Secondary Magazine 145

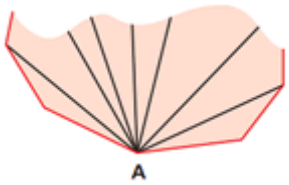
National Centre for Excellence in the Teaching of Mathematics

Welcome to Secondary Magazine 145. As we roll perilously close to Christmas, we highlight some of the joys of teaching Core Maths (including arguments for the non-existence of Santa), and offer some support and commentary on using our new KS3 Mastery Assessment Materials.

Don't forget that all previous issues are available in the Archive.

This issue's featured articles

Core Maths
In this article we are exploring a couple of great modelling activities designed and road-tested by Core Maths pioneers. New funding, announced in the budget to encourage a greater uptake of post-16 mathematics study, 'has potential to really boost the uptake of Core Maths' says NCETM Director, Charlie Stripp. Here we give more of a flavour of what it's about and why schools and colleges should teach it.



Assessing Mastery at KS3
This article hears from some teachers who have been using the recently published Key Stage 3 Mastery Assessment Materials. Maybe you've already had a chance to have a look at them? We also feature some of our favourite questions, suggest different ways they might be used and share some of the rationale behind the materials. Will this publication be as popular as the Primary Mastery Assessment Materials (downloaded over 1.3 million times since publication in 2015)?

And here are some other things for your attention:

- Since our last issue, we've launched NCETM podcasts, and the two episodes so far are firmly in secondary territory. In the first, our director Charlie Stripp, is challenged by two secondary maths teachers, to defend his assertion that students with GCSE Grade 5 should be allowed to study A level maths. In Episode 2 - just out - we hear what the exam boards learnt when they marked all those new GCSE exam papers last summer
- Please consider completing the MEI survey on the uptake of mathematics AS/A levels and Core Maths in your school. The closing date for responses is **Wednesday 13 December**
- A set of five-minute videos, providing ideas for A level Maths/Further Maths lessons, is now on the FMSP website. Each video covers an overarching theme (e.g. proof) and is accompanied by resources both for use in the classroom, and for professional development with colleagues. Alternatively, you could join their PD Video Club and take part in a Twitter discussion around a video, each Monday night. On Monday 4 December, it's Mechanics: Kinematic Graphs
- The NCETM has responded to the budget announcement last week, of extra money for schools/colleges that increase their numbers studying AS level, A level and Core Maths
- DfE-commissioned research, published on 27 November, reports on what constitutes effective delivery and teaching of Maths and English to students aged 16-18 who did not achieve A*–C at GCSE
- The Science and Technology Facilities Council (STFC) is seeking 10-12 maths teachers for a focus group to assist in the design of educational resources showcasing the science and technology the council is involved in. All expenses paid and preferential access to the resources offered.

Image credit: Page header by Kelly Sikkema (adapted), in the public domain

Core Maths

The question to ask is "Why wouldn't you teach it?", says Colin Prestwich, Yorkshire Ridings Maths Hub Lead.

As schools and colleges around the country are picking up the baton of Core Maths (known by a different name in each exam board but identifiable as 'Level 3 Core Maths') we take a look at a couple of activities that demonstrate what Core Maths can offer that traditional maths courses do not.

What is Core Maths?

The Core Maths initiative is aimed at increasing the number of post-16 students studying mathematics. It fills a gap, encouraging capable mathematicians for whom A level is too hard, too abstract, or just too great a commitment, to continue to study maths.

Core Maths is about students doing meaningful mathematical problems to increase their confidence in using mathematics to become better equipped for the mathematical demands of other courses, higher education and employment.

Core Maths is the new Level 3 qualification for students who achieved at least a Grade 4 (formerly a Grade C) at GCSE mathematics, and wish to develop their practical skills so they may apply these in work, study or everyday life.

You can view the different exam specifications on their websites:

- AQA: Mathematical Studies
- City and Guilds: Using and applying mathematics
- Edexcel: Mathematics in Context
- Eduqas: Mathematics for Work and Life
- OCR: Quantitative Reasoning/Quantitative Problem Solving.

There's more information and resources from the STEM Learning Core Maths webpage, and in FE Week, Paul Glaister, Professor of Mathematics at the University of Reading, makes a case for the provision and funding of Core Maths.

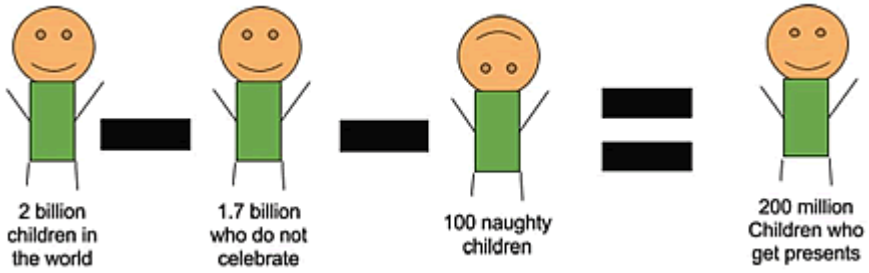
In our research for this article, it has become clear that Core Maths has generated some fierce advocates amongst its teachers. When we tentatively put out a call for contributions, we were surprised by the number of busy teachers keen to share their experiences and resources. Here, Colin Prestwich (Harrogate Grammar School) and Dominic Nice (West Sussex College, tweeting as @NiceMaths) each share one of their most successful series of lessons. First, Dominic's task asks hypothetically whether Santa could exist. This task contains the sort of frivolity and light-heartedness that both students and teachers appreciate at this time of year, while addressing the 'Modelling Cycle' aspects of the curriculum. Second, by asking students to reflect on what they spend money on, Colin's task has them calculating what gross salary they will need to earn to finance their lifestyles, addressing the 'Personal Finance' aspect of the curriculum.

Dominic Nice: Is Santa Real?

At West Suffolk College, we are in our second year of running Core Maths, with a Year 2 cohort of 18 students and a Year 1 cohort of 22. Over these first two years we've been building up resources unique to Core Maths, which engage students in a different way than we are used to from GCSE.

One such activity which students enjoyed last year was our task, 'Is Santa Real?'. Students were given the very broad task of determining mathematically whether the Father Christmas we are all familiar with could feasibly exist (there's also an accompanying sheet of teacher notes). They might choose to discover whether he could possibly travel fast enough to visit every home in the world, or might calculate how many reindeer it would take to pull a sleigh containing thousands of tons of presents.

How many reindeer will be needed to pull a sleigh?



CTRL+click image to see students' reasoning

They can make any assumptions they like (provided they're stated) and can use computers to research anything they need to. At the end of the project, they present their findings to the group. This task should preferably be left as open as possible, to allow students to find their own inspiration and to encourage unique projects, though a prompt sheet of possible ideas is provided for teachers needing to offer students more guidance.

Some of our students blew me away with what they produced: I had students researching the origins of Saint Nicholas and the population density of the Roman Empire in the fourth century; some were looking into employing a team of elves; while others were finding the number of calories in a billion mince pies, and what amount of exercise could possibly burn that number of calories before the next Christmas (click the graphic above to view their PowerPoint presentation). They each took the question in a different direction, and while more support was needed for some groups, they all engaged with the activity and produced something really impressive.

If students need more guidance, there are countless examples of similar calculations online, which students can look up themselves, and of course they can always ask for a bit of help from their teacher to point them in the right direction. However, I feel this really does work best with as little teacher intervention as possible, at least until students have decided what they want to discover. I was initially wary of giving them so much freedom, thinking they wouldn't know where to start, but from my experience with the task, they engaged with being able to direct their own projects, and discover and calculate whatever they thought might be interesting.

In terms of tying this activity to Core Maths objectives, it certainly had students thinking about how to set out a mathematical argument, research independently, critically analyse, and to an extent, Fermi estimation came in to play too. Not all Core Maths topics lend themselves to this type of task, and not all of our lessons are run this way. However, our ethos is to plan tasks to be this kind of open-ended investigation wherever possible, where students can each reach a different answer or conclusion, and can each be right for a different reason. It's a breath of fresh air to teach, which I genuinely enjoy planning and teaching every week.

Colin Prestwich: How Much Do I Need to Earn?

How can students leave school without being able to calculate net salary from gross salary? Or indeed manage their debts properly?...

Students readily recognise the importance of understanding personal finance issues, the ability to analyse large amounts of information and the confidence to tackle problems that are new to them.

Here is a series of three lessons based on 'How much do I need to earn?' that, using a problem solving thread, links a number of key big ideas:

- estimation
- web interrogation
- use of assumptions
- wisdom of the crowd
- summary calculations
- fluency with net income calculations
- calculating and using percentages in context
- use of spreadsheets
- appreciation of household expenditure.

(designed to meet AQA 1350 Mathematical Studies specification paper 1 refs 3.1 Analysis of Data, 3.2 Maths for Personal Finance and 3.3 Estimation).

Summary
You are 25, single, and looking for a sales and marketing job in Leeds. What advertised salary should you be targeting?

*Text in **bold** takes us through the steps of the project, indented text describes how my students responded to the task.*

The first step is to agree what a 25-year old will spent money on. It is a good idea to ask a student to chair this discussion.

> The students found this fun and engaged readily. The discussion was very enlightening, and I was surprised at the student responses - they had not thought about this before! They came up with: rent, food, entertainment, clothes, energy, transport, phone. There was plenty of disagreement before they realised that all their views could be represented.

Next, students are asked to come up with estimates for all the spending categories using real life searches, pure guesses, or other methods that they can justify. The 'wisdom of the crowd' principle is useful for dealing with pure guesses and Marcus du Sautoy's YouTube **clip is an excellent addition here. This is a rich task and will involve many discussions.**

> I let students come up with their own estimates and supported individuals as necessary.
>
> "What are rates for?" "Why do you need to pay council tax?" "I thought that water was free!" were just a few of the questions my class asked as they went along.

So how are the results to be summarised?

> The students found this hard to answer. Eventually putting the results in a spreadsheet was suggested.
>
> "Sir, what is a spreadsheet?"
>
> I was shocked at how few of them were able to do this efficiently. Indeed, many simply used it as a table in which to enter values - even the totals and means were calculated by hand! By allowing them to do this, inefficiently at first, they were then able to appreciate the power of a spreadsheet when shown some of the basic functions. At this stage I gave them some example questions to help them understand the data and bring in different types of percentage work.

By lesson two, students should have agreed a net monthly income required.

> My students settled on £1140 by taking averages of their various sums. The students thought that this was a king's ransom! They started to panic.

Then students are set on job searches on the internet for marketing jobs. Local newspapers, corporation websites etc can all be used – the students first need to find them.

Working backwards from net income to gross income is very challenging and it is valuable to discuss possible approaches.

Initially, my students decided to select a gross income from the adverts and then see if it gave a net salary that would be sufficient. We divided up the calculations and pooled the results. After encouragement the students realised that a spreadsheet would enable these calculations to be done much more quickly.

In lesson three, students set about producing a spreadsheet to do the appropriate calculations using up-to-date tax and National Insurance bands.

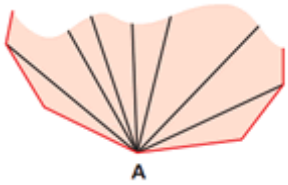
My students found this very hard and we needed a number of attempts, but I did not show them an example until all had tried hard to come up with a working spreadsheet with a number of gross and net incomes so that they could sensibly address the original question.

With a well-constructed spreadsheet, it is easy to scroll down to exceptional salaries and provoke meaningful discussion about tax contributions of high and low earners.

For my class, this was a big wow factor. I took the spreadsheet and scrolled down to an income of £100 000, £250 000, £500 000, £1 million: at this point they were shouting for more: a five-minute discussion about footballer, film star and maths teacher salaries took place.

The final step is in asking the students to reflect on what they have achieved.

Many of my students quickly came to the conclusion that most people would just try to get a job and then look at their pay packet at the end of the month and just hope it was enough, rather than working backwards as we had done. The project allowed them to see how they could use mathematics to support their everyday life and enable them to do the calculations required to plan ahead – to be proactive rather than reactive with their finances.



Assessing Mastery at KS3

The NCETM has recently produced a set of Key Stage 3 Mastery Assessment Materials to complement those available for primary teachers. In this article we look a little at the rationale behind the materials and share some experience of teachers who have used them.

Have you read the recent EEF guidance report, Improving Mathematics in Key Stages Two and Three? It offers eight key recommendations with a focus on 'improving the quality of teaching.'

The first of these recommendations is to

- *use assessment to build on pupils' existing knowledge and understanding*

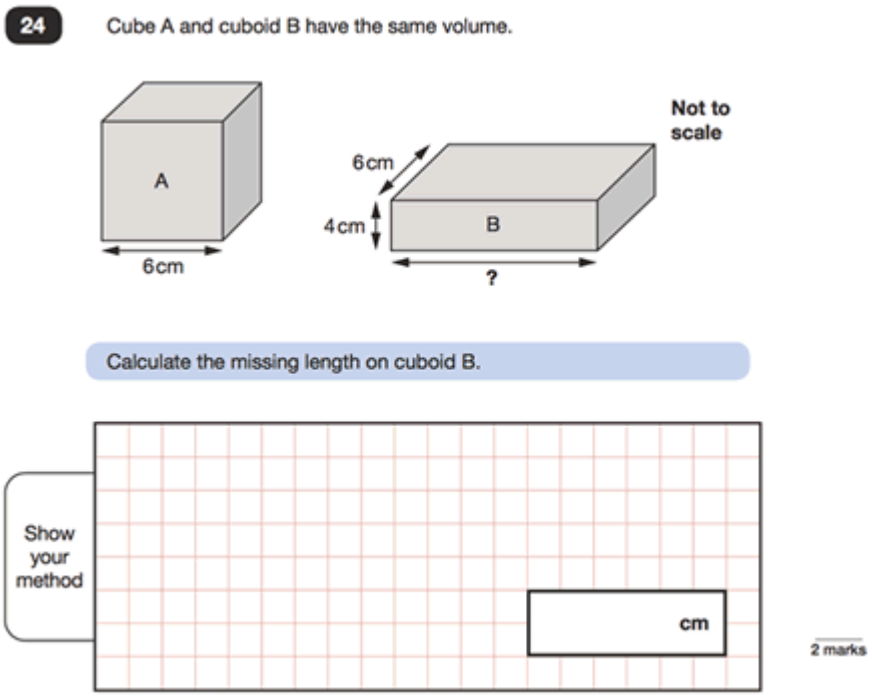
and the eighth recommendation is to

- *support pupils to make a successful transition between primary and secondary school.*

How do we, as secondary teachers, support pupils as they move from KS2 to KS3? How do we find out what they know and build on it? Are we aware of the high expectations of the new primary curriculum? You may have already noticed the impact that the increased focus on fluency in primary school has had on your current Year 7s.

As well as building fluency in calculation, primary pupils are expected to reason mathematically.

Have a go at this question from this summer's KS2 SATs:



Did you calculate the volume of the cube, or did you use reasoning to simplify the calculation? What do you think your Year 7 students did when faced with this question last summer?

The EEF guidance report mentions that

> *'one large national study of primary attainment in England found that, at the end of Year 7 - a full year after the transition to secondary school - pupils' performance on a test of primary numeracy was below their performance at the end of Year 6'*

and poses the question:

> *'Are primary and secondary schools developing a shared understanding of curriculum, teaching, and learning?'*

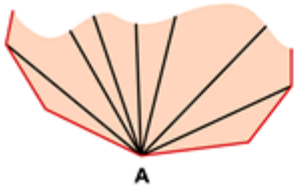
In order to help develop a shared understanding, and to support teachers of KS3 to smoothly continue the journey started at primary school, the KS3 Assessment Materials have been explicitly designed to follow on from the NCETM's Primary Assessment Materials that have proved immensely popular with teachers, now downloaded over 1.3 million times. The questions allow teachers to find out what children in their class already know and so build on this in their lessons. Questions that offer pupils a chance to reason mathematically, and to expose what they know so that the teacher can build on it, are time consuming to create so the Assessment Materials are designed to assist teachers in this.

The KS3 Mastery Assessment Materials document (published as a PDF for easy printing) is divided into sections that address many of the content statements in the KS3 Programme of Study. It is designed to be used in different ways, each giving students an opportunity to show what they know or have learned.

Used as pre-assessment to find out what pupils know already, you might predict what errors you think the pupils will make, then use the questions to find out if they do. Focus on listening to what they're saying rather than correcting mistakes for a while, then use this to inform the focus of the content, and the starting point, when you come to teach it. It's worth noting that this not only shows what pupils can't do, but also gives an opportunity to find out what they can do.

An example of using the questions in this way comes from one teacher who was preparing to teach her class a unit on angles in polygons and who used this question from the *Geometry and Measures* section:

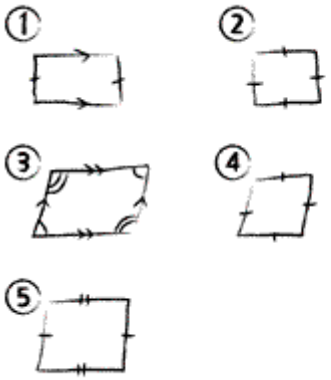
The diagram below shows part of a polygon. This polygon had been divided into triangles by joining every vertex to point A.
A part of the polygon has been torn off.
How many edges did the polygon have originally?
Explain how you know.



During the discussion that followed, she unexpectedly found that a number of students in the class shared the unusual misconception that a regular polygon with ten sides should have angles of 10°. She was able to tailor the next lesson to explicitly explore and address this misconception with the class.

She plans to return to the same question midway through the unit, or towards the end of the unit and ask the students to correct or improve their work, or talk about how they've changed their mind.

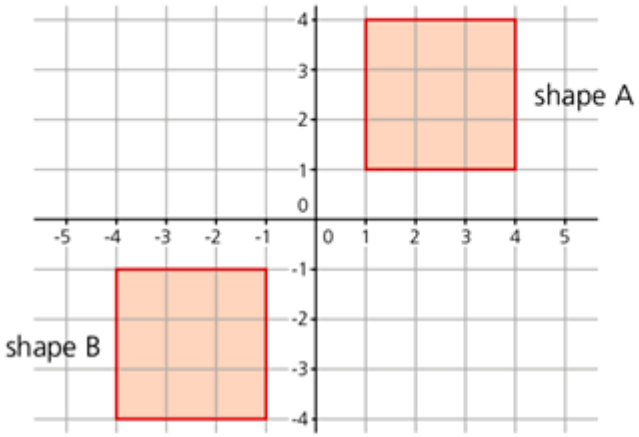
While this misconception was not one that the teacher had come across before, some misconceptions are common and can be predicted and explored using questions like those in the booklet. An interesting task at a department meeting might be to choose one or two questions, such as those below, and discuss the misconceptions that might be exposed when they are given to a class:



These shapes are drawn very sketchily and are intended to be quadrilaterals. They are marked correctly. What shapes could they be? How did you decide?

or:

Shape B is a transformation of shape A.

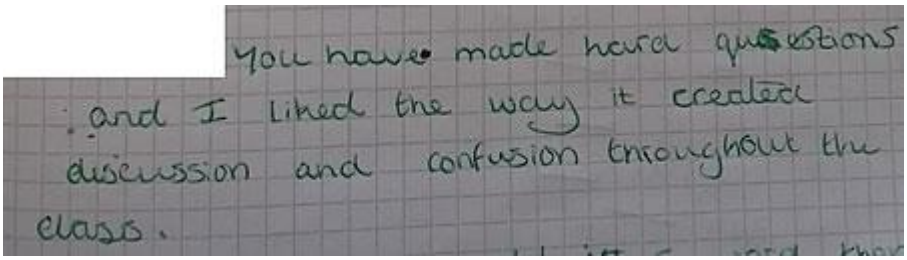


Alex says, "Shape A has been reflected to make shape B"
Berenice says, "Shape A has been rotated to make shape B"
Claudia says, "Shape A has been translated to make shape B"
Deepak says, "Shape A has been enlarged to make shape B"
Are any of them correct? If so, who?
Explain your thinking.

You might choose to use some of the questions in the booklet as an assessment task at the end of a section – maybe through homework questions for pupils to give a written explanation of their methods, or to stimulate a discussion between pupils to challenge their understanding. One teacher who used this question:

> Five people tried to calculate 18.3% of 541. Here is what they typed into their calculators:
>
> a) 541 x 1.83 b) 0.183 x 541
>
> c) 541 x 0.183 d) 541 ÷ 0.183
>
> e) 541 ÷ 100 x 18.3 f) 541 x 18.3
>
> Which of these will give me the correct answer? For the incorrect answers, can you write a percentage question that their calculation does give the answer to?

as a consolidation task at the end of a lesson on finding percentages of amounts found this comment in one of the pupils' books at the end of the lesson:



The document is intended to provide a source of questions that expose misconceptions and show pupils' understanding. A good starting point might be to show one of these questions on the board in the last five minutes or so of a lesson, sit on your hands and listen to what your class think. In listening carefully to their responses, you may find that you're better equipped to start the next lesson, building from what they already know.