MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

4 Sequences and graphs

4.2 Graphical representations

Guidance document | Key Stage 3

Making connections

The NCETM has identified a set of six 'mathematical themes' within Key Stage 3 mathematics that bring together a group of 'core concepts'.

The fourth of these themes is *Sequences and graphs*, which covers the following interconnected core concepts:

4.1 Sequences

4.2 **Graphical representations**

This guidance document breaks down core concept *4.2 Graphical representations* into three statements of knowledge, skills and understanding:

4.2.1 Connect coordinates, equations and graphs

4.2.2 Explore linear relationships

4.2.3 Model and interpret a range of situations graphically

Then, for each of these statements of knowledge, skills and understanding we offer a set of key ideas to help guide teacher planning.

Please note that these materials are principally for professional development purposes. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Overview

In Key Stage 2, students should have become familiar with coordinates in all four quadrants. They should have made links with their work in geometry by both plotting points to form common 2D quadrilaterals and 'predicting missing coordinates using the properties of shapes' (Department for Education, 2013)[†]. These skills are developed further in Key Stage 3. A key focus will be thinking about *x*- and *y*-coordinates as the input and output respectively of a function or rule, and appreciating that the set of coordinates generated and the line joining them can be thought of as a graphical representation of that function.

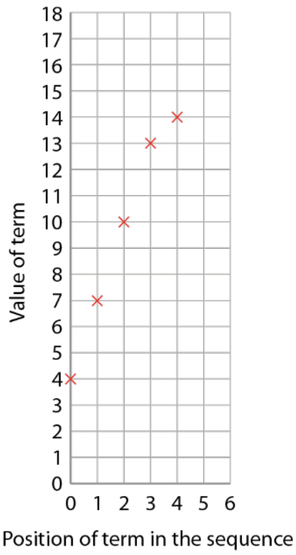
Significant attention is given in this core concept to exploring linear relationships and their representation as straight line graphs. Students should appreciate that all linear relationships have certain key characteristics:

- a specific pair of values or points on the graph; for example, where $x = 0$ (the intercept)
- a rate of change of one variable in relation to the other variable; for example, how the *y*-value increases (or decreases) as the *x*-value increases (the gradient).

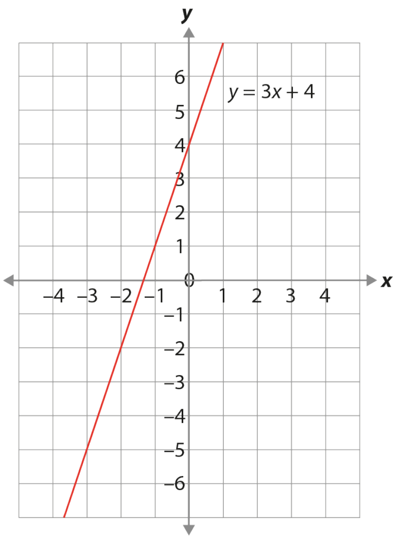
Students should be able to recognise these features, both in the written algebraic form of the relationship and in its graphical representation.

Similarities can be drawn with students' work on arithmetic sequences (see 4.1.2), where each sequence can be characterised by an initial term and a common difference. When working with linear relationships, it is important that students are aware that, while discrete points can be marked on a set of axes to represent an arithmetic sequence, when a function or equation is represented by a set of points, a continuous line (stretching to infinity in both directions) can be drawn to represent it, and the *x*- and *y*-coordinates of every point on the line will satisfy the function.

Diagram showing the terms of an arithmetic sequence



Graph of a linear equation



[†] Department for Education, 2013, *National curriculum in England: mathematics programmes of study, Key Stages 1 and 2*, Year 6

The Key Stage 3 programme of study states that students should be taught to 'move freely between different numerical, algebraic, graphical and diagrammatic representations' and to 'express relationships between variables algebraically and graphically'. In order to develop a deep understanding and achieve fluency, students should explore the connections between equations of lines and their corresponding graphs, including those presented in a non-standard form, such as $ax + by = c$, as well as the more standard $y = mx + c$.

After thoroughly exploring the structure of linear relationships in this way, students should have experience of other functions and relationships (particularly quadratic ones), be able to use graphs to solve problems in real-life contexts and understand how linear graphs can be used to find solutions to simultaneous equations.

Much of this learning is new and is built upon significantly in Key Stages 4 and 5. It is therefore essential that students are given time to develop a secure and deep understanding of these ideas, concepts and techniques.

Prior learning

Before beginning to teach *Graphical representations* at Key Stage 3, students should already have a secure understanding of the following from previous study:

Key stage	Learning outcome
Upper Key Stage 2	• Describe positions on the full coordinate grid (all four quadrants) • Find pairs of numbers that satisfy an equation with two unknowns • Enumerate possibilities of combinations of two variables
Key Stage 3	• 1.4.1 Understand and use the conventions and vocabulary of algebra, including forming and interpreting algebraic expressions and equations • 1.4.5 Rearrange formulae to change the subject **Please note:** Numerical codes refer to statements of knowledge, skills and understanding in the NCETM breakdown of Key Stage 3 mathematics.

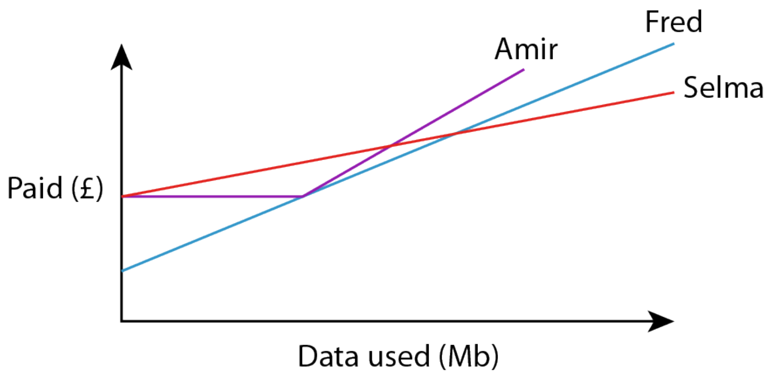
You may find it useful to speak to your partner schools to see how the above has been covered and the language used.

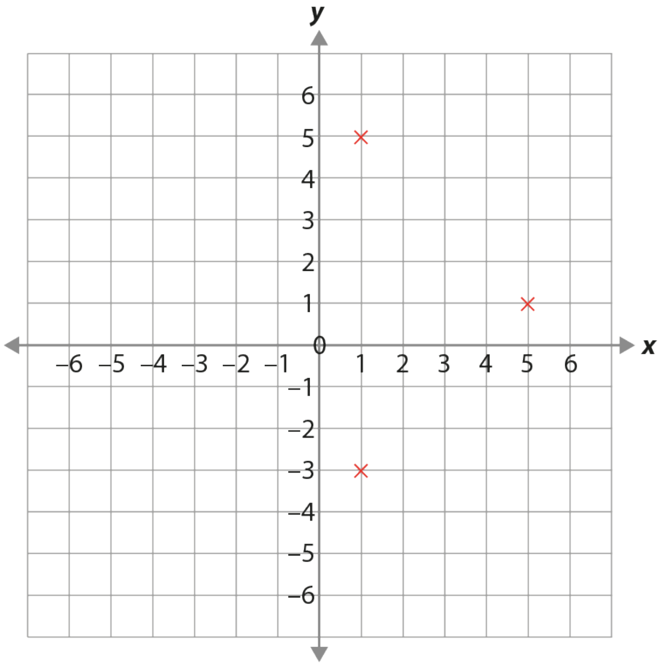
You can find further details regarding prior learning in the following segments of the NCETM primary mastery professional development materials[1]:

- Year 5: 1.27 Negative numbers: counting, comparing and calculating
- Year 5: 1.28 Common structures and the part–part–whole relationship

Checking prior learning

The following activities from the NCETM primary assessment materials[2] and the Standards & Testing Agency's past mathematics papers[3] offer useful ideas for assessment, which you can use in your classes to check whether prior learning is secure:

Reference	Activity
Year 6 page 39	*Three mobile phone companies each have different monthly pay-as-you-go contracts.* *Phil's Phones: £5 fee every month and 2p for each Mb of data you use.* *Manish's Mobiles: £7 fee every month and 1p for each Mb of data you use.* *Harry's Handsets: £7 fee every month and 200Mb of free data, then 3p for each Mb of data after that.* *Amir, Selma and Fred have mobile phones and they have recorded for one month how much data they have used (in Mb) and how much they have paid (in £). They have represented their data on this graph.*  *With which company do you think Amir has his contract?* *With which company do you think Selma has her contract?* *With which company do you think Fred has his contract?* *Explain each of your choices.*

2018 Key Stage 2 Mathematics Paper 3: reasoning Question 10	*Layla draws a* ***square*** *on this coordinate grid.* *Three of the vertices are marked.*  *What are the coordinates of the missing vertex?* Source: *Standards & Testing Agency* Public sector information licensed under the Open Government Licence v3.0

Key vocabulary

Term	Definition
Cartesian coordinate system	A system used to define the position of a point in 2- or 3-dimensional space: 1. Two axes at right angles to each other are used to define the position of a point in a plane. The usual conventions are to label the horizontal axis as the *x*-axis and the vertical axis as the *y*-axis, with the origin at the intersection of the axes. The ordered pair of numbers (*x*, *y*) that defines the position of a point is the coordinate pair. The origin is the point (0, 0); positive values of *x* are to the right of the origin and negative values are to the left of the origin; positive values of *y* are above the origin and negative values below the origin. Each of the numbers is a coordinate. The numbers are also known as 'Cartesian coordinates', after the French mathematician, René Descartes (1596–1650). 2. Three mutually perpendicular axes, conventionally labelled *x*, *y* and *z*, and coordinates (*x*, *y*, *z*) can be used to define the position of a point in space.

gradient	A measure of the slope of a line. On a coordinate plane, the gradient of the line through the points (x_1, y_1) and (x_2, y_2) is defined as $\frac{(y_2 - y_1)}{(x_2 - x_1)}$. The gradient may be positive, negative or zero depending on the values of the coordinates.
intercept	1. To cut a line, curve or surface with another. 2. In the Cartesian coordinate system, the positive or negative distance from the origin to the point where a line, curve or surface cuts a given axis. **OR** On a graph, the value of the non-zero coordinate of the point where a line cuts an axis.
linear	In algebra, describing an expression or equation of degree one. Example: $2x + 3y = 7$ is a linear equation. All linear equations can be represented as straight line graphs.
quadratic	Describing an expression of the form $ax^2 + bx + c$ where a, b and c are real numbers. The function $y = ax^2 + bx + c$ is a quadratic function; its graph is a parabola.
simultaneous equations	Two linear equations that apply simultaneously to given variables. The solution to the simultaneous equations is the pair of values for the variables that satisfies both equations. The graphical solution to simultaneous equations is a point where the lines representing the equations intersect. Example: $x + y = 6$ and $y = 2x$ is a set of simultaneous equations. The solution is the value of x and y which satisfies both simultaneously – i.e. $x = 2$ and $y = 4$.

Collaborative planning

Below we break down each of the three statements within *Graphical representations* into a set of key ideas to support more detailed discussion and planning within your department. You may choose to break them down differently depending on the needs of your students and timetabling; however, we hope that our suggestions help you and your colleagues to focus your teaching on the key points and avoid conflating too many ideas.

Please note: We make no suggestion that each key idea represents a lesson. Rather, the 'fine-grained' distinctions we offer are intended to help you think about the learning journey irrespective of the number of lessons taught. Not all key ideas are equal in length and the amount of classroom time required for them to be mastered will vary, but each is a noteworthy contribution to the statement of knowledge, skills and understanding with which it is associated.

The following letters draw attention to particular features:

D Suggested opportunities for **deepening** students' understanding through encouraging mathematical thinking.

L Examples of shared use of **language** that can help students to understand the structure of the mathematics. For example, sentences that all students might say together and be encouraged to use individually in their talk and their thinking to support their understanding (for example, *'The smaller the denominator, the bigger the fraction.'*).

R Suggestions for use of **representations** that support students in developing conceptual understanding as well as procedural fluency.

V Examples of the use of **variation** to draw students' attention to the important points and help them to see the mathematical structures and relationships.

PD Suggestions of questions and prompts that you can use to support a **professional development** session.

For selected key ideas, marked with an asterisk (*), we exemplify the common difficulties and misconceptions that students might have and include elements of what teaching for mastery may look like. We provide examples of possible student tasks and teaching approaches, together with suggestions and prompts to support professional development and collaborative planning. You can find these at the end of the set of key ideas.

Key ideas

4.2.1 Connect coordinates, equations and graphs

Students should be fluent at both reading and plotting coordinates involving negative and non-integer x- and y-values in all four quadrants. They should be confident in solving problems that require them to be analytical, and be able to go beyond finding an answer to being able to give clear reasons based on the relationships between the coordinates (a key element in this core concept).

For example, in the second question in 'Checking prior learning' (above), in order to determine the coordinates of the missing vertex, students could:

- identify the gradient of one of the sides of the square and infer the gradient of the opposite side

or

- use the fact that the diagonals of the square are perpendicular and of equal length.

A sound understanding of the relationships between the x- and y-values of pairs of coordinates provides the basis for more sophisticated analysis of the features of linear functions and their graphs, which students will need to develop throughout Key Stage 3.

By graphing sets of coordinates where the x- and y-values are connected by a rule, students will become aware of the connection between a rule expressed algebraically and the graph joining the set of points. Students will then also need to think about horizontal and vertical straight line graphs where the functions ($x = a$ and $y = b$) are of a particular form, and relate the concepts of gradient and intercept to these. This work should lead students to appreciate the important two-way connection, that is:

- if the x- and y-values of the coordinates fit an arithmetic rule, then they will lie on a straight line
- if the coordinates lie on a straight line, then their x- and y-values will fit an arithmetic rule.

4.2.1.1 Describe and plot coordinates, including non-integer values, in all four quadrants

4.2.1.2 Solve a range of problems involving coordinates

4.2.1.3* Know that a set of coordinates, constructed according to a mathematical rule, can be represented algebraically and graphically

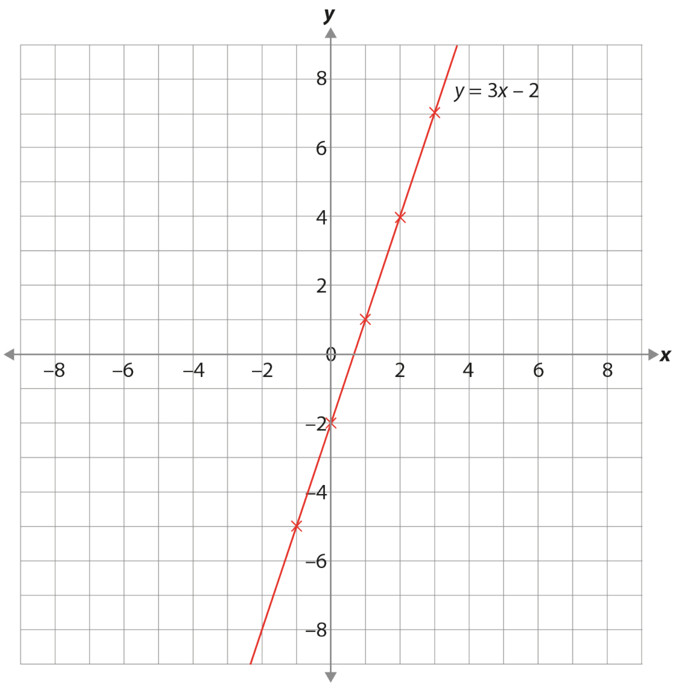
4.2.1.4 Understand that a graphical representation shows all of the points (within a range) that satisfy a relationship

4.2.2 Explore linear relationships

Students will have begun to explore simple algebraic relationships and number patterns in Key Stage 2. This is taken further in Key Stage 3, where students will write the relationship between the *x*- and *y*-values in a set of coordinates using algebra and recognise when it is a linear relationship.

They should become fluent at plotting and identifying straight line graphs and make connections between the equation of the line and the coordinates of points on the corresponding line. To achieve this, students should be given equations presented in a range of forms and opportunities to think about how many points are required to plot a straight line and to choose appropriately scaled axes.

Students should also be given opportunities to explore the connections between the equation of a line, its gradient and its *y*-intercept. By looking at the features of particular graphs, the corresponding set of points and the equation of the line, certain key features can be identified and discussed. For example, students could be presented with a graph, such as this:



x	...	–1	0	1	2	3	...
y	...	–5	–2	1	4	7	...

and asked questions, such as:

- *'How quickly is the graph rising (or falling)?'*
- *'As x increases by one each time, how is y increasing (or decreasing)?'*
- *'How does this relate to its equation?'*
- *'What does the '– 2' in the equation signify? Can you explain why this is so?'*

This will support students to become aware that the two significant features of any straight line which enable it to be drawn uniquely – the rate at which *x* changes with respect to *y* (the gradient) and where the line is positioned in the plane (the intercept) – can be inferred by looking at the equation of the line.

When students are confident transitioning between a graph and its corresponding equation written in the standard form $y = mx + c$, they should be encouraged to do the same when the equation is written in a different form, such as $ax + by = c$.

4.2.2.1 Recognise that linear relationships have particular algebraic and graphical features as a result of the constant rate of change

4.2.2.2 Understand that there are two key elements to any linear relationship: rate of change and intercept point

4.2.2.3* That writing linear equations in the form $y = mx + c$ helps to reveal the structure

4.2.2.4 Solve a range of problems involving graphical and algebraic aspects of linear relationships

4.2.3 Model and interpret a range of situations graphically

Students should explore graphs in given contexts, such as distance–time graphs, and be able to match graphs with specific scenarios. They should also not only develop algebraic and graphical fluency when understanding linear functions, but also experience simple quadratic functions. Students should build on what they have learnt when plotting straight line graphs and apply this knowledge to quadratic functions. This is a key skill that is developed further in both Key Stages 4 and 5, so it is important that students are given time to develop secure foundations for this future work.

Students should begin to explore the idea of two linear graphs intersecting and recognise that the point of intersection is the solution to a pair of simultaneous equations. This will help prepare students for future learning in Key Stage 4 when solving two linear simultaneous equations algebraically. In order to gain a deep understanding of this concept, students must also experience scenarios where there is no point of intersection and be able to explain why this is so by making reference to the gradients.

4.2.3.1 Understand that different types of equation give rise to different graph shapes, identifying quadratics in particular

4.2.3.2 Read and interpret points from a graph to solve problems

4.2.3.3* Model real-life situations graphically

4.2.3.4* Recognise that the point of intersection of two linear graphs satisfies both relationships and hence represents the solution to both those equations

Exemplified key ideas

4.2.1.3 Know that a set of coordinates, constructed according to a mathematical rule, can be represented algebraically and graphically

Common difficulties and misconceptions
When working with linear equations and graphs, it is not uncommon for students to accept that integer coordinates fit the rule given by an equation. What students may not appreciate is that the line is representing an infinity of points, all of which fit the rule. It will be important for students to experiment with coordinates in between integer points they have used to construct the line, and to verify that these coordinates also fit the rule. This should lead students to the important awareness of the key idea that if a set of coordinates lies on the same straight line, then there is a consistent relationship between the *x*- and *y*-values that can be expressed algebraically as the equation of the line. Students should be encouraged to plot the coordinates themselves to confirm that the coordinates do, in fact, lie on a straight line, but also to think deeply about why this is so and not just rely on practical demonstration. These more probing explorations will support students in reaching two important awarenesses: • the line represents the infinity of points satisfying the rule and therefore 'captures' or represents that rule in the same way the algebraic equation does • the line divides the plane into points that fit the rule and points that do not. Some students may find it challenging to express the relationship between the *x*- and *y*-values algebraically. Asking students to test a given algebraic relationship by generating another coordinate and testing whether this lies on the same straight line can help them to overcome this difficulty. Encouraging the use of precise language will also help students to overcome difficulties; establishing the relationship and articulating it using key vocabulary will enable students to discuss and reason with clarity. Prompting students to describe the relationship in words by considering how the *x*-value is being operated on in order for it to match the *y*-value, will help students identify the relationship before formally expressing it in algebraic form.

What students need to understand	Guidance, discussion points and prompts
Identify an additive relationship from a set of coordinates. *Example 1:* *For each set of coordinates:* *Find the relationship between the x- and y-values.* *Can you draw a straight line which passes through them?* *(i) (0, 2) (1, 3) (2, 4)* *(v) (0, –3) (5, 2) (–3, –6)*	**V** In *Example 1*, students find the relationship between the *x*- and *y*-values in sets of coordinates where that relationship is additive. In parts (i), (ii), (iii) and (iv), the equations are in the form $y = x + c$. Parts (i), (ii) and (iii) start with a coordinate that has an *x*-value of zero. Students might find this helpful when starting to identify the relationship between the *x*- and *y*-values. Part (iv) starts with a coordinate that has an *x*-value of –3 to test whether students can accurately identify the relationship without zero as a starting point.

(ii) *(0, 3)*
(1, 4)
(2, 5)

(iii) *(0, 4)*
(1, 5)
(2, 6)

(iv) *(−3, −1)*
(0, 2)
(3, 5)

(vi) *(−2, −3.5)*
$\left(\frac{1}{2}, -1\right)$
(9, 7.5)

(vii) *(−3, −3)*
(1, 1)
(4, 4)

(viii) *(2, 6)*
(4, 7)
(−1, −3)

Find another point on each line. Do the x- and y-values have the same relationship?

V In *Example 1*, students find the relationship between the *x*- and *y*-values in sets of coordinates where that relationship is additive.

In parts (i), (ii), (iii) and (iv), the equations are in the form $y = x + c$. Parts (i), (ii) and (iii) start with a coordinate that has an *x*-value of zero. Students might find this helpful when starting to identify the relationship between the *x*- and *y*-values. Part (iv) starts with a coordinate that has an *x*-value of −3 to test whether students can accurately identify the relationship without zero as a starting point.

Parts (v) and (vi) have equations in the form $y = x - c$. In part (v), the first coordinate has an *x*-value of zero to allow students to more easily identify the relationship, but this is not the case in part (vi), where both a fraction and decimals are used.

In part (vii), the equation is $y = x$. This part provides an opportunity for students to explore whether this is the same as $y = x + 0$ or $y = x - 0$.

Part (viii) has no linear relationship. This part will help to assess whether students are testing the relationship with all the coordinates provided and may help them understand why this is necessary. Plotting the coordinates may support this understanding.

In all parts, a range of coordinates has been provided, including those with negative and fractional values. The order in which they have been written also varies, as students need to become adept at selecting the easiest coordinates to work with first before testing the relationship with the others. Using the first coordinate listed for each part might not be the most efficient option.

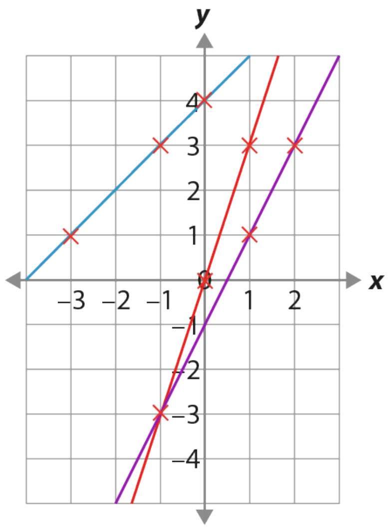
L Encourage students to use precise language when describing the relationship between the *x*- and *y*-values. In part (i), students may identify that the *y*-values and the *x*-values are both increasing by one. It is important, however, that students can verbalise the relationship between the *x*- and *y*-values and

		in multiple ways (i.e. that the *y*-value is two more than the *x*-value; *y* subtract two equals the *x*-value; two added to the *x*-value is always the *y*-value, etc.).
	D **R**	Asking students what the coordinates from each part of *Example 1* would look like visually will promote a deeper understanding of the concept and reinforce the idea that if there is a linear relationship, then all coordinates will lie on the same straight line. Further questions to promote a deeper understanding could include: • *'Can you find another coordinate on the same straight line?'* • *'(3, y) also lies on the line. What is the value of y?'* • *'(x, 0.5) also lies on the line. What is the value of x?'*
Example 2: *Here are some coordinates:* *(−2, 0) (9, 11) (−4, −2)* *Jamila says that the equation of the line passing through these coordinates is* $y = x + 2$. *Kuba says the equation is* $x = y + 2$. *Lillie says the equation is* $x + 2 = y$. *Who is correct? Justify your answer.*	**D**	*Example 2* is designed to give students an opportunity to explore and discuss the way in which a verbal relationship is written algebraically. Many students will identify the relationship as '+ 2' and they should be challenged to explain what they mean by this, using key vocabulary. You could ask questions, such as: *'Does anyone think the relationship is "– 2"?', 'Is there more than one way to write the relationship algebraically?', 'Is one way preferable to another?'.* Discussing these key ideas is important if students are to become fluent at writing equations in multiple forms and to develop a deep and secure understanding.

Understand how many points are required to determine a linear relationship. *Example 3:* *Mohammed says that the equation of the straight line going through the point (3, 6) is $y = x + 3$.* *a) Is he correct?* *b) Do any other straight lines pass through this coordinate?* *c) Are there any other relationships between 3 and 6 which are not 'Add 3'?*	**PD** Consider asking students how many points are required to test whether a line is straight or not. Can they reason that $y = x + 3$ is an equation of a line passing through the given coordinate, but that there are an infinite number of others? Can they come up with other equations of lines? Can they use a graph to support their reasoning? What other activities and questions would help students to understand that there are multiple lines passing through one point, but that two points define a line?
Identify a multiplicative relationship from a set of coordinates. *Example 4:* *For each set of coordinates:* *a) Find the relationship between the x- and y-values.* *Can you draw a straight line which passes through them?* *(i) (6, 12) (−2, −4) (0, 0)* *(ii) (−1, −3) (7, 21) (4, 12)* *(iii) $(\frac{1}{2}, 2\frac{1}{2})$ (−2, −10) (2, 10)* *(iv) (4, 2) (−3, −1.5) (5, 2.5)* *(v) (3, −3) (−6, 6) (−1.5, 1.5)* *(vi) (−3, 6) (2, −4) (0, 0)*	**V** In *Example 4*, students find the relationship between *x*- and *y*-values in sets of coordinates where the relationship is multiplicative. In parts (i), (ii) and (iii), the *x*-value is multiplied by a positive integer. In part (iv), the *x*-value is multiplied by $\frac{1}{2}$ (or divided by two). This provides an opportunity to discuss the different ways in which the equation can be written. In parts (v) and (vi), the *x*-value is multiplied by a negative integer.
Example 5: *a) (−10, −2) (−2, 6) (6, 14)* *Charlie thinks the equation of the line passing through these coordinates is $x = y + 8$.* *Explain why Charlie is wrong.* *b) (10, 2) (1, 5) (−3, −15)* *Mia thinks the equation of the line passing through these coordinates is $y = 5x$.* *Explain why Mia is wrong.*	**V** *Example 5* gives students an opportunity to explore misconceptions when finding the relationship between *x*- and *y*-values in sets of coordinates. **PD** Students often lose sight of the fact that an equation represents a relationship between two values. Asking students to each write down a coordinate where the *y*-value is three more than the *x*-value, and then representing all the coordinates generated on a coordinate grid, can help to reconnect them with this

	idea. When might it be useful to revisit such an activity?
Example 6: *Find the equation of the line which passes through these sets of coordinates.* *a) (10, 5) (−6, −3) (4, 2)* *b) (6, 2) (0, 0) (9, 3)* *c) (−10, −1) (6, 0.6) (−1, −0.1)* *d) (12, −3) (−4, 1) (0, 0)* *e) (3, 30) (−1, −10) (2.5, 25)*	**V** In *Example 6*, students are continuing to identify multiplicative relationships but with more complex equations, some of which can be written in multiple ways. Encouraging students to write the equations in as many ways as possible will help them to develop procedural fluency and a deeper conceptual understanding. In parts a), b) and c), the *x*-value is divided by a positive integer (or multiplied by a unit fraction). In part d), the *x*-value is divided by a negative integer. In part e), the *x*-value is divided by 0.1 (or multiplied by 10).
Example 7: *(17, 17) (−4, −4) (8, 8)* *Ally says that the relationship is '+ 0' each time.* *Ben says that the relationship is '× 1' each time.* *Cathryn says that the relationship is '÷ 1' each time.* *Who is correct? Justify your answer.*	**PD** Providing students with opportunities to reason using given statements is a useful strategy as it ensures students can engage with a concept without relying on them coming up with the statements themselves. Students often feel more confident discussing 'fictional' misconceptions rather than exposing their own. Can you devise some more examples of this use of misconceptions for students to discuss? **V** Asking students to identify who is correct, when all statements are true, will challenge them to explore all options. *Example 7* shows students that there may be more than one way to express the relationship between the *x*- and *y*-values, but that all lead to the same equation.
Visualise the relationship between sets of coordinates. *Example 8:* *Find the coordinates of the point that is exactly halfway between these pairs of coordinates.* *a) (0, 12) and (0, 16)* *b) (13, 5) and (21, 5)* *c) (13, 12) and (21, 16)*	**V** In *Example 8*, the coordinate pairs have been carefully chosen so that they are probably too large to draw accurately (drawing should be discouraged unless absolutely necessary), but not so large that they cannot be visualised by students. By picturing the midpoint of the coordinates, students will visualise the line joining them and the relative positions of the three points.

d) (14, 22) and (18, 10)	Note that in parts a) and b), the pairs of coordinates are parallel to the axes, and that in part c) these pairs of coordinates are combined; part d) offers a negative gradient for students to consider. **D** Students could be challenged to think more deeply by asking questions, such as: • *'What would happen if the second point were the midpoint? What would be the third coordinate in the set?'* • *'Can you find a fourth or fifth coordinate that is also on the line?'* • *'How would you explain to someone else how to find the midpoint of a pair of coordinates?'*
Identify a two-step relationship from a set of coordinates. *Example 9:* *Find the equation of the line which passes through these sets of coordinates.* *a) (1, 3) (2, 5) (3, 7)* *b) (1, 2) (4, 11) (0, –1)* *c) (1, 5.5) (3, 15.5) (–2, –9.5)* *d) (2, 2) (5, –1) (0, 4)*	**V** In *Example 9*, students are given sets of coordinates, written in a list horizontally instead of vertically. Students might find it more challenging to spot the relationship when the x- and y-values are not lined up visually. The coordinates all have a linear relationship but, unlike in previous questions, they are two-step relationships (for example, in part a), multiply by two and then add one). In parts a), b) and c), the initial coordinates all have x-values of one. In part a), the x-values increase by one each time to support students to see the relationship more easily. Part a) has the equation $y = 2x + 1$, and part b) has the equation $y = 3x - 1$. The coordinates in part c) contain decimals; the equation is $y = 5x + 0.5$. In part d), the multiplier is negative; the equation is $y = -x + 4$. **PD** Students may find it helpful to make connections to sequences here, but they should be made aware of the differences between the discrete nature of sequences versus the continuous nature of equations. Is that a connection you currently explore with your students?

Example 10: *Which of these equations pass through this set of coordinates?* *(−2, −2) (5, 12) (0, 2) (4, 10)* *a) $y = 2x + 2$* *b) $y = 2(x + 1)$* *c) $y = 3x - 3$* *d) $y = x$*	**D** Examples such as *Example 10* are useful in challenging students' understanding of a concept. Students must check that the equation holds true for all coordinates in the set. Students who consider only one coordinate could select an incorrect equation, as every equation is true for one of the coordinates given. Students who correctly identify equation a) should be asked whether any of the other equations could also be true. Discussing why both equation a) and b) are correct is useful in exploring the different ways in which equations could be written, making connections to other areas of algebra.
Example 11: *Consider the graph below. Look at the three coordinates marked on each line and use them to identify the equation of each line.* 	**R** *Example 11* allows students to continue to make connections between the procedure and the concept. It will also support future learning on equations of linear graphs. **D** Asking students to find other coordinates on the lines outside of the regions of the graph, for example (10, *y*), will increase students' awareness of the flexible relationship between coordinates and equations – they can find the equation from coordinates and find coordinates from the equation.

Solve problems where there is more than one answer and there are elements of experimentation, investigation, checking, reasoning, proof, etc. *Example 12:* *Find possible values for a and b, $a > b$, if the equation of the straight line going through the point (a, b) is $y = 2x - 3$.*	**PD** Encourage students who have demonstrated a secure understanding to go deeper by solving more complex problems, such as exploring all possibilities, creating their own examples and testing conjectures.
Example 13: *Is it always, sometimes or never true that linear graphs are horizontal (vertical)?*	**PD** Can you create some other 'missing number' or 'Always, Sometimes, Never'-type problems which would be suitable for your students and would allow you to identify key misconceptions?

4.2.2.3 That writing linear equations in the form $y = mx + c$ helps to reveal the structure

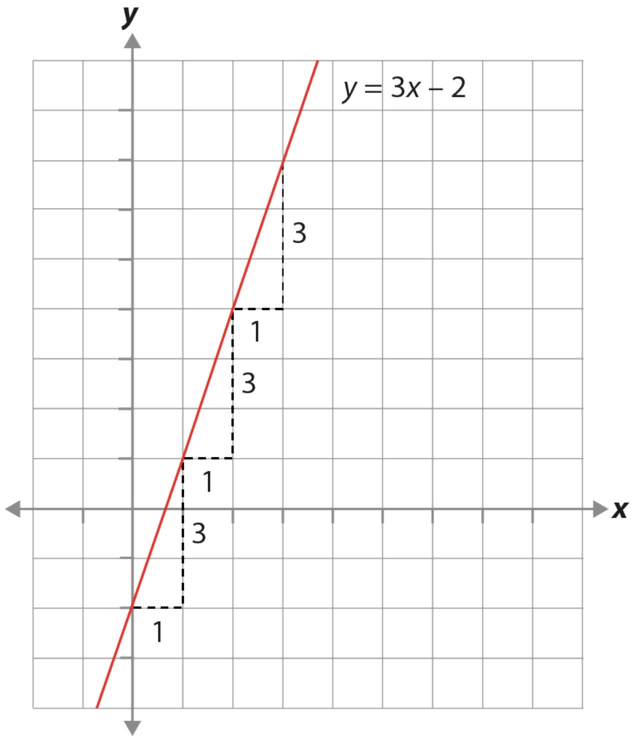
Common difficulties and misconceptions

When examining a set of coordinates, particularly when offered in a table such as this:

x	…	–1	0	1	2	3	…
y	…	–5	–2	1	4	7	…

students may attend to the 'add 3' in the sequence of *y*-values and conclude that the equation of the line is $y = x + 3$, rather than the correct $y = 3x - 2$.

Drawing students' attention to the relationship between the *x*- and *y*-values, and using a graph to illustrate the role that the '3' is playing, will support students in overcoming these difficulties.



Students should understand the key idea that the gradient is a measure of the rate at which the function is changing (i.e. as *x* increases by one, how is *y* increasing – or decreasing?) and that the *y*-intercept is a fixed point (i.e. the value of *y* when *x* is zero). Students should be aware that these two pieces of information uniquely define any straight line.

Another difficulty is the perceived randomness of '*m*' and '*c*' to represent the value of the **g**radient and ***y***-**i**ntercept. Why not $y = gx + i$? Exploring the historical and cultural connections, such as 'm' representing the French word 'monter' (to climb or ascend) and 'c' representing the French word 'commencer' (to start), helps students to understand this mathematical convention and make connections with $y = ax + b$ used in statistics.

<table>
<tr><th>What students need to understand</th><th>Guidance, discussion points and prompts</th></tr>
<tr><td>
The value of the constant term is the y-intercept when the equation is in the form $y = mx + c$.

Example 1:

Find the value of the y-intercept for each of these equations:

a) $y = 3x + 4$
b) $y = 3x - 4$
c) $y = 3x$
d) $y = 3$
e) $y = 0.5 - 3x$
f) $3x + 4 = y$
g) $2y = 4x - \frac{1}{3}$
</td><td>
V In Example 1, students are asked to find the value of the y-intercept. The equations are originally given in the standard $y = mx + c$ form, but this changes as the questions progress. These equations have been carefully chosen to draw out the following nuances:

For part a) the y-intercept is +4. Students who might initially state an intercept of ‘4’ might not realise the importance of the sign; this will become clear in part b), if students continue to state +4 rather than –4.
Part c) has a y-intercept of zero.
Part d) has a y-intercept of three (it is a horizontal line). Note, you may wish to ask students, ‘What is the gradient of this graph?’ at this point.
Part e) has a decimal y-intercept and the order of terms has changed.
Part f) has the same values as part a) and is designed to challenge students’ understanding of an equation.
Part g) has a fractional y-intercept and the equation needs rearranging to get it into the form $y = mx + c$.

All of the numbers chosen are small, so that students can visualise the lines, and so that you can sketch the graphs easily if needed.

R Students might find it beneficial to sketch or be shown sketches of these graphs and to use the images to support their mathematical thinking.
</td></tr>
<tr><td>
Equations with a y-intercept of zero pass through the origin.

Example 2:

Which of these straight lines pass through the origin?

$y = 0x + 4$ $y = -2x$
$y = 3x + 0$ $3y + 5x = 0$
$2y = 4$
</td><td>
Example 2 has been designed to allow students to demonstrate whether they understand the connection between a straight line which passes through the origin, and the y-intercept.

V The options have been carefully chosen, and there are three correct answers. Students might choose part a) or part c) if they confuse the y-intercept with the gradient and, consequently, think the coefficient of x
</td></tr>
</table>

should be zero. In part a), the gradient is explicitly shown as zero, whereas in part c) there is no *x* term.

Parts b), d) and e) are all correct, but are all slightly different. Part b) explicitly shows a *y*-intercept of zero and is the most obvious correct answer. Part d) has no constant term, and part e) needs some rearranging to get it into the standard $y = mx + c$ form.

R Asking students to draw a pair of axes and mark the origin may help to support them in making the connection between the origin and a *y*-intercept of zero.

D Questions with more than one correct answer are worth spending time on, and challenging students to find another correct answer will help them to think more deeply. Asking students to discuss the misconceptions behind the incorrect answer(s) will also help them to develop a deep understanding of what the concept is and what it is not.

PD Can you write similar questions with more than one correct answer to help students develop a secure understanding of the concept?

The value of the coefficient of *x* is the gradient, when the equation is in the form $y = mx + c$.

Example 3:

Find the gradient for each of these equations.

a) $y = 2x + 5$

$y = x + 3.2$

$y = -x - 7$

$y = -2x - 7$

$y = -5$

$y = 10 + 0.5x$

$2x + 5 = y$

$2y = \frac{1}{3}x + 4$

V In *Example 3*, students are asked to find the value of the gradient. Again, equations are originally given in the standard $y = mx + c$ form, but this changes as the questions progress. The equations have been carefully designed to draw students' attention to key features:

- For part a), the gradient is positive (2).
- Part b) has a gradient of 1. Some students may find it challenging if there does not appear to be a coefficient for the *x* term.
- In part c), the gradient is –1. Can students apply understanding of part b) to part c)?
- For part d) the gradient is –2. This might be a good time to stop students and ask them what they notice about parts a) to d). What is the same? What is different?

	• Part e) has a gradient of zero. Some students may need to visualise the horizontal line to understand why this is true. • Part f) has a decimal gradient, and the order of the terms has changed. • Part g) has the same values as part a) and is designed to challenge students' understanding of an equation. • Part h) has a fractional gradient and the equation needs rearranging to get it into the form $y = mx + c$. **R** Students might find it beneficial to sketch or be shown sketches of these graphs and use the images to support their mathematical thinking. **D** Students could use the gradients found in this example to put the equations into a logical order; sketches of the graphs might support this. They should be encouraged to discuss different methods of doing this (ascending or descending order, or by steepness, irrespective of whether it is a positive or negative gradient).
Example 4: *Match lines which have the same gradient.* *If there are any equations that do not have a match, write an equation with the same gradient.* *a)* $y = 4x - 3$ $y = 8 - 4x$ $2y = 4x + 3$ $y - 4x = 3$ $2y + 8x = 0$ $y = -4x + 3$	**V** *Example 4* targets students' understanding of the gradient of a straight line. It has been written so that students might think they are matching pairs of equations; however, three lines – parts b), e) and f) – have the same gradient (–4); another pair – parts a) and d) – share the same gradient (+4); and one line – part c) – does not match any of the others (it has a gradient of 2). The numbers have been selected so that similar integers are used throughout.

Example 5: *a) Grace thinks that the gradient of the line with equation* $y = 7x + 3$ *is* $7x$. *Dylan says it is* $+3$. *Who is correct?* *Evie thinks the gradient of the line with equation* $4y + 2x = -5$ *is* $+2$. *Ffion says it is* -2. *Who is correct?* *Billy thinks the gradient of the line with equation* $x = 7$ *is 0.* *Haider thinks the gradient is 7.* *Who is correct?*	**V** *Example 5* has been chosen to address common misconceptions relating to the gradient. In part a), students may think that the x term is the gradient rather than the coefficient of x. This example can not only be used to assess whether students know the difference between the gradient and the y-intercept, but also whether they spot that Grace has incorrectly stated the x term rather than the coefficient of x. Part b) requires students to rearrange the equation correctly to match the form $y = mx + c$. Again, neither statement is correct, but the example could stimulate discussion about how best to rearrange the equation. Part c) challenges the understanding of an undefined gradient as occurs in a vertical line. Students often mistakenly identify vertical lines as having a gradient of zero. A sketch can help them to understand why this is not the case. **PD** Asking students to discuss the statements of others is one method that allows them to engage with misconceptions without having to admit to having them themselves. Can you write similar statements to help students discuss other common misconceptions?

Identify the gradient and the y-intercept from equations in various forms.

Example 6:

Complete this table by finding the gradient and the coordinate of the y-intercept for each of the equations given.

	Equation	Gradient	Coordinate of y-intercept
a)	$y = 6x - 4$		
b)	$y = -3x + 1$		
c)	$y = 9x$		
d)	$y = -3$		
e)	$y = \frac{1}{2} - 3x$		
f)	$y = -4 + 0.25x$		
g)	$-2 - 5x = y$		
h)	$y = ax + b$		
i)	$y = t + (u + 3)x$		

V *Example 6* asks students to identify both the gradient and the coordinate of the y-intercept from a series of equations. This is slightly different from *Example 1*, as students will need to express the y-intercept as a coordinate and not just read the constant value from the equation.

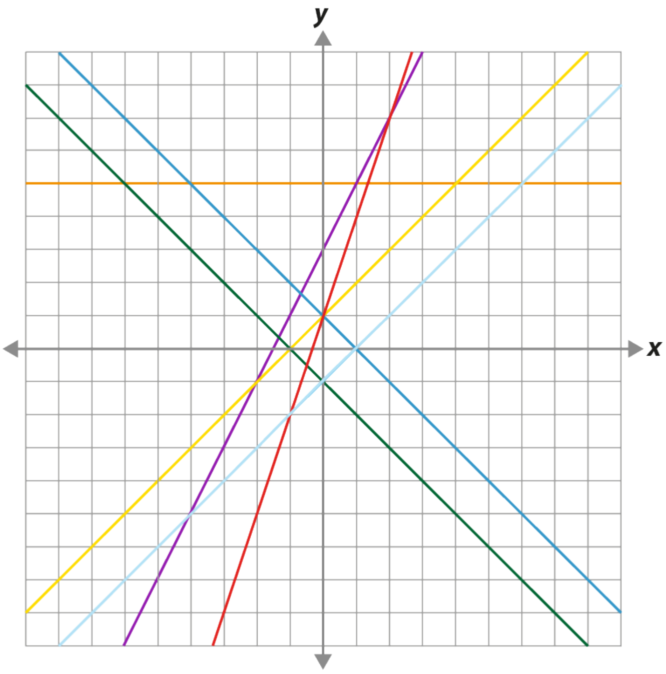
The equations have been carefully chosen to cover a range of options with positive, negative and zero gradients and y-intercepts, as well as integer and decimal values. Part g) challenges students' understanding of an equation. Can students verbalise why swapping the left and right sides of an equation in their entirety does not affect the gradient or y-intercept?

D Parts h) and i) are opportunities for students to explore the use of algebraic variables. These examples draw attention to the fact that it does not matter whether the coefficient of x or the constant term are known values or variables.

Example 7:

Match these equations to the lines on the graph below.

$y = 3x + 1$	$y = x - 1$
$y = 2x + 1$	$y = 1 - x$
$y = 2x + 3$	$y = 5$
$y = x + 1$	



Which equation cannot be one of the lines? Explain how you know.

R It is important that students can make the connection between the gradient and the *y*-intercept of a line from an equation and what this looks like on a graph. Problems such as *Example 7*, where no scales are given on the axes, really challenge students to consider the links between equations and lines.

L Encouraging students to verbalise their mathematical thinking by asking them to explain why something cannot be true is key to developing their understanding of a concept.

There are many ways that students could approach this example and it is important to explore the different strategies. Some students may begin by paying attention to the *y*-intercepts and see that there are not enough lines passing through what could be (0, 1). Others may begin by looking at the gradients and see that there are two pairs of parallel lines – one set with a positive gradient, and another with a negative gradient – yet only one equation has a negative gradient.

PD Problems such as this one, where the initial wording suggests that all the equations should match with lines and yet one equation does not, encourage students to question their understanding. What prompts could you offer students to help them with this?

Example 8:

$y = cx + m$

Kayla says that the gradient is m and the y-intercept is c, because the gradient is always m and the y-intercept is always c.

Luka says that the gradient is c and the y-intercept is m, because the gradient is always the coefficient of x and the y-intercept is always the constant term.

Are either Kayla or Luka correct?

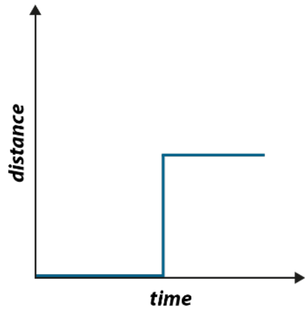
Justify your answer.

D *Example 8* is designed to challenge students' understanding that the letter name is irrelevant; it is the role it plays in the formula that is important. Students may identify Luka's statement as correct, but it is important to explore his reasoning. Asking students whether it is *always* the case and encouraging them to consider examples when it is not (depending on the format of the equation) will help to develop a deep and secure understanding.

4.2.3.3 Model real life situations graphically

Common difficulties and misconceptions

Students are often familiar with plotting graphs from an equation or from given data but can still find interpreting the 'story' told by the graphs a challenge. Hart (1981) explored students' understanding of distance–time graphs: while they were able to identify key features of these graphs (such as 'different rates of travel' and 'arrival times'), several had 'incorrect perceptual interpretations of the graph' and identified the graphs as a 'picture' of the journey.



For example, rather than identifying the graph here as an impossible journey, they offered descriptions based on the shape such as, 'went along a corridor, then up in a lift, then along another corridor' or 'going east, then due north, then east'.

Challenging students to interpret real life graphs that do not describe a journey or to identify non-examples of journeys such as the one above may help them identify the key points and connect the graphical representation to the context it models.

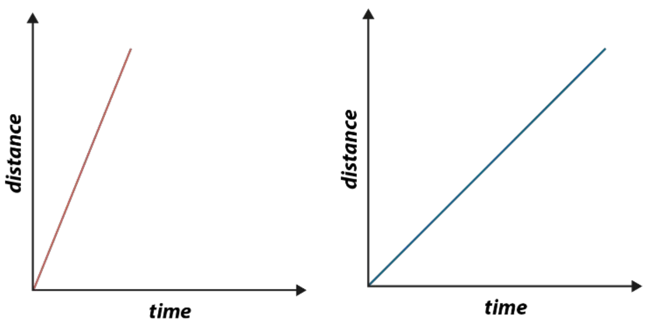
What students need to understand

Understand and interpret the gradient in context.

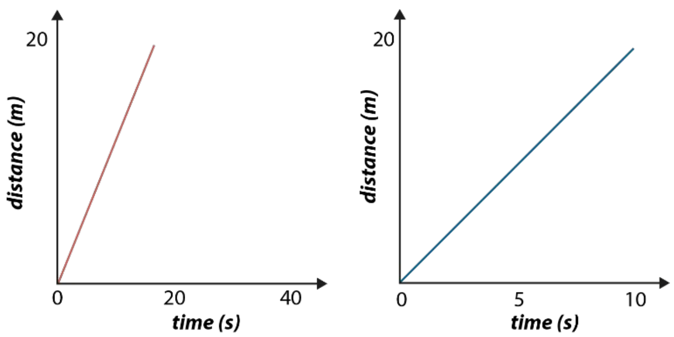
Example 1:

Tilly is 11 years old, and Willow is 4 years old. They run a race over 20 m.

a) *Which graph do you think belongs to Tilly, and which to Willow?*



b) *Here are the same graphs with some more information. Which graph do you think belongs to Tilly, and which to Willow?*



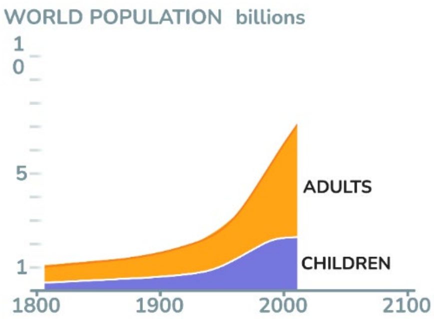
Guidance, discussion points and prompts

V *Example 1* offers an opportunity to explore the meaning of the gradient and consider what it means to describe the 'steeper' line. In part a) students are likely to assume that Tilly, as the eldest, is the fastest child and that the red line describes her race because the line is steeper. Part b) then uses the same graphs but with additional information, showing that the scales are not the same and in fact the blue line represents the faster runner. To interpret the meaning of the gradient in this context, the students must identify that if the blue line was plotted on the same axis it would be steeper because the runner has travelled the same distance in a shorter time.

D Ask students to consider whether straight lines are likely in this context and give them an opportunity to sketch how more accurate lines might look.

Example 2:

This graph shows the change in the world population over about 200 years.



a) *Between 1800 and 1900, which grew faster – the number of adults or the number of children?*

b) *Which is now growing faster, the number of adults or the number of children?*

c) *Why do you think this might be?*

L In this graph the context of population growth is used to demonstrate gradient as a rate of change. Encourage students to refer to the gradient of the lines to describe how they know the number of adults is growing faster than the number of children. Challenge them to explain why a faster growth rate results in a steeper graph.

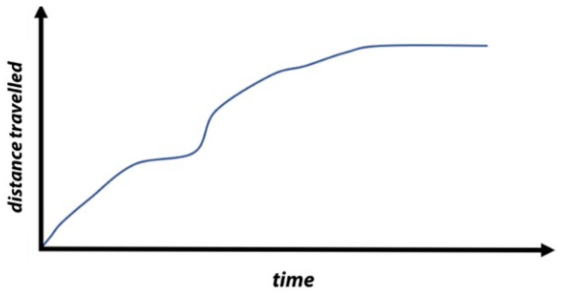
V Part a) of *Example 2* should draw students' attention to the way the lines share the same gradient, meaning that the growth rate is almost the same for adults and children. This offers a useful comparison to more recent years where the rate of change for adults is significantly greater than that for children.

(Source: free material from www.gapminder.org)

Example 3:

Nicola goes out for a bike ride.

This graph shows the distance she travels. There is a very steep hill on Nicola's route.

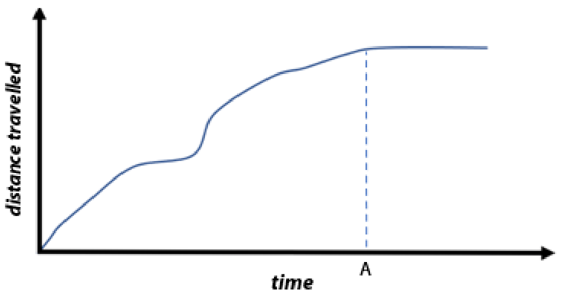


a) *Where on the graph do you think she is going uphill?*

b) *Where do you think she's going downhill?*

c) *Explain how you know.*

d) *Nicola's bike ride takes her about an hour. Use this information to estimate what time is represented by A on this second graph:*



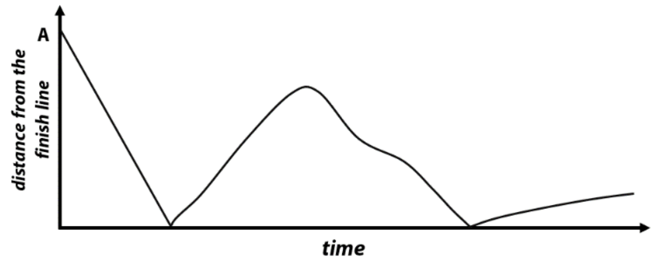
R *Example 3* is designed to explicitly address the misconception students may have about the shape of the line mapping out the journey. The episodes of travelling uphill and downhill can be identified by Nicola's speed, and some students may find it counterintuitive that the gradient is least when Nicola is travelling uphill.

In part d), students should be aware that the line is horizontal after time A. This means Nicola is no longer travelling, and so time A can be estimated to be the end of her bike ride after about 1 hour. Students should understand that a gradient of zero shows that while one variable (in this case, time) is changing, the other remains the same (the distance travelled).

Understand and interpret the intercept in context.

Example 4:

This graph shows the final 10 m of a sprint race.



a) *What is the value of the point marked A?*

b) *Priya says, 'That's odd! The graph shows that the runner reached the finish line then instantly turned around and ran back a few metres.' She is not correct. What do you think the graph shows?*

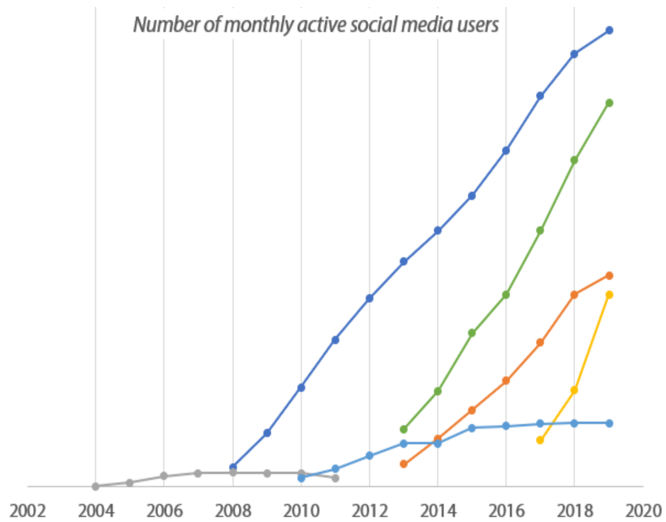
R *Example 4* requires students work with the *y*-axis of the graph. Initially students need to recognise that the intercept identifies the distance from the finish line at the start of the graph and connect this with the text above. In part b) they might expect the line to continue below the horizontal axis. This is another opportunity to stress that the line is not a 'picture' of the situation, rather it is a mathematical construct.

D Although the focus of this example is the intercept and interpretation of the *y*-axis, students might be asked to consider the speed of the runner at different points. For example, what is different about the gradient when the runner crosses the line for the second time?

Understand and interpret a graph in context.

Example 5:

The graph below shows the number of active social media users for different platforms between 2004 and 2019.



a) *Which line do you think represents Facebook? Explain how you know.*

b) *Which line do you think represents TikTok? Explain how you know.*

c) *Which line do you think represents MySpace? Explain how you know.*

Example 5 provides a context for students to identify key points and interpret them to make conjectures.

Students may be unfamiliar with lines that stop (as in the lower line here) or that do not start at the origin. This is an opportunity to review what's the same and what's different between graphs plotted from real life information and those plotted from an equation.

PD Three lines on the graph have been used as a focus for parts a), b) and c). What features might you draw attention to from these lines? What other features might you draw attention to from the remaining lines?

Answers:

a) *Dark blue – Facebook*

b) *Yellow – TikTok*

c) *Pale grey – MySpace*

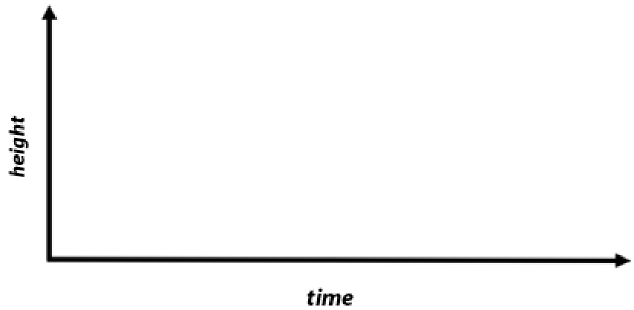
d) *Green – WhatsApp; orange – Instagram; light blue – Twitter*

(Source: Esteban Ortiz-Ospina (2019) "The rise of social media". Published online at OurWorldInData.org. Retrieved from: 'https://ourworldindata.org/rise-of-social-media)

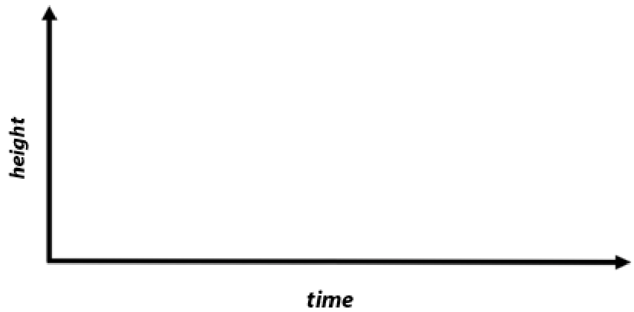
d) What platforms might the other lines show? Explain how you know.

Example 6:

a) Sachin throws a ball straight up in the air and catches it again. On this set of axes, sketch how the height changes with time.



b) Sachin then throws the ball to Catriona who is standing close by. On the same set of axes, sketch how it changes with time.

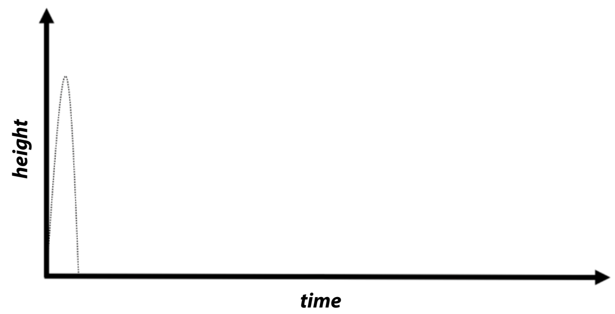


V *Example 6* provides an opportunity to explore how students either understand graphs as representations or misunderstand them as a 'picture' of the situation.

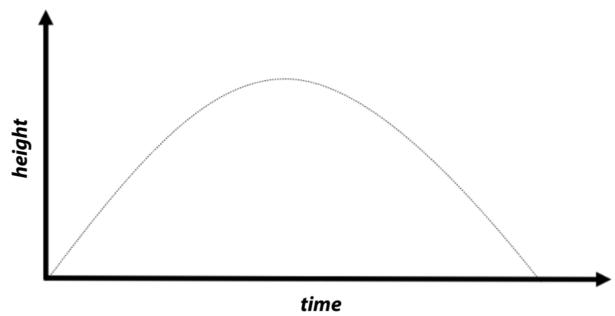
For the situations in parts a) and b), the graph is likely to look similar. However, some students may sketch the first situation (with the vertical throw) with a narrower base than the second situation, mirroring the horizontal distance travelled by the ball rather than the time elapsed since the ball was released.

For example:

a)



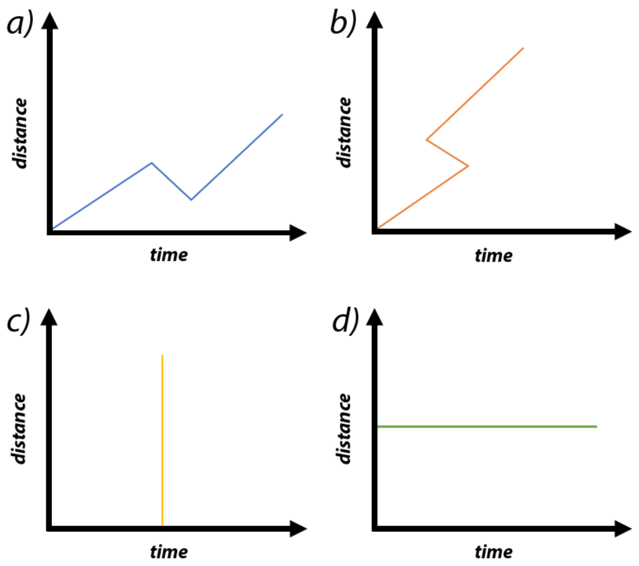
b)



D Students might usefully consider whether the different heights of Catriona and Sachin would impact on the shape of the graph and how this would be represented. What has been assumed to be 'zero height' in this example?

Example 7:

Which of these graphs show journeys that are impossible?



D *Example 7* (which draws on examples used by Hart, 1981) gives another opportunity to explore whether students understand that a graph is not a 'picture' of a situation. They should interpret that the situations with the orange and the yellow line are impossible, as they either involve travelling backwards through time, or instantaneously changing distance. The key idea to stress is that the line is not a sketch of a path.

4.2.3.4 Recognise that the point of intersection of two linear graphs satisfies both relationships and hence represents the solution to both those equations

Common difficulties and misconceptions

This key idea builds on *2.2.1.3 Understand that a solution is a value that makes the two sides of an equation balance*. This idea is explored here with explicit reference to Cartesian graphs.
A key understanding here is that a line such as $y = 3x + 5$ splits the plane into three parts: the region where $y > 3x + 5$; the region where $y < 3x + 5$; and the line itself that contains all possible points where $y = 3x + 5$.

The focus of this key idea is on the graphical representation so algebraic manipulation is not a feature of the examples offered. Rather, the focus here is on the different ways in which the lines and any crossing points might be understood.

What students need to understand

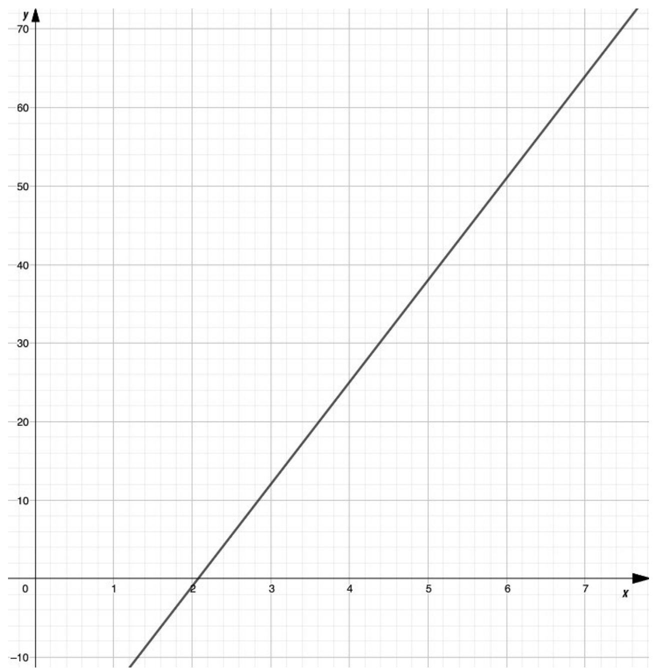
Identify regions on the plane.

Example 1:

This graph shows the line $y = 13x - 27$.

a) *Use the graph to write down the answer to $13 \times 7 - 27$.*

b) *Use the graph to estimate $13 \times 3.5 - 27$.*



Guidance, discussion points and prompts

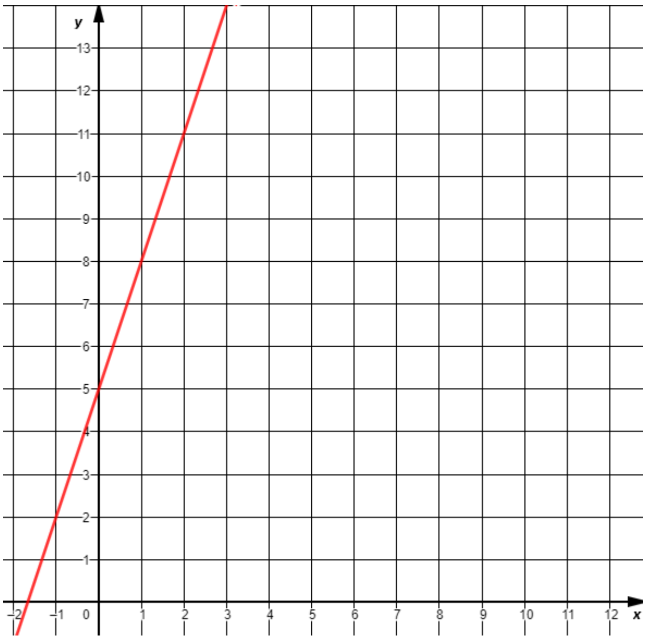
In *Example 1* students use a graph in an unfamiliar context (to complete a calculation). The equation is intended to be challenging enough that students will be discouraged from calculating and will instead use the graph.

R It is likely that students will be familiar with using graphs to find values and this question might commonly be phrased as *'Find the value of y when x = 7'*. However, it may be that they have not connected the graphical representation and its equation with the calculation.

V Part b) asks students to estimate the solution to a calculation that can be quite accurately seen on the graph. The intention here is to encourage them to see that all points on the line fit the given equation. You might ask students to write more calculations and approximate solutions, for example asking how many ways they can complete a frame such $13 \times$ ___ – 27 = __ (or maybe $13 \times$ ___ – 27 ≈ __).

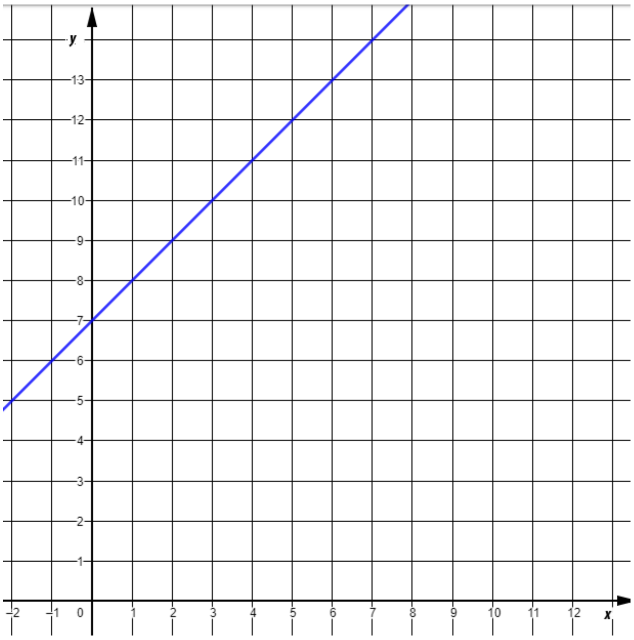
Example 2:

1. *This graph shows the line $y = 3x + 5$.*



   a) *Mark three points where $y > 3x + 5$.*
   b) *Mark three points where $y < 3x + 5$.*
   c) *Mark three points where $y = 3x + 5$.*

2. *This graph shows the line $y = x + 7$.*



   a) *Mark three points where $y > x + 7$*
   b) *Mark three points where $y < x + 7$*
   c) *Mark three points where $y = x + 7$.*

Examples 2 and 3 are designed to be used together, to give students a context in which they can identify the different regions of the plane and their meanings.

V By identifying three positions for each region, students build a range of their own examples. These can be further developed through teacher questioning (for example, you might introduce some further constraints such as marking three points where $y > 3x + 5$ and $x = 1$, or where $y > 3x + 5$ and $y = 1$). These constraints might challenge students to go beyond the range of the given graph (such as write down three coordinates so that $y > 3x + 5$ and $x > 100$, or $y = 3x + 5$ and $x > 100$). The key awareness here is that all points on the line are connected by one relationship.

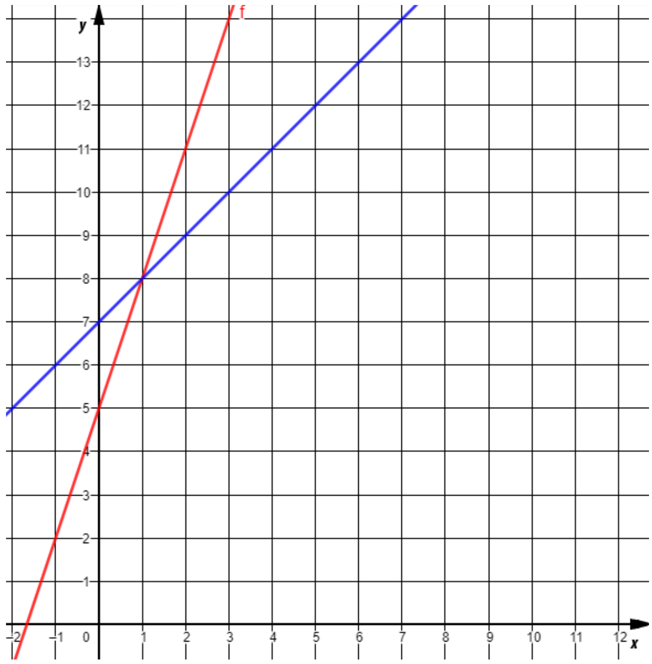
R V Part 2 of *Example 2* uses the same prompts as part 1 and students might be encouraged to describe what's the same and what's different about the two examples.

Before showing *Example 3*, ask students to consider the two lines used in *Example 2*, whether there is a point where the two lines cross, and what value x and y might have at that point.

PD In this set of examples, inequalities are used to help students make sense of the graphical representation, particularly homing in on the equality of both equations at the point of intersection. Do you feel that this a useful connection to make? How explicitly are graphical inequalities linked to initial work on graphs in your scheme of work? What's the thinking behind your decision?

Example 3:

This graph shows the lines $y = 3x + 5$ *and* $y = x + 7$.



On the diagram:

a) Mark three points where $y > 3x + 5$ ***and*** $y = x + 7$.

b) Mark three points where $y < 3x + 5$ ***and*** $y = x + 7$.

c) Mark ***one*** *point where* $y = 3x + 5$ ***and*** $y = x + 7$.

Example 3 builds on *Example 2* and shows the two lines used on the same axis.

V Similar prompts are used to those in *Example 2*, and only the constraints around $y = 3x + 5$ change in each question. The intention here is that pupils again focus on the fact that all points on the line $y = x + 7$ are connected by that relationship.

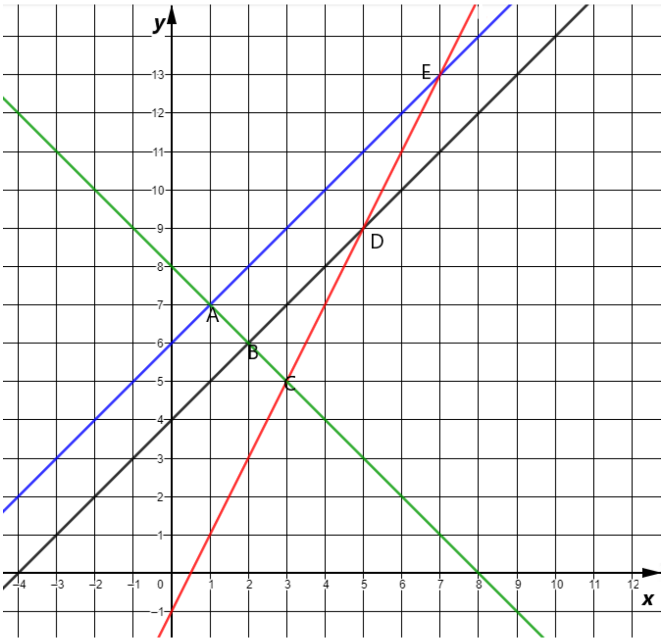
It may be productive to ask students why part c) only asks for one point while all other prompts in *Examples 2* and *3* have asked for three points, to support their reasoning around straight line graphs and the uniqueness of the crossing point.

Understand that the point of intersection is a solution to both equations.

Example 4:

This graph shows the lines of these equations:

$$y = x + 4$$
$$y = x + 6$$
$$y = 2x - 1$$
$$y = 8 - x$$



a) *When $x = 1$, $y = 7$ is a solution to two of the equations. Which two?*

b) *Two equations have the same value for y when $x = 5$. Which two equations?*

c) *Write down the values of x and y that are a solution to both $y = 8 - x$ and $y = 2x - 1$.*

PD The variation in the way that parts a), b) and c) ask similar questions with different wording is designed to draw attention to different awarenesses that students must have to make sense of solving simultaneous equations. Part a) introduces that the crossing point is a ‘solution’ to two equations, part b) draws attention to the equality of the values at the solution and part c) then builds on this to identify the crossing point as a solution to both equations.

Points B and E on the graph are not used in the three questions. You could ask students to write a question in the style of a), b) or c) for which B or E is the answer.

D You might usefully explore whether **all** the pairs of lines in *Example 4* will have a solution, and if not what that might tell us about those pairs of lines.

Weblinks

[1] NCETM primary mastery professional development materials
https://www.ncetm.org.uk/resources/50639

[2] NCETM primary assessment materials
https://www.ncetm.org.uk/resources/46689

[3] Standards & Testing Agency past mathematics papers
https://www.gov.uk/government/collections/national-curriculum-assessments-practice-materials#key-stage-2-past-papers