Fractions

This document is part of a set that forms the subject knowledge content audit for Key Stage 1 and Key Stage 2 maths. Each document contains: audit questions with tick boxes that you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications; explanations; and further support links. At the end of each document, there is space to type notes to capture your learning and implications for practice. The document can then be saved for your records.

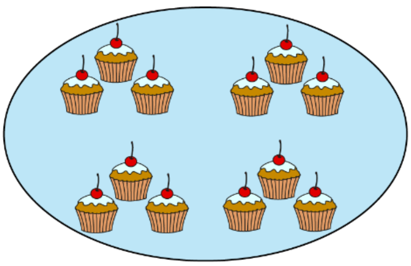
Question 11

How confident are you that you understand and can support children to recognise that equivalent fractions share the same proportional (multiplicative) relationship between the numerator and denominator?

1 ☑ 2 ☐ 3 ☐ 4 ☐

How would you respond …?

a. Sabijah and Jess go to the café. They share some cakes.



Sabijah eats $\frac{1}{4}$ of the cakes.

Jess eats $\frac{3}{12}$ of the cakes.

Sabijah says, *'Jess ate more than me!'*

Jess says, *'We ate the same amount.'*

Can you explain how to prove who is correct?

b. Using the language 'numerator and denominator', can you explain why all these fractions are equivalent?

$$\frac{1}{3} \quad \frac{2}{6} \quad \frac{3}{9} \quad \frac{4}{12} \quad \frac{5}{15} \quad \frac{6}{18}$$

c. Can you identify how the denominator relates to the numerator in this set of equivalent fractions?

$$\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25} = \frac{12}{30}$$

Responses

Note your responses to the questions here before you engage with the rest of this section:

Did you notice that…?

a. In this example, Jess is correct. Explanations may include:

- referring to the picture to prove that Sabijah and Jess ate the same amount
- referring back to fractions of quantities work, explaining that as $\frac{1}{4}$ of 12 is 3 and $\frac{3}{12}$ of 12 is also 3, they ate the same amount.

Some children may look at the numerals in the fraction $\frac{3}{12}$ and (incorrectly) think that $\frac{3}{12}$ must be more than $\frac{1}{4}$ because the numerator and denominator are larger so Jess ate more.

b. These fractions are equivalent because of the relationship between the numerator and the denominator:

$$\frac{1}{3}\ (\times 3) \quad \frac{2}{6}\ (\times 3) \quad \frac{3}{9}\ (\times 3) \quad \frac{4}{12}\ (\times 3) \quad \frac{5}{15}\ (\times 3) \quad \frac{6}{18}\ (\times 3)$$

This can be explained as: *'The denominator is always three times the numerator or the numerator is one-third of the denominator.'*

This vertical relationship is one to focus on because, in this case, all the fractions have the same proportional relationship to each other which is why they are equivalent.

c. This is another example of focusing on the vertical relationship and recognising the proportional relationship between the numerator and the denominator. In this example, the denominator is two and a half times the numerator. This might be a little trickier to spot so a good strategy would be to consider what would happen if the numerator was one.

$$\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25} = \frac{12}{30}$$

Equivalent fractions

The idea that when finding an equivalent fraction, the numerator and denominator are both multiplied by *x* but the fraction is not, can be quite challenging. The fundamental differences between these two concepts need to be understood:

$$\frac{2}{7} = \frac{6}{21} \quad (\text{numerator} \times 3,\ \text{denominator} \times 3)$$

$\frac{2}{7} \times 3 = \frac{6}{7}$

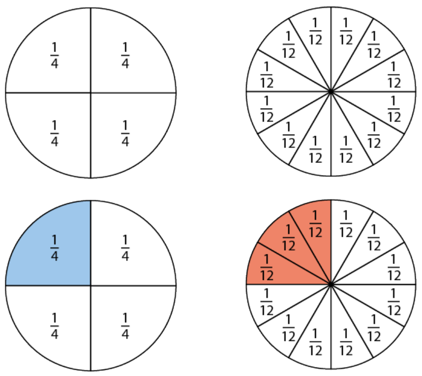
$\frac{6}{7}$ is 3 times larger than $\frac{2}{7}$.

$\frac{6}{21}$ is NOT 3 times larger than $\frac{2}{7}$. It has the 'same value' as $\frac{2}{7}$.

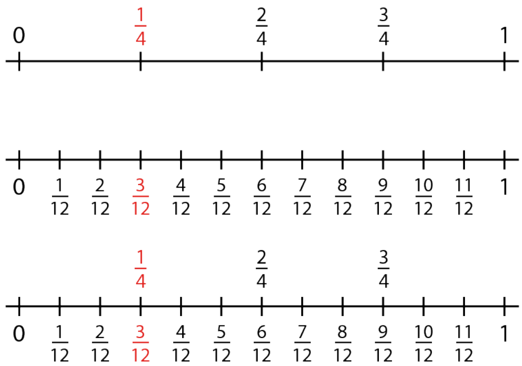
It is important that practitioners understand the relevance of the proportional relationship between the numerator and the denominator. This relationship will be easier to identify when dealing with unit fractions and it is something children will have encountered when looking at $\frac{1}{2}$ as having the same value as $\frac{2}{4}$ in previous practical contexts.

Children's attention will have been drawn to the relationship between a part (numerator) and whole (denominator), for example using informal language to discuss whether a part was *'Quite a large part of the whole'*, or *'Quite a small part of the whole'*. As they develop their understanding, children will use the language of comparison. Some children may have already made the link that if a fraction has the same value as a half, then the numerator is half the denominator, or similarly that the denominator is double the numerator.

Children will need to have experiences of seeing where fractions are equivalent in a range of representations; they should not just be presented with equivalent expressions.



$\frac{1}{4} = \frac{3}{12}$



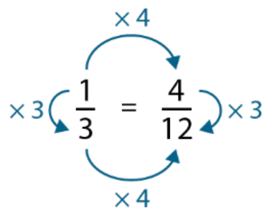
$\frac{1}{4} = \frac{3}{12}$

The idea that the multiplicative relationship between the numerator and denominator determines the value of a fraction, rather than the absolute values of the numerator and denominator, is a really challenging idea for children. Fractions are essentially multiplicative comparisons between a part (expressed by the numerator) and a whole (expressed by the denominator). Children will need to understand that it is the sizes of these relative to each other that determine the value of a fraction. For example, $\frac{7}{11} < \frac{5}{6}$ even though the numerals in the numerator and denominator in the first fraction are larger than those in the second fraction.

$\frac{7}{11}$ is less than $\frac{5}{6}$ because seven is a smaller part of eleven than five is of six.

Consider another example: $\frac{5}{6} = \frac{10}{12}$. Here, the numerator and denominator in $\frac{10}{12}$ are both larger numerals than those in $\frac{5}{6}$. However, $\frac{10}{12}$ is equal to $\frac{5}{6}$. These fractions are equal because the multiplicative relationship between ten and twelve is the same as the multiplicative relationship between five and six.

In families of equivalent fractions, the ratio between the numerator and the denominator stays the same. Children also need to be provided with opportunities to explore the horizontal and vertical relationships within a pair of equivalent fractions. In this example, the numerators and denominators have both been scaled by a factor of four but the denominator is also three times the numerator in both fractions.



Common errors in this area may include:

- children multiplying the fraction rather than recognising the equivalent value
- children thinking $\frac{3}{12}$ is larger than $\frac{1}{4}$ because they are looking at the size of the numbers, rather than their relationship to the whole.

What to look for

Can a child:

- create families of equivalent fractions, not just by 'continuing the pattern' but by explaining the proportional relationship?

Links to supporting materials:
NCETM Primary Professional Development materials, Spine 3: Fractions: • Topic 3.7: Finding equivalent fractions and simplifying fractions

Notes:
Key learning from support material and self-study:
What I will focus on developing in my classroom practice: