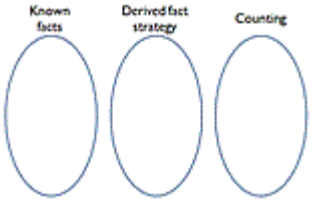
Primary
& Early Years
Magazine 95



National Centre
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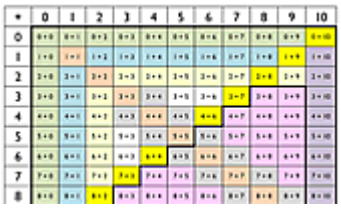
Teaching of Mathematics

Welcome to Primary Magazine 95. In this issue we look at the importance of fluency in Y1/2 addition and subtraction facts, with suggested teaching sequences and strategies. And we review a recently published report into 'The Parent Factor' – examining the influence of parental involvement in a child's maths education. The article includes links to plentiful resources and ideas for schools wanting to increase the involvement of their parents.

Don't forget all previous issues are available in the Archive.



Achieving Fluency in Addition and Subtraction Facts

How is your school addressing the fluency strand of the National Curriculum? This article provides evidence-based justification for the importance of fluency in addition and subtraction facts – achieved not simply through memorisation but through the explicit teaching of strategies for quick calculation. Clare Christie uses her extensive experience in the classroom, and through interviewing children, to explain not only why fluency is important, but suggests sequences and strategies (used in her own department) for achieving fluency in addition and subtraction facts, for all children.

Count on Us: Parental Engagement Programme

Parents make a real difference to maths engagement and enthusiasm of their children! No surprises there, but how do schools actually go about encouraging the less engaged parents? Taking on board the findings of National Numeracy's 'Count on Us' study, we find out how the study schools were helped to get parents involved. And link to extensive, publicly-available resources, refreshed for the purposes of the study.

And here are some other things to draw to your attention:

- In case you missed it, the DfE has confirmed that a times-table test for Year 6 pupils will be introduced in summer 2019. The School Standards Minister, Nick Gibb, confirmed this when he spoke to an MPs committee last month, as reported by the BBC.
- With next term's Y6 SATs in mind, you might be interested in this blog from @thatboycanteach, a vice-principal in a two-form entry inner city primary school in West Yorkshire, offering insights on last year's tests, the questions for which you can find here.
- Fed up with pizzas and cakes? Seeing fractions in many different pictorial representations can help children gain a deeper understanding. In this blog, Mark Horley provides some good ones, and signposts many more. He also welcomes more contributions from you!

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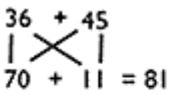
Achieving Fluency in Addition and Subtraction Facts

by Clare Christie, maths lead for a federation of two Bristol schools, and Teaching for Mastery Lead for Boolean Maths Hub

Why we focus on fluency in addition and subtraction facts

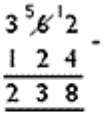
- A defined set of addition and subtraction facts builds the basis of all additive calculation, just as times tables are the building blocks for all multiplicative calculation. For example:



Informal/mental addition by partitioning:

Root addition facts

3 + 4, 6 + 5, 7 + 1, 0 + 1



Formal subtraction with column method

Root subtraction facts

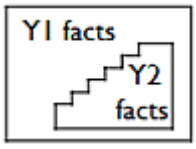
12 – 4, 5 – 2, 3 - 1

- If children are not fluent in these facts, then when they are solving more complex problems, the working memory is taken up by calculating basic facts, and children have less working memory to focus on solving the actual problem (see Is It True That Some People Just Can't Do Math? by the cognitive scientist Daniel Willingham). So fluency in basic facts allows children to tackle more complex maths more effectively.
- The importance of fluency is recognised in the national curriculum, and since 2016, children's fluency is more heavily tested by SATs.
- Children need to be taught strategies to solve these facts. Conferencing I have done over the last few years show that most children don't magically become fluent in these facts even in KS2, particularly for those facts which bridge 10. If they aren't explicitly taught to solve e.g. 6 + 7 by thinking 'double 6 and one more' or to solve 12 – 8 by using 'find the difference' strategies, then many children will get stuck on inefficient counting based approaches.
- Counting on approaches are not only less efficient, they are associated with lower attainment in maths as well. Research by Tall and Gray (1994) found what our own extensive conferencing has shown: higher attainers tend to use known facts or derived fact strategies, and lower attainers are much more likely to use counting based approaches to solve addition and subtraction facts.

Which facts have we focused on?

The full set of addition facts is:

+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10
5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10



And here are the corresponding subtraction facts:

-	0	1	2	3	4	5	6	7	8	9	10
1	1−0	1−1									
2	2−0	2−1	2−2								
3	3−0	3−1	3−2	3−3							
4	4−0	4−1	4−2	4−3	4−4						
5	5−0	5−1	5−2	5−3	5−4	5−5					
6	6−0	6−1	6−2	6−3	6−4	6−5	6−6				
7	7−0	7−1	7−2	7−3	7−4	7−5	7−6	7−7			
8	8−0	8−1	8−2	8−3	8−4	8−5	8−6	8−7	8−8		
9	9−0	9−1	9−2	9−3	9−4	9−5	9−6	9−7	9−8	9−9	
10	10−0	10−1	10−2	10−3	10−4	10−5	10−6	10−7	10−8	10−9	10−10
11		11−1	11−2	11−3	11−4	11−5	11−6	11−7	11−8	11−9	11−10
12			12−2	12−3	12−4	12−5	12−6	12−7	12−8	12−9	12−10
13				13−3	13−4	13−5	13−6	13−7	13−8	13−9	13−10
14					14−4	14−5	14−6	14−7	14−8	14−9	14−10
15						15−5	15−6	15−7	15−8	15−9	15−10
16							16−6	16−7	16−8	16−9	16−10
17								17−7	17−8	17−9	17−10
18									18−8	18−9	18−10
19										19−9	19−10
20											20−10

Note that in subtraction facts not all subtractions within 20 are root facts, e.g. 17 - 5 is not considered a root fact (7 - 5 is the root fact for this).

The majority of these facts are learned in Years 1 and 2.

In Reception, children become fluent in working with totals to 5 (though not presented as number sentences), e.g. "Show me 5 on your hands. Now show me 5 in a different way."

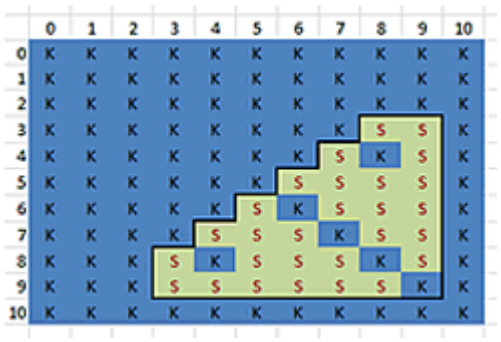
Year 3 continue to focus on securing fluency in facts which bridge 10, and subtraction facts which bridge 10 in particular. Although this is a Year 2 objective, my feeling after many hours teaching and reflecting on factual fluency is that aiming for real fluency in subtraction facts such as 14 – 9 and 13 – 5 (where fluency is an answer in three seconds) for each and every child in Y2 is unrealistic. We feel that unless we are honest about that and accept the need to secure these facts in Year 3, we risk having children who never become secure in this.

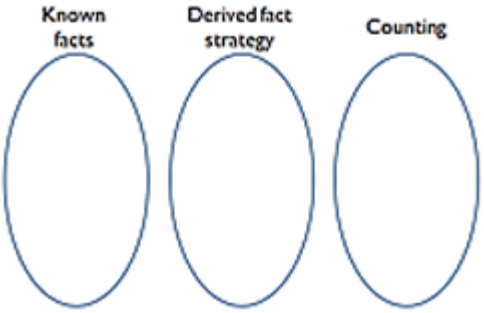
Does fluency just mean memorisation?

Not necessarily – when you conference adults on how they solve addition and subtraction facts, almost all adults rely on very quick use of strategies to solve some of them. Reflect carefully on the set of addition and subtraction facts shown: which have you memorised and which are you very quickly deriving? We've taken fluency to mean 'getting an answer pretty quickly and with limited demands on working memory', aiming for an average of three seconds or less per fact. My work conferencing fluent children in KS2 who were working at this speed showed:

- Most facts which didn't bridge 10 were memorised – the children reported 'just knowing' that 4 + 5 = 9 or 2 + 6 = 8 for example.
- For facts which bridge 10 the picture is more complex, and many of the facts which bridge 10 were quickly derived using strategies (but still in less than three seconds!).
    - Double 6, 7. 8 and 9 were always memorised in fluent children
    - Many fluent children also reported 'just knowing' that 9 + 3 = 12 and 8 + 4 = 12 and related this to their times table/skip counting knowledge.
    - Fluent children in the year groups conferenced (up to Year 4) generally reported using strategies for many of the other facts. 8 + 9 is an example of a fact that actually very few people (either adults or children) have memorised. Of the many hundreds of teachers I have asked, only about five per cent report 'just knowing' that 8 + 9 = 17. Most fluent people solve this through very quickly applying a strategy: bridging through ten, near doubles or compensating (adding 10 and subtracting 1).

As a reference point, this grid is a good example of the approaches taken by a fluent, high attaining Year 4 child to each of the addition facts: he doesn't use a counting approach for any of the facts, but he has certainly not memorised them all either (K= Known fact; S= Strategy):





Why not try this on some of your children? Find out how they solve each of the 121 facts. They are all written out on this PDF – just print and cut them out. I use these sorting circles with the children, and have found they very quickly get the idea once I have given them an example of a known fact (the vast majority of KS1 and KS2 children will just know that 5 + 5 = 10 for example), a strategy fact (e.g. calculating 6 + 5 by relating it to 5 + 5) and a counting based approach. If children say they would use counting, don't bother getting them to solve the fact – you will be there all day. If they say strategy, I find it interesting & helpful to ask what strategy they have used: my notes on the child whose grid is shown above for example showed that he added numbers with a difference of 2 by relating to doubling the number in between (e.g. 6 + 8 = 7 +7; 5 + 7 = 6 + 6). It takes about 15 minutes to conference one child, and is time very well spent.

How do children become fluent?

As mentioned above, children need to be **TAUGHT** strategies to derive the facts! An interesting piece of research (Thornton, 1978) showed that teaching strategies is more effective in securing fluency in addition and subtraction facts than taking a rote memorisation approach. That is to say, even if your aim is memorisation, the most effective way to get there is through the teaching of strategies. There is a huge amount to unpick in this and you need to consider how children are going to become fluent in each and every fact. For example, we want children to just know that 4 + 2 = 6 and 9 – 2 = 7 etc so we need to teach children that when we add 2 or subtract 2 we are moving to the next/previous even number (if starting on an even) or odd number (if starting on an odd). Without being taught this, many children will count. e.g. for 9 – 2, "nine, eight, **seven**". Being able to do this without counting requires being able to count fluently in odd numbers (as well as in the more commonly practised even numbers), something we realised we were not previously teaching our children to do.

We mapped out a teaching progression so we could identify when every individual fact was being taught, and discussed and agreed teaching approaches for each of these fact groups. In Year 1 we teach strategies for facts within 10 (steps 1 – 7) and in Year 2 we teach the bridging ten facts (steps 8 – 11).

1. Adding 1 (e.g. 7 + 1 and 1 + 7)
2. Doubles and near double of numbers to 5 (e.g. 3 + 3, 4 + 5, 5 + 4)
3. Adding 2 (e.g. 4 + 2 and 2 + 4)
4. Number bonds to 10 (e.g. 8 + 2 and 2 + 8)
5. Adding 0 to a number (e.g. 3 + 0 and 0 + 3)
6. Adding 10 to a number (e.g. 5 + 10 and 10 + 5)
7. The ones without a family 5 + 3, 3 + 5, 6 + 3, 3 + 6 (these pairs of facts are the only ones which don't fit in any of the other families, though the last two can be related to counting in 3s)
8. Doubles of numbers to 10 (e.g. 7 + 7)
9. Near doubles (e.g. 5 + 6 and 6 + 5)
10. Bridging (e.g. 8 + 4 and 4 + 8)
11. Compensating.

The last three of these strategies can often be used interchangeably, e.g. for 8 + 9, some people will use near doubles (e.g. 8 + 8 + 1), some will use bridging (e.g. 8 + 2 + 7) and some will use compensating (8 + 10 – 1).

NB: before the children are ready to learn bridging as a strategy, they need to be able to partition all single digit numbers. Adding 8 + 5, for example, by bridging through 10 requires children to partition 5 into 2 and 3. We do an enormous amount on partitioning single digit numbers all through Year 1.

There is even more pedagogy involved in supporting children to become fluent in subtraction facts then there is for addition facts, beyond the scope of this article.

Once children have been taught the strategies, they need to move on to **PRACTICE** of the facts, Remember, for many facts the ultimate aim of the practice is memorisation, while for others the aim of the practice is increasing speed and fluency in the applied strategy. We have a software package which children use to practice which encourages them to aim for an average of 3 seconds or less per fact. This means that in a two-minute practice session the children should be recalling at least 40 facts: the more you practice the quicker they get, and the quicker they get, the less time it takes out of the lesson.

Generally, for practice we focus on:

- Practising the set of facts being learnt (or just learnt) in isolation for a few days
- Mixing these up with all previously learnt facts.

We use a mixture of a software package, practice sheets in class and flash cards/smartboards to give the children practice, in fact very similar approaches to those we take in phonics sessions. In fact, we've found thinking about the structured and systematic approach we take to the teaching of phonics in general is a good analogy for thinking about structured and systematic teaching and learning of these strategies and facts.

Although working on securing fluency in addition and subtraction facts might sound basic and dry, we have found children really enjoy both the discussion and reasoning that the learning of strategies involves, and the confidence they get from having these building blocks in place. In one memorable conferencing session with a middle attaining Year 4 child, who had been part of a class revisiting the teaching of addition facts for a month, I asked how she would work out 6 + 9:

> *"I'd probably add one to make 10, then add the other 5 to make 15," she said before continuing, "but of course the other way you could think about it is that 6 is two groups of 3 and 9 is 3 groups of 3, so if you add them together you get 5 groups of 3 which is 15."*

This was a child who just four weeks before had been reliant on counting based strategies and had solved 6 + 9 by putting "nine in my head and counting on." It was really exciting to see she had gained not just greater confidence in basic arithmetic, but had come to see patterns and connections with a much much wider reach than this.

- Read more about those with gaps

References:

Gray, E.M. and Tall, D.O. (1994): Duality, Ambiguity and Flexibility: A Proceptual View of Simple Arithmetic.

Thornton, C.A. (1978): Thinking Strategies in Basic Fact Instruction.

Willingham, D.: Is It True That Some People Just Can't Do Math?.

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'The Parent Factor' – The Importance of Parental Involvement in Children's Maths Learning and How to Engage That Involvement

No teacher will be surprised at the conclusion of a recent study into parental involvement in their children's maths learning:

> *The volume and statistical significance of the evidence, both quantitative and qualitative, strongly demonstrates that parental engagement leads to increased confidence and improved behaviours in maths and raises standards of attainment. Parents are indeed a significant factor in determining how well children learn maths, and schools should work towards improving the level and quality of parental engagement.*

The study, Count on Us: Parental Engagement Numeracy Programme (main findings in the infographic in page 5), was carried out by National Numeracy, and funded by The Mayor's Fund for London and MAN Group.

But how, exactly, did the schools in the study get parents involved?

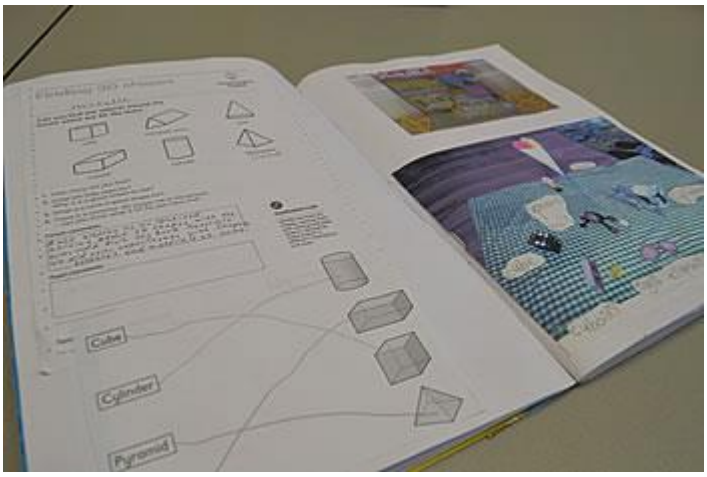
What is less clear to teachers, is how to engage the parents that are more inclined to leave their child's education to the school.

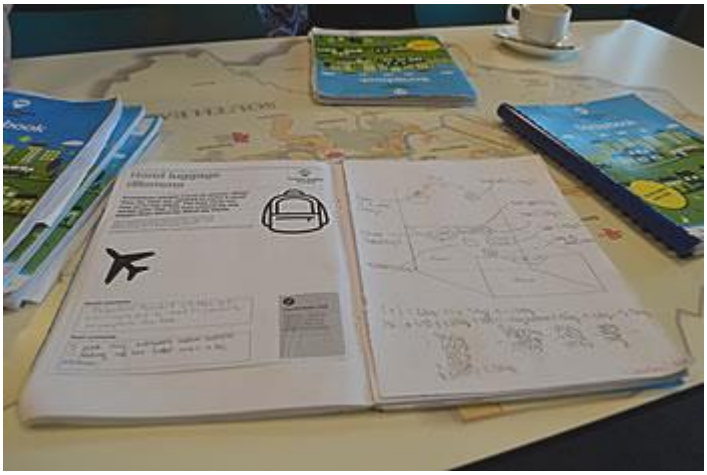
Appendix 4 (p36) of the study's report gives more detail, but among methods used were:

- Parents' workshops,
- Inviting parents into lessons
- Maths games/puzzles clubs
- Guides for parents,
- Drop-in maths surgeries
- Bring a Dad day
- Family numeracy classes.

There is plenty more information from National Numeracy on engaging parents, including addressing barriers for parents such as: negative experience of school themselves (or specifically of maths), poor English, lack of time, or just not knowing what they can do to best help their child.

From the classroom end, all schools in the study used National Numeracy's Family Maths Scrapbook – an A4-size book of blank pages to encourage children to work freely, with their parents, on various activities (provided in PDF form). Schools were encouraged to set this as an alternative to traditional homework, rather than as an optional extra.





What can *my* school do to get the parents involved?

The report states:

> *Whilst the schools received £2,000 to implement the project, sustainability of the parental engagement strategy does not require this level of financial support. Once embedded in whole school policy and practice, the ongoing cost is minimal.*

In addition to this, all of the resources used by schools in the study are available on National Numeracy's Family Maths Toolkit website. The site has a huge number of resources and ideas for getting parents and carers involved in their children's maths education. There is an Advice For Families section, as well as plenty of activities for children, with guidance about what they will be learning at school at various ages. There is also a section aimed at schools wanting to get their parents interested.

Here are some examples from the PDF packs used by schools in the study and available, with the scrapbooks:

- Bug football
- Expedition
- Hogwart's Olympics

- Monkey addition fun
- Star Wars!
- Teddies in a queue
- The witch's spell.