Secondary Magazine Issue 67

National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 67 of the Secondary Magazine.

## Contents



*Contributors to this issue include: Barbara Jaworski, Mary Pardoe, Peter Ransom and Heather Scott.*




## From the editor

Welcome to this issue of the NCETM Secondary Magazine. As we look forward to an exciting and challenging new school year, in 5 things to do we have some suggestions and links to help you plan your professional development during this year.

You will find starting points for a variety of classroom activities in the Focus on learning with pegboards, in An Idea for the classroom – polygon puzzles, and in the profile of Barbara Jaworski, which you can reach via The interview.

If you missed the 'special' Issue 66 while you were away, you might like to look in it at what some professional mathematicians have to say about the nature of mathematics and mathematical activity.

At this time of the year, as we consider trying out new activities and ways of working, and we each think about how we plan to continue our professional development this year, it may be helpful to reflect on the following extract from Starting Points about the role of the teacher:

> *What is the role of the teacher?*
> *How can he create the conditions in which creative and independent work can take place?*
>
> *The heart of the matter lies in the nature of the relation between the teacher and the pupil. Ultimately in the relation of the teacher to himself. Teaching soon becomes a moral voyage. But one in which the traveler has to choose his own map.*
>
> *'The most important lesson-preparation is to prepare oneself'.*

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# It's in the News!
# Big Burger

The fortnightly *It's in the News!* resources explore a range of mathematical themes in a topical context. The resource is not intended to be a set of instructions but as a framework which you can personalise to fit your classroom and your learners.

This fortnight's resource explores the idea that television advertising exaggerates in order to convince us that the product is bigger and better than it is in reality. A recent advertisement for a chicken burger was banned by the Advertising Standards Agency because, in the three real burgers it bought, the thickness, the quantity of additional fillings and the overall height of the product was "considerably less" than that advertised."

This resource offers a context for students to access the data handling cycle by exploring how they might investigate such a claim and offers an opportunity for students to model a situation using both data handling and proportional reasoning.

This resource is not year group specific and so will need to be read through and possibly adapted before use. The way in which you choose to use the resource will enable your learners to access some of the key processes from the Key Stage 3 Programme of Study.

Download this *It's in the News!* resource - in PowerPoint format



## The Interview

**Name:** Barbara Jaworski

Barbara is Professor in Mathematics Education at Loughborough University. She has recently received a prestigious invitation to take up, for three months next year, the F. C. Donders Chair, in the Freudental Institute at the University of Utrecht in the Netherlands.

Many of you will have been inspired by Barbara in your contacts with her in her present position, or in the past when she was a head of mathematics in a secondary school, during her time as a lecturer at the Open University and then at the University of Birmingham, while she was Reader in Mathematics Education at the University of Oxford, or as Professor in Mathematics Education at the University of Agder in Norway.

You may also know Barbara through her former roles as President of the European Society for Research in Mathematics Education, as the Editor in Chief of the Journal of Mathematics Teacher Education, or as Chair of the British Society for Research into Learning Mathematics.

It is also likely that you are among the thousands of mathematics teachers and educators whose professional development has been greatly enhanced by reading her many books and other publications about mathematics teaching.

Possibly your present concerns are similar to those that Barbara described when in 1994 she was reflecting on her own experiences as a classroom mathematics teacher:

> "Teaching mathematics was difficult, because students found learning mathematics difficult. I started to question what it actually meant to learn mathematics. Explanations or exposition seemed very limited in terms of their effect on students' learning, so I found myself seeking alternative approaches to teaching mathematical topics, especially for students who were not inclined to like or be successful with mathematics."

Barbara generously responded to our invitation to give us an interview for this issue by writing a fascinating account of her experiences up to now, and including one of her own very rich classroom activities. You can read the whole article on the National Centre portal.

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# Focus on…learning with pegboards

Coloured pegs and pegboards are useful resources that are often overlooked.

Learning about numerical and algebraic ideas and relationships can be stimulated and aided in an enjoyable way by creating and exploring arrangements of coloured pegs placed in pegboards. This is true for learners of all ages and at all stages, from primary through secondary to university level.

Pegs may be placed so as to:

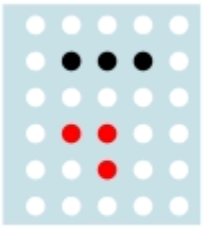
- create and continue patterns
- satisfy particular conditions
- modify arrangements in particular ways.

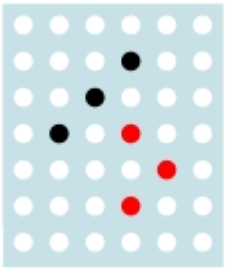
Coloured pegs and pegboards are attractive materials. When using them learners are able to set their own rules and conditions, experiment freely, make and test conjectures, generalise and reason. It is easy to shift 'to and fro' between arranging and thinking about pegs on pegboards and making and studying coloured marks on dotty paper. Learners can quickly and easily record 'transient' peg arrangements on dotty paper.

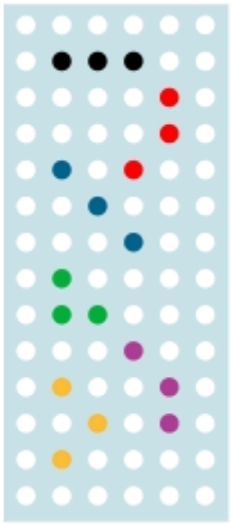
The possibilities for explorations and activities are endless – as are the kinds of examples and phenomena that learners may create and investigate. In this *Focus on…* only a few ideas are briefly suggested.

How many different arrangements of three pegs placed next to each other are possible?

The arrangements that are possible depend on our decisions about the meaning of 'next to' and our criteria for sameness. What different decisions lead to each of these sets of possibilities?

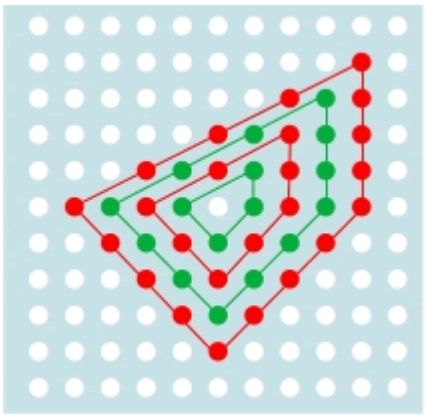






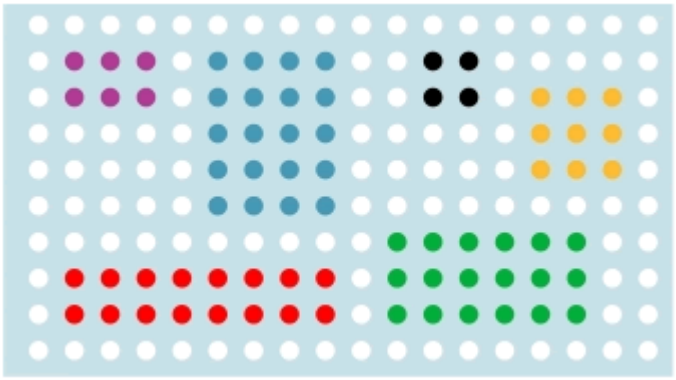
Under each of these three different sets of assumptions what will be the possible arrangements of four pegs, of five pegs…?

Make a shape with some pegs, and then place pegs in all the holes next to its boundary. Continue to 'grow' new shapes in this way. For example:



Count the pegs on the boundary of the original, innermost, shape (4). Count the pegs that were added at the next stage (8). Continue to count the pegs added at each stage. What is happening numerically? Describe the number sequence that is being generated?

Make 'filled in' rectangles:



Ask yourself questions about composite and prime numbers.

Explore some number sequences that you can generate, such as the sequence of squares, 1, 4, 9, 16…

What happens when you explore 'tilted' rectangles?

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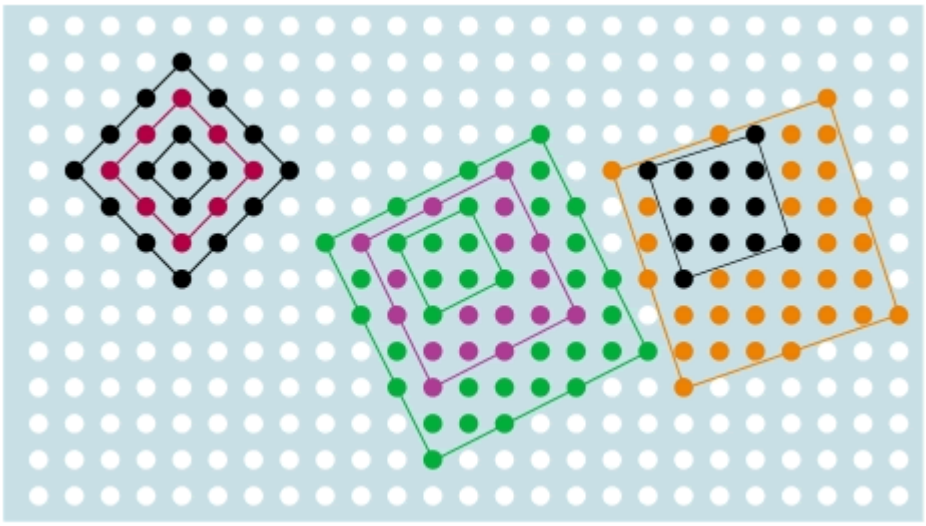
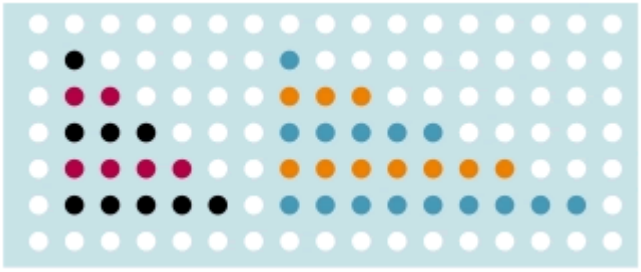


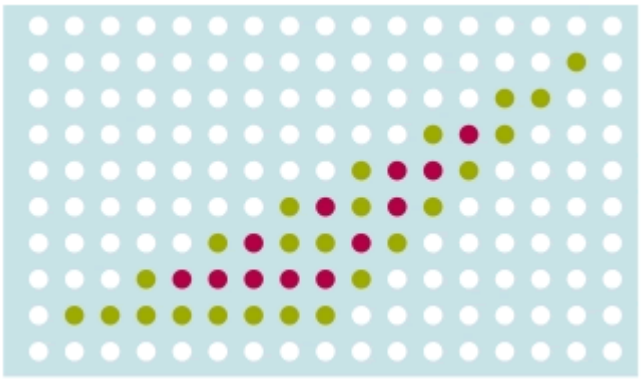
Look for number patterns, and generalise!

Make triangular arrangements of pegs:



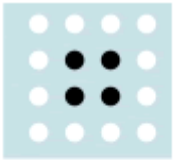
If we assume that the first triangle in the sequence of triangles of the left-hand kind in the diagram above consists of one peg, the number of pegs in the *n*th triangle is the sum of the first *n* whole numbers. What is it in the sequence of triangles of the right-hand kind? How can this situation be generalised further?

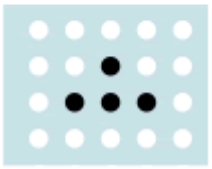
Suppose *any* arrangement of three pegs may be the first triangular arrangement in a sequence of peg triangles…



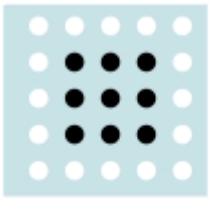
...what happens in various different cases?

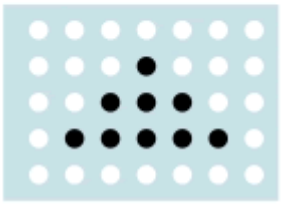
How many moves are required to change a square arrangement of four pegs...





...to this triangular arrangement?





And how many moves to change this...

...to this?

Investigate number sequences generated by continuing in this way.

The black pegs in this board make a pattern of parallelograms:

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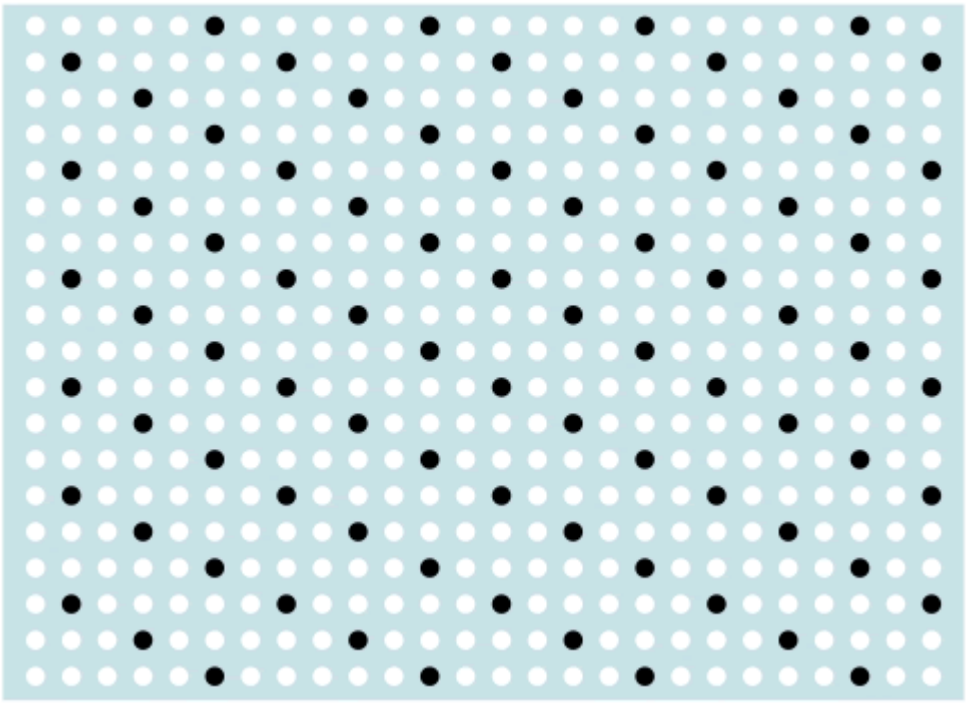
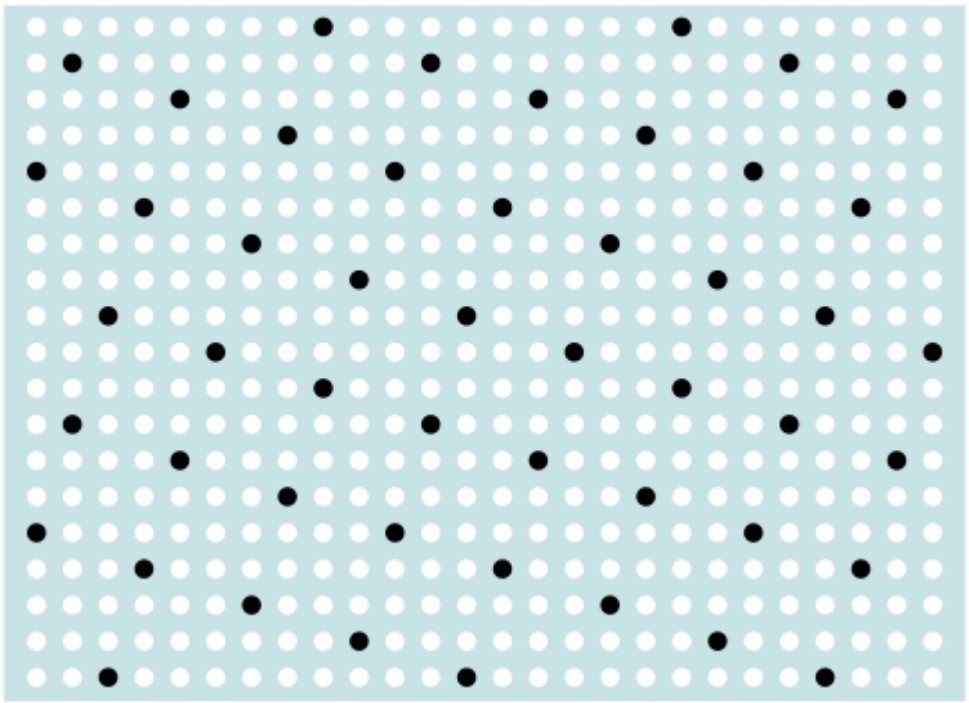


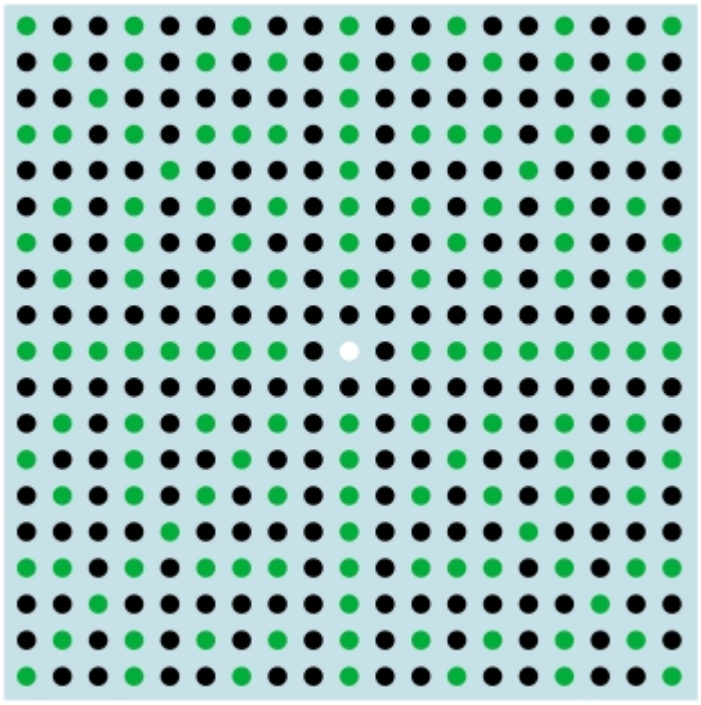
Each row has a peg in every ***sixth*** hole, and every row slips ***two*** places to the right.

If, instead, we put a peg in every ***tenth*** hole of each row, and make every row slip ***three*** places to the right, we create a pattern of squares.

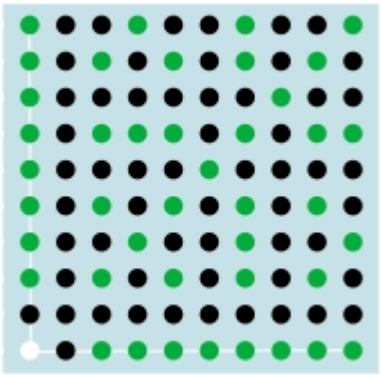


Investigate in general placing a peg in every *n*th hole of each row and slipping each row *m* places to the right. For which values of *n* and *m* do we create squares? What has Pythagoras' Theorem got to do with this?

Place a special peg (a white peg in this diagram) in a hole. Working outwards from the white peg, place a black peg in every hole that is 'visible' from the white peg – a hole is 'visible' from the white peg if there is no peg already on the straight line joining it to the white peg. Place a green peg in every hole that is 'obscured':



As the pattern is symmetrical, we might look at just one quadrant:



We find that, if we regard the white peg as the origin, the coordinates $(a, b)$ of every black peg, and of no green peg, are relatively prime – and so the fraction $b/a$ that we could associate with each black peg is a fraction in its lowest terms. By going further with these ideas we can discover Farey Sequences in the coordinates of the peg positions.

The Farey Sequence, $F_n$, is the sequence in increasing size order of all the irreducible fractions with denominator no greater than n that are no greater than one.

The arrangement of 'visible' pegs can help learners explore the number of terms in each Farey sequence. Generalising leads to Euler's Totient Function.

When using pegs and pegboards to help you explore ideas and situations printed sheets representing arrangements of pegboard holes are useful.


The ATM has re-issued, as a PDF, a frequently requested booklet in which starting points for some rich activities using pegboards are described – Activities for children of all ages with PEGBOARDS.

Seventeen activities are introduced.


You will also find some excellent suggestions in Starting Points by Banwell, Saunders and Tahta.

After an extensive search I can find on the web only one – rather 'babyish looking' – free interactive pegboard. Click on the black and white peg dog!

You will find a better interactive pegboard in Oral and Mental Starters Y7, ISBN 978 0 340 88360 0, which was originally published by Hodder Education. It is number 22 on the disc, which unfortunately is not one of the three interactive environments in this free Y7 interactive demonstration.

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# An idea for the classroom – polygon puzzles

If you are thinking about helping students to learn deeply about a particular topic it is helpful to plan a range of activities that continually challenge their thinking and surprise their minds. The topics of shape and angle properties are two that are rich with opportunities to use and devise tasks that enable students to learn in greater depth. Some situations that students can explore are described here – for you to try out and adapt, and perhaps to inspire you to develop your own ideas.

Start with some activities in which students are motivated to discover as much as they can about properties of regular polygons. Make a large display-chart on a classroom wall to record all the information that students can deduce or find out. Give students a small chart to aid them in looking for information systematically and by themselves.

Students who are acting mathematically will try to find facts, observe patterns, look for reasons, and then generalise where possible. Use the following questions as prompts to support their learning:

- what are the names of the regular polygons?
- how many triangles can be put together to form each regular polygon?
- what patterns do you notice?
- can you find a generalisation about…?
- can you explain why the generalisation must be true?
- if you have found the exterior angle from knowing the interior angle, is there a different way of finding the exterior angle?
- if you draw diagonals in your polygon is there any way of predicting how many diagonals there will be?
- remember that some questions may not be easily answered – it may take you years to discover a generalisation about the number of regions in a polygon…

This work can be extended in various directions. In the following activities, students might use the shape templates and also draw accurately their own regular polygons.

- draw mystic roses or polygons with different number of sides.
- use the shape templates to draw in some diagonals of each shape. Now find as many angles as you can without using a protractor. Is it possible to find every angle in the polygon? Is this easier in some polygons than in others? For example, here is a regular hexagon with some of the diagonals drawn in…how many angles can you find?

- which regular polygons tessellate? You could use the Mathomat to draw the polygons. You can also use ATM activity tiles to explore this problem – or make your own regular polygons with card. Extend this activity by asking students to find pairs of regular polygons that tessellate.



## 5 things to do this fortnight

- Now may be a good time for you to think about planning to take part in the ATM Easter Conference 2011 – Celebrating Gattegno, which will take place at the Telford Campus of the University of Wolverhampton from 18-21 April 2011. Caleb Gattegno, who founded the Association of Teachers of Mathematics (as the Association for Teaching Aids in Mathematics), invented geoboards, and introduced teachers to Cuisenaire rods - which was our focus in Issue 60, and to the mathematical films of Nicolet, which we featured in the Focus on short image sequences in Issue 59.

- You might also be thinking now about applying for a Goldsmiths' Grant, in order to carry out a project that will enhance your personal and professional development during 2011 and 2012.

- The British Science Festival, one of Europe's largest science festivals, will be held in Birmingham from 14-19 September this year. The festival is in a different location in the UK each year, bringing to teachers the latest in science, technology and engineering. Many sessions will focus on mathematical ideas. For example, *Pi Hunting*, on 15 September, will explore the amazing history of π. The cost is £5. You can download the whole programme and book tickets or email for more information.

- Why not explore the programme of Gresham College lectures 2010-11, many of which will have a mathematical theme - and which are free?

- With a view perhaps to using it with your students, you could watch the video of The mathematics of Trains and Boats and Planes. This is a previous Gresham College lecture by John Barrow, Professor of Mathematical Sciences at the University of Cambridge, which includes interesting examples of substituting numbers in simple formulae. There are resources to accompany this film that include a transcript of the lecture and downloadable lecture notes.

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# Diary of a subject leader

**Issues in the life of an anonymous Subject Leader**

Why is it that whenever I cut the grass it never works out practically as well as it does theoretically? It's the first Sunday of the summer holidays and even though I know the theory behind grass-cutting, the mower never seems to go easily into the corners and there's never a nice pattern left in the moss. Anyway, it gives me time while sweating under the sun to reflect on the last few weeks of term – my last days as a subject leader.

I completed my 'Leading from the Middle' course run by the National College and reported back on the work that I did in school on Staff Voice – an exciting development this year that has resulted in people now more openly suggesting improvements and being more aware of what is going on so they can make them happen. At first there had been some reluctance to say anything for fear of it being thought of as whining. But, by having regular slots in staff meetings we have all benefitted from using those opportunities to talk things over: categorising people's observations as 'what works well' and 'even better if' made things more positive! We have, for example, made great progress with 'Rewards and Sanctions' (for students that is!) and with reorganising the calendar.

I spent one Saturday with a group of committed mathematics educators in Birmingham, selecting questions for the UK Junior and Intermediate Mathematical Challenges. We worked through everyone's questions (each of us having previously sent in a few we had written), to aid us in reviewing last year's papers and setting next year's challenges. A great sense of humour prevailed in getting the wording sorted precisely and adding a bit of subtlety while producing a balanced interesting paper. At my school, we generally enter about 60 students from each year group and they enjoy receiving the bronze, silver or gold certificates if they have done well enough.

The set lists are all done ready for next term. This year we have had only Y6 teacher assessment levels to help us set the Y7 students, without KS2 science and mathematics results – but not all schools have provided the same data. Some have only given a level (such as 4), others have given a sublevel (such as 4B), others nothing. Anyway we have allocated each student to a set, and our decisions will be reviewed during the first half of next term.

In the penultimate week of term, I did some work with mathematics and science teachers at a local science centre, in which we used Texas Instruments handheld technology to explore data logging and try to bring mathematics and science closer together. We know from their evaluations that we did well with those present, and hope that the work will be disseminated in schools.

My car got serviced and passed its MoT, though it does work out at rather more than I had hoped, so I haggle a bit and get the price reduced by just over 10%! It pays to be nice about things, but also to see what can be done in the present economic climate.

The last week of term was our 'Enrichment Week'. Many of our students are on visits abroad to places like Paris, for culture; the battlefields, for history; Italy, for classical studies; or Spain, for sports. Those remaining in school have been allocated to half days in faculties. Mathematics has two half-day sessions

with Y10 and one with Y9. This year we decided that students would make and use historical mathematical instruments – in particular, proportional dividers and parallel rulers. The latter were featured in Secondary Magazine Issue 62.

The proportional dividers in brass and steel are at the bottom of this picture. These are replica instruments and are available from Clipperlight. Students can see what the actual instruments look like and discover how to use them. Then they make their own proportional dividers from sheets of card on which we have already printed the scales. The dividers use a central slider to set the scales so that they can be used in four different ways: to divide the circumference of a circle into a number of equal parts, from 6 to 20, to enlarge lines using a scale factor from 1 to 10, to enlarge areas in a scale factor from 1 to 10, or to enlarge solids in a scale factor from 1 to 10. We make two proportional dividers: one to divide the circumference of a circle into 7 parts and the other to enlarge with a scale factor of 2. That involves some ratio work, but there is not space here for further elucidation!

This is my final contribution as a subject leader – I have left my school after twelve years and two terms to become a freelance mathematics educator and consultant, which will allow me the freedom to work in all sectors of mathematical education.