Secondary Magazine 108

National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 108 of the Secondary Magazine

Spring is sprung, the grass is ris, I wonders where the birdies is (anon)

In this spring issue of the Secondary Magazine, there are plenty of green shoots including some ideas to consider as a mathematics teacher educator, the next article in the *Key Ideas* series focuses on reasoning with decimals and the matching set of problems that connect decimals and measurement. As the Easter break approaches, you may find some interesting things to read and do in this issue.

Contents

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From the editor: NCETM support for the new National Curriculum

Have you started to plan changes to the scheme of work in your department or plan changes to your own teaching in response to the new National Curriculum? Over the next few years there will be some significant changes to mathematics in the Secondary school. To support you in adapting to these changes, NCETM is developing a range of support materials; the best way to access these materials is to follow the link from the blue button on the homepage:

By following this link you will find the latest facts and guidance for the new National Curriculum that includes an interview with NCETM director Charlie Stripp and an interview with Primary director Debbie Morgan

As a Secondary specialist you may not have looked at some of the links from this page that have direct reference to the Key Stage 3 curriculum so here are a couple of items that are directly relevant to teachers of Secondary school pupils:

- the National Curriculum resource tool has links to the Year 6 curriculum: by clicking on one of the blue buttons (try the *Fractions (including decimals and percentages)* button) you can access that part of the curriculum for Year 6, but can also link to Making Connections, which allows you to see and make connections to this topic in Year 5 and Key Stage 3, and also gives some real life and cross-curricular links. Activities provides some suggestions for suitable rich tasks. There are also links to articles, videos and exemplification of the programme of study statements.
  The Key Stage 3 buttons within this resource tool will be 'live' with material specifically aimed at Year 7, 8 and 9 teachers within the next few weeks.

- New curriculum videos links to a collection of 60 video clips. The Algebra and Multiplicative reasoning links have videos from Key Stage 3 classrooms. The videos can be downloaded to ensure that they can be easily viewed and may be a good focus for discussion in a departmental meeting or used to inform planning for the new curriculum

The NCETM Essentials page Implementing the new Curriculum, in common with all the NCETM Essentials pages, provides a synthesis of various resources on our extensive website that may be useful in supporting your work.

Key Ideas in Teaching Mathematics – Reasoning with decimals in Key Stage 3

In this and other issues, the Secondary Magazine will feature a set of six articles, written by Anne Watson, Keith Jones and Dave Pratt, the authors of the recent publication Key Ideas in Teaching Mathematics. While not replicating the text of this publication, the articles will follow the themes of the chapters and are intended to stimulate thought and discussion, as mathematics teachers begin to consider the implications of the changes to the National Curriculum. This article is the fourth in the series and focusses on Reasoning with decimals in Key Stage 3. Future articles will feature Place Value, Algebra and Probabilistic Reasoning. Previous articles focussed on similarity, ratio and trigonometry in Key Stage 3, Geometric and spatial reasoning in Key Stage 3, and statistical reasoning in Key Stage 3.

A recent discussion on the TES mathematics teaching forum concerned the following question that a teacher had set for homework: how many tenths in 1.5? A response that might immediately be expected is 15 tenths. Another suggested response was 5 tenths, with the explanation that 1.5 is one unit and 5 tenths. A further response was that there are 10 tenths, in that there are always 10 “tenths” in whatever is the “whole”. One suggested way forward was that perhaps such variation in response could be avoided if the original question was worded how many tenths is 1.5; but then perhaps the variation in response provoked by the original question can provide a valuable teaching opportunity.

The debate illustrates that there is more to reasoning with decimals than might be expected, given that children are exposed to ideas associated with place value from an early age (from early counting through, for example, to everyday experience of measures). In the mathematics curriculum, the notion of place value, and of decimal place value in particular, begins in the early primary school years and continues through secondary school – with a progression from decimal notation and equivalents towards terminating and recurring decimals, while taking in ideas of rounding and significant figures. To be successful, students need to coordinate place value ideas with aspects of whole number and fraction knowledge. Making the transition to being able to reason with decimals relies on pupils having a thorough understanding of ideas previously met and these earlier ideas becoming fully integrated with new information. If pupils persist in trying to treat decimals as if they were whole numbers, then pupil reasoning with decimals can remain uncertain - as exemplified by the following:

- “more digits means bigger” (e.g. 0.21345 is larger than 0.3); this can occur when pupils use a method that is successful for whole numbers (ie 21345 is larger than 3).
- “more digits means smaller” (e.g. 0.31345 is smaller than 0.25); this can be a reaction to learning that “more digits means bigger” is incorrect, or it can result from incorrectly generalising that because one hundredth is smaller than one tenth (and so on) then the more digits the smaller the number.
- “zeros to the right of a decimal number increases the size of that number” (e.g., 0.400 is larger than 0.40, which is larger than 0.4); this is related to the case above of treating the decimal part as a whole number.
- “zeros on the left can be ignored” (e.g. 0.4 is the same as 0.04, and both are the same as 0.004); this occurs from incorrectly generalising that just as the number 8 is not changed by placing a zero in front of it (ie 08), the zeros after the decimal point can also be disregarded.
- “the decimal point can be ignored” (e.g. $0.2+4 = 0.6$, and variants: $0.07+0.4 = 0.11$, $6\times0.4 = 24$, and $42\div 0.6 = 7$); this also occurs when pupils use arithmetic methods that are successful for whole numbers.

In all these cases, what is needed is for pupils to build their knowledge in such a way that they come to recognise the features of whole numbers that are similar to decimal fractions and those that are unique to whole numbers. Although it is clear that pupils struggle with understanding rational numbers, in general it is decimals that present one of the greatest challenges. TIMSS data, for example, indicate that pupils do not always perform as well on questions involving decimals compared to those involving ordinary fractions - perhaps because of the difficulty some students have in shifting from understanding fractions as 'parts of something' to understanding them as single numbers.

With time and direct effort, pupils can learn to distinguish whole number from rational number concepts (at times where a distinction needs to be made) and develop a meaningful understanding of how fractions and decimals are represented symbolically. In secondary mathematics, a key idea is the transition from additive to multiplicative reasoning and the move pupils need to make from seeing fractions and decimals as two numbers to seeing them as either as numbers in their own right or as the result of dividing where the divisor does not divide exactly into the dividend.

There can be two approaches to teaching decimals. One approach tends to emphasise integrating decimals with other proportion concepts such ratio and percentages. Another approach focuses on building meaningful understanding of decimal numeration based on place value, perhaps through a link to measurement. The two approaches are not opposing camps; rather both approaches seek to construct meaningful links between related ideas such as between measurement, fractions, decimals and percentages. The teaching of topics such as probability can support the linking between decimals and fractions. As with much of the mathematics we teach, there is no unique path in sequencing different ideas to arrive at the understanding that decimals and fractions can be pure numbers or proportional operators, but percentages can only ever be proportional operators.

It can be that over-reliance on one approach ends up exacerbating the problem. For example, over-reliance on the context of money can lead to difficulties in comparing pairs of decimals such as 4.4502 and 4.45. Children over-relying on halves and quarters can struggle with other fractional proportions. For example, they can struggle with tenths because tenths cannot be achieved by halving and more halving. Here money does not help because a penny does not look like a tenth of a 10p piece. Even with length measures, which can be a useful context for work on decimals, pupils have been known to identify a 1-place decimal with cm and 2-place decimals with mm. While in some countries it can be acceptable for learners to say 0.85 as what would translate as 'nought point eighty five', because they are thinking about hundredths or about mm, focussing on the size of the parts, in this case comparing cm and mm in general, can lead such pupils to conclude that 8.1 is larger than 8.15 because centimetres are larger than millimetres. Nevertheless, refining measurement units to measure quantities more precisely can be a useful approach in teaching. Use of the “double number line”, integrating two units of metric measures such as metre and centimetre, can also be beneficial. Pupils who tackle contextualized problems can build up their understanding of decimals by utilising their ‘everyday’ knowledge as it relates to the meaning of decimal numbers and the results of decimal calculations

Even so, the use of the decimal system to express large numbers (such as a population) or in commonplace metric measures can mask the continuous nature of decimals, including the notion of the density of the real numbers (these being the field of all rational and irrational numbers). As a result, pupils can have difficulty in imagining what happens with all the number that they can think of between, say, 12.849 and 12.850. A range of other issues relating to real numbers tends to be glossed over in school mathematics. For example, converting an ordinary fraction into a decimal fraction frequently gives rise to infinite decimals, yet little curriculum time is generally devoted to infinite decimals or clarifying their representation by never-ending decimals. The result is that well-known questions like the following are usually left unaddressed: “is 0.999 . . . exactly equal to 1, or only approximately?”. What is masked is the jump from finite to infinite processes, and how infinite processes can be safely treated mathematically. Similarly, while pupils encounter numbers like $\sqrt{2}$, π, and e (the latter at A-level), there is little time at secondary school for a consideration of irrational numbers.

While it is beyond school mathematics to resolve the contradiction between the continuous nature of the number line continuum and the discrete nature of the numbers themselves (the apparent contradiction can be resolved by defining Real numbers using either Dedekind cuts or Cauchy sequences), infinite processes (the basis of the transition from Calculus to Analysis) do occur in school mathematics; and one topic in which such infinite processes occur is within a consideration of decimals. In touching on notions of the density, and completeness, of the real numbers, reasoning with decimals is revealed as a key idea in school mathematics; one that continues beyond school into higher level mathematics.

Keith Jones, Dave Pratt and Anne Watson

In keeping this series of articles brief, there is no space for full references; these can be found in the book Key Ideas in Teaching Mathematics

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A resource for the classroom – problems connecting measurement and decimals

This issue of the magazine has an article linked to the recent publication, Key Ideas in Teaching Mathematics. There is a website that accompanies the book which provides links to some relevant resources. Our article this month is related to reasoning with decimals, so the resources for the classroom are a suite of problems connecting measurement and decimals. (The "proportion" aspect of decimals is considered in the "ratio and proportional reasoning" section, where relevant ideas of "quantities", whether expressed as fractions or decimals, are considered). Some of these problems may be familiar whilst others may be new to you; all have been chosen to develop and deepen understanding.

The website identifies six themes within the context of connecting measurement and decimals which it lists as measurement scales, compound measures, geometric measures, measurement and proof, angle measures and estimating measures and states:

> *While measurement is often linked with geometry, there are good arguments for better connecting it with decimals. In terms of teaching, neither measurement nor decimals are best served by being taught as a set of simple skills; rather, each is a complex combination of concepts and skills that develops over a number of years.*

The evidence indicates that the principles of measurement and of decimals are not straightforward for many learners and may require more attention in school than is sometimes given.

The individual problems are:

- The Greenest Route
- Containers
- More Miles for Your Money
- Stretchiness
- Accessible Spaces
- Changing Areas, Changing Perimeters
- Crescents and Triangles
- Angle A
- Does This Sound about Right?

What will you do now?

You could:

- select a problem and try it out with a particular class
- select a problem and work with a colleague to consider how you can use the problem to develop understanding for a group of pupils
- include some of these problems in your scheme of work
- consider how these problems develop the powerful aspects of the curriculum.

Do tell us what you find out...

Image credit

5 things to do

Download the BBC Radio 4 series A Brief History of Mathematics. In these ten short episodes Professor Marcus du Sautoy considers the lives and contributions of different mathematicians from Newton and Leibniz to Hardy and Ramanujan.

Watch this YouTube clip to show an experiment using a Tornado tube. Another illustration of clockwise and anticlockwise in an interesting context.

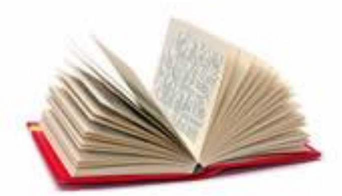
If you are a regular user of Twitter you may already have seen these patterns. And don't forget you can also follow the NCETM on Twitter - and even join in our weekly Twitter chats every Tuesday, 7-8pm.

As a lover of shape and space, you may be interested in the patterns in this blog and the mathematical questions that can be asked about them.

You may like to investigate the Rolls-Royce Science Prize. This is an annual awards programme that helps teachers implement science teaching ideas in their schools and colleges. There is a total of £120 000 in prizes to be won each year. For 2014, the competition has been extended to include mathematics. Judges will be looking for entries which demonstrate that mathematics is integral to everyday work in the field of science. They will be looking to reward excellence in mathematics teaching across the education system and support teacher continuing professional development. This new element to the Rolls-Royce Science Prize is in partnership with the Institute of Mathematics and its Applications (IMA) and the NCETM. Further information about the prize can be found on the Rolls-Royce website.

The application form for mathematics-based entries is now live. Complete and submit your action plan by **26 May**.

Image credit



Tales from the classroom: changing practice

I've just bought a new family car. Well I'm a teacher so it's not new! Just new to me. It is more than a new car however, it is a new era. I've just hit the age where I know the meaning of life and the answer to universe and everything - not that my wife or three children would agree - and probably you neither. But the new era is a change of brand. So it's a quarter of a century driving with one brand, loyal to the end. The problem now being that their loyalty to me did not stretch to providing seven seats, the only way our shared child care can work. My new family car is very coveted by my neighbour and a couple of friends. However, I still choose to use my "trusty rusty old banger" of brand type 1 in place of the family car whenever I can (albeit leaking door seal, no rear wash-wipe, and piercing wind whistle at anything over 35mph). So when I offered my Danish neighbour and her little girl a lift home, their edgy Danish giggles of excitement at going in the 'new' car, rapidly turned to tears as I opened the doors of "old trusty rusty". My neighbour continued "You're so funny. You have a lovely new car all wrapped up in blankets in the garage, and you use this thing."

Initially I thought the 'you' was meaning the English. Glancing across however, her infectious smile clearly meant 'no it's just you and you alone' - bringing forth my West Country rumble to harmonise the Danish giggle now coming from the back seat. It wasn't raining and it was only a short ride through lanes limited to 30mph so all was well.

Having dropped off my European charges and returning home I was initially musing about how much my own relationships over the years with friends and colleagues from different cultures have really enhanced my understanding of my own life and being, but then I drifted onto a conversation about learning I had last week with a colleague and a Bulgarian student.

The conversation was regarding a new IT based package. I have very little to do with it aside from ensuring that the IT is available and that teachers have rooms etc. I'm far from uninterested in it: I just like to give my colleagues space and responsibility to learn. They know I am there for help and advice if needed. I was just checking that it was working in terms of the IT, rooms etc. However, impatient and inquisitive I could not resist scratching a little deeper. I just asked "is it worth doing?" I was hoping for a "Yes".

I didn't get a "No", but neither did I get a "Yes". My colleague said "it's got a bit of buy-in". The student added "it's new and different so I guess it's a bit more fun", the teacher qualified it all with "it's early days yet". And that was probably all I needed to know. I'm very confident that my colleague is looking critically. Suggesting "initial buy-in" shows me that although students appear to enjoy it, she is looking for something deeper; suggesting it is "still early days" shows she is willing to persist and look for that something deeper happening in the long term. There are lots of things we could do to make maths "more fun" and get a buy-in. And in my own practice that tends to be a personality thing and developing a charismatic bond with students that gets the "buy-in" which then allows me to motivate, capture and teach in a way that deepens understanding. Expecting to use "new technologies" just for the "buy-in" is an expensive approach - financially, and also in terms of the time taken in learning to use something new.

So when I look for something new, it has to be something that fundamentally changes the way a student may understand a concept, otherwise the change is peripheral and in time will wear away, and I will still be teaching that concept in the way I always have. Practice of "Brand type 1" will prevail. I've always been a bit sceptical of "new technologies" not really seeing a wow factor where others might. But this revelation has helped me. I think that without really consciously deciding, I was being discerning about the point of the "new technology". Can it fundamentally change my core practice to enhance the understanding of my students in a way that progresses their learning. If not, then why use it?

Two days after the conversation I looked up from helping one of our least able Year 11 students who was struggling with some very simple equivalent fractions. I saw one of his classmates totally absorbed in trying to explain to another how to convert a top-heavy fraction to a mixed number. To my amazement and deepest pleasure she had gone to the scrap paper draw then found some scissors and was cutting up fractions of circles to make “wholes” and “left-overs”. Now there is somebody who is using technology – albeit well used- to really change core understanding. I liked it because it is just what I would have done. I’m not sure she picked that up from me: it would be nice if she had. But it made me realise that I too struggle to change my core practice. A decade ago I would have reached for the paper and scissors, and ten years on, I still do. True to the core!

So is my struggle to embrace the new car, just a reflection of that difficulty so many of us have in adjusting our "core"? Or is it that actually “shiny smart new” cannot actually do anything “old trusty rusty” could not do either? - thereby unable to add any real benefit to me in moving out of my comfort zone? And is that the same with the “new technology”? Will it be a peripheral embellishment that in time will quickly tarnish, lose its shine and eventually fall away? Or, will that “new technology” change the core of who I am? And professionally that is a fundamental question. Will it change the way I teach, the way my students learn?

When I think back to how I have changed in my practice that is the real challenge for those of us that have a responsibility to educate, whether it be students or teachers. How do we have an impact on that core being? Judging by my own resistance to my new car, it really is a challenge.

The author is a mathematics subject leader and assistant principal working in the South West