioning the number of groups.

In this example, the five groups of eight have been partitioned into three groups and two groups. This could be expressed in the following ways.



$$\begin{aligned} 5 &= 3 + 2 \\ 5 \times 8 &= 3 \times 8 + 2 \times 8 \\ &= 24 + 16 \\ &= 40 \end{aligned}$$

'Five is equal to three plus two, so five times eight is equal to three times eight plus two times eight.'

$$\begin{aligned} 3 &= 5 - 2 \\ 3 \times 8 &= 5 \times 8 - 2 \times 8 \\ &= 40 - 16 \\ &= 24 \end{aligned}$$

'Three is equal to five minus two, so three times eight is equal to five times eight minus two times eight.'

To deepen children's understanding of partitioning the number of groups and writing associated equations, they should have a range of opportunities to practise this in different contexts.

Felicity's summer holiday is eight weeks long. How many days is this?

- 'Interpret the problem: 8×7 .'
- 'Apply the distributive law: $7 = 5 + 2$.'

$$\begin{aligned} 8 \times 7 &= 8 \times 5 + 8 \times 2 \\ &= 40 + 16 \\ &= 56 \end{aligned}$$

- 'Interpret the answer: Felicity's holiday is 56 days long.'

Two part problems with common factors can be efficiently solved by applying the distributive law as only one calculation is required rather than two. For example $9 \times 4 + 9 \times 5$ can be more efficiently solved as 9×9 . Apply this principle to the question, 'Last week, Sam ran five kilometres each day on Monday, Tuesday and Wednesday. This week, she ran five kilometres on both Monday and Tuesday. How far has she run altogether?'

The second way in which distributive law can be applied is to derive multiplication facts beyond known times tables.

In this example, an array representation leading to a grid method is used. This is to expose the structure of the mathematics, not to be used as a tool for calculation. It demonstrates how distributive law can be applied when the multiplication extends beyond what is known.

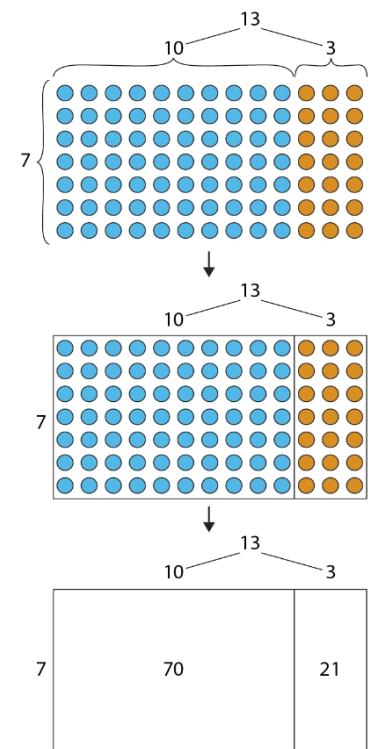
Children should be reminded of the 'ten and a bit' structure of teen numbers, to support partitioning teen numbers into a ten and ones.

When children are confident of the structure, remove the array or grid and encourage children to use a concise set of equations.

$$\begin{aligned} 7 \times 13 &= 7 \times 10 + 7 \times 3 \\ &= 70 + 21 \\ &= 91 \end{aligned}$$

There are many ways children could apply the distributive law, not just by partitioning into tens and ones. For example, for higher teen numbers children could multiply by 20 and then subtract. Alternatively, they could partition even numbers into two equal parts using halving and doubling strategies to find the product.

The idea is to provide children with a range of strategies they could apply, promoting flexibility in calculation and encouraging them to select the most efficient method.



Common errors in this area may include:

- children not accurately partitioning the factors
- children having no known multiplication facts to draw upon to support distributive law.

What to look for

Can a child:

- apply distributive law to a range of contexts, understanding how to partition factors for efficient calculation?
- apply a suitable strategy in a given context, reasoning their choice?

Links to supporting materials:

NCETM Primary Professional Development materials, Spine 2: Multiplication and Division:

- Topic 2.10: Connecting multiplication and division, and the distributive law

Notes:

Key learning from support material and self-study:

What I will focus on developing in my classroom practice: