Secondary Magazine 110

National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 110 of the Secondary Magazine

Now the summer term is underway and the examination season is upon us, do be sure to take a break from all your exam preparation to read this issue of the Secondary Magazine – and to enjoy the summer weather of course!

Contents

From the editor: Seeing the mathematics

If you are a loyal listener to the BBC Radio 4 programme More or Less, you will know that Tim Harford regularly explains and investigates the mathematics used in current items of news, entertainment and political debate. For instance, in a recent programme broadcast on 2 May 2014, the topic was the anniversary of Sir Roger Bannister's four minute mile. This is how the programme was introduced:

> *Sir Roger Bannister became the first man to run a mile in under four minutes 60 years ago. It's one that the passage of time has shrouded in legend. Was the four-minute mile really considered an 'impossible' physical barrier? Are motivational speakers like Anthony Robbins right to claim that the year after it was broken, the power of positive thinking helped dozens of runners to break the four-minute barrier. More or Less speaks with Sir Roger Bannister to separate myth from reality and find out exactly what propelled him to his famous feat.*

You will be familiar with the phenomenon that, for instance, if you were to buy a new silver car you would suddenly start to notice all the other silver cars on the road – this could lead to you feeling quite a trend-setter, but it is really an example of heightened perception?

As teachers of mathematics, we are perhaps more acutely aware of the mathematics inherent in everyday situations than some other people. When hearing of the sad death of a celebrity 'aged 62' do you instantly find yourself calculating his year of birth for example? Or when hearing a percentage quoted on the television you may find yourself thinking16% of what? And how do you cope with large numbers? A recent report about a possible HS2 link to Eurostar quoted:

> It would have cost an estimated £700m. The cost of the total project is currently estimated at £42.6bn.

As teachers of mathematics we may have a ready understanding of large numbers which may not be shared, particularly by the pupils that we teach.

In earlier issues of the Secondary Magazine, there were regular *Up2d8 Maths* features (later replaced by *It's in the news!*), which aimed to pick out and explore the mathematics in a topical situation. You could look at The world's fastest texter, A marathon a day, or Crop circles to whet your appetite.

Sometimes some of the best ideas are your own. When you see some mathematics in a situation that is relevant to your students in their lives, this would be an ideal context to explore the mathematics within that situation and make it real for your students. Why not share your good idea?

Secondary Magazine 110

National Centre for Excellence in the Teaching of Mathematics

Key Ideas in Teaching Mathematics – Probabilistic reasoning

In this and other issues, the Secondary Magazine features a set of six articles, written by Anne Watson, Keith Jones and Dave Pratt, the authors of the recent publication Key Ideas in Teaching Mathematics. While not replicating the text of this publication, the articles will follow the themes of the chapters and are intended to stimulate thought and discussion, as mathematics teachers begin to consider the implications of the changes to the National Curriculum. This article is the fourth in the series and focusses on Reasoning with decimals in Key Stage 3. Future articles will feature Place Value, Algebra and Probabilistic Reasoning. Previous articles focussed on similarity, ratio and trigonometry in Key Stage 3, Geometric and spatial reasoning in Key Stage 3, statistical reasoning in Key Stage 3, reasoning with decimals in Key Stage 3, and Algebraic reasoning. The opinions expressed in this article are those of the authors, and not of the NCETM.

In science, pupils are taught how to formalise their experiences of cause and effect. In mathematics, they are taught deductive systems of logic that draw on precisely defined representations, which often turn out to be helpful in science. However, determinism and logic are insufficient for the management of uncertainty. When we shake two dice, we might predict a total of 7 knowing that this is the most likely, but we would not be surprised if we were wrong. When we hang an object on a spring, there will be uncertainty about exactly how far the spring might stretch because of measurement error but, by focusing on the physics of the situation, we might predict the stretch with some confidence. The level of confidence we feel in making predictions about most situations in professional and everyday life falls between these two examples. Consider the following examples, involving both causality and uncertainty.

Professionals such as actuaries use probabilistic reasoning to compute appropriate premiums for life assurance based on life expectancies; meteorologists offer probabilistic weather forecasts; doctors provide an assessment of the risks in an operation; media give publicity to research on the dangers of eating particular foodstuffs. Every day, citizens are receivers of such wisdom and need to be empowered to evaluate that information.

Probabilistic reasoning is therefore an essential tool for citizens and professionals alike, and one that is becoming ever more significant as scientific communication becomes more available and society more technological. Looked at from this perspective, the newly released National Curriculum for Mathematics in England appears somewhat impoverished. At Key Stages 1 and 2, learners are now offered no opportunity to consider chance. As a result, while their thinking about cause and effect will have developed considerably by the time they enter secondary school, opportunities for them to account for non-causal variability in observed data will have been lost, and early intuitions of relative frequency (which research has shown to exist even before children start school) will have been allowed to wither. The task therefore for Key Stage 3 teachers to introduce pupils to probabilistic reasoning presents an immense challenge.

Research has shown how peoples' intuitions for chance are often misguided. For example, there is a tendency for people to judge chance by trying to evoke occurrences of similar events from memory. Unfortunately, the memories evoked are likely to be those most salient rather than those most frequent. This could for example explain why some people find it difficult to accept the relative safety of travelling by air when any aeroplane crash is always headline news. Or it might explain why people believe that a 6 on a die is less likely to occur than other numbers, perhaps based on painful memories of trying to obtain a 6 to get started in a board game. There is also a tendency to expect a random situation to self-correct so that outcomes that are so far underrepresented in a sequence of results are now more likely to occur. For example, a gambler might expect the roulette wheel that has just created a run of red numbers to be more likely to throw up a black number. There is also a strong tendency for people not to recognise situations as amenable to probabilistic reasoning and instead regard the situation as simply a matter of

luck. Nevertheless, a strategic approach based on probabilities will be more successful in the long term than one that relies on good luck.

The lack of any reference to probabilistic reasoning in the new Key Stage 1 or 2 curriculum will mean that misguided intuitions such as those above will have been firmly established by the time pupils enter secondary school. It is possible to go about everyday life without recognising the missed opportunities for applying probabilistic reasoning. Whereas young pupils might develop some basic sense of cause and effect from everyday perception, the stochastic cannot be perceived in such an immediate way since the effects are based on proportions and only noticeable in the longer term. Probabilistic reasoning is more likely to emerge through systematically organised experience in school.

Mathematics teachers at Key Stage 3 need to introduce pupils to the use of probabilistic reasoning by helping them to recognise their naïve or misguided intuitions. Situations that involve uncertainty can be analysed by asking pupils to make predictions and to account for differences between their expectations and what actually happens. Pupils' attention will need to be drawn not only to short-term variation but also to the sense in which the longer-term aggregated view is in fact predictable – at least in a probabilistic sense. Typically, pupils will not appreciate that the Laws of Probability can be used to explain the behaviour of a wide range of phenomena. The knowledge they hold about such matters will develop within particular situations. For example, the pupils might eventually realise that 'the more times you toss the coin, the more even are the numbers of heads and tails' (in a proportionate rather than absolute numerical sense). However, it does not follow that the children will realise that 'the more times you throw the die, the more even are all six outcomes'. Some effort has to be made to widen the domain of knowledge to both coins and dice and subsequently to a general Law of Large Numbers. The Key Stage 3 teacher should also expect that such knowledge when it first emerges will be overgeneralised; after only a small number of throws of a die, a pupil might be surprised that the frequencies of the scores are not at all equal. Simulations offer opportunities for pupils to generate large sets of data quickly and so facilitate systematic observation beyond what is possible in practical experiments.

It would be easy to interpret the four bullet points under probability in the new National Curriculum in a narrow literal way that did not address any of the above issues. The previous paragraph represents pedagogic knowledge about teaching probability that is not set out in the curriculum but which research would suggest is essential.

Most tasks that have developed for the teaching and learning of probabilistic reasoning have in the past focused on coins, spinners and dice. However, these situations do not embrace common partially-determined situations such as the examples given at the start of this article. The actuaries, meteorologists, doctors and researchers draw on models where uncertainty and determinism co-exist. Key Stage 3 and 4 teachers might ask their pupils to build their own probability-based models by exploiting new modelling tools (for example, TinkerPlots).

For example, pupils might enjoy trying to create their own weather forecaster that uses factors such as today's weather but also recognises that to some extent weather is unpredictable. Or they might like to model sporting outcomes such as future football results based on the form of the competitors but again recognising the uncertainty in such situations. By modelling with probabilistic reasoning in such software, pupils might not only engage with meaningful everyday decision-making, they might also see connections between probability and statistics that are perhaps less evident in the behaviour of coins, spinners and dice.

Keith Jones, Dave Pratt and Anne Watson

In keeping this series of articles brief, there is no space for full references; these can be found in the book Key Ideas in Teaching Mathematics

Image credits
Page header by Ella's Dad (adapted), some rights reserved

A resource for the classroom – problems involving probabilistic reasoning

This issue of the magazine has an article linked to the recent publication, Key Ideas in Teaching Mathematics. There is a website that accompanies the book which provides links to some relevant resources. Our article this month is related to probabilistic reasoning so the resources for the classroom are a suite of problems that involve reasoning about uncertainty. Some of these problems may be familiar whilst others may be new to you; all have been chosen to develop and deepen understanding.

The website identifies six themes within the key idea of reasoning about uncertainty; intuitions, misconceptions, simulations, modelling, distribution and risk. It states:

> *Lack of information creates uncertainty. In some circumstances, such as card games or sport, this uncertainty can be compelling for both players and spectators. By reasoning about uncertainty, a player can adopt a strategic approach that will improve his or her success rate in the long term, even if it appears 'unlucky' in the short term.*

The individual problems are:

- Chance maker
- Games leading to ideas on probability
- Using probability computer games
- Evaluating probability statements
- Design a Board Game
- Which Spinners?
- Rabbits
- How risky is life?
- Deborah's Dilemma.

What will you do now?

You could:

- select a problem and try it out with a particular class
- select a problem and work with a colleague to consider how you can use the problem to develop understanding for a group of pupils
- include some of these problems in your scheme of work
- consider how these problems develop the powerful aspects of the curriculum and the links between them.

Do tell us what you find out...

Image credit
Page header by boviate (adapted), some rights reserved

5 things to do

You may like to consider a free two-week download of Number Gym software.

For fans of Batman...

Have you heard about the online STEM engagement event I'm an Engineer, Get me out of Here!? The aim is to get Maths, D&T and Science secondary students talking with engineers online, for two weeks at a time. It's completely free for schools and shows students how they can use what they learn in the classroom in the real world. The next event is 16 - 27 June. There are four 'zones': Apprentice, Cities, Food, and Health.

The British Society for the History of Mathematics is running a weekend event, *Counting and calculation - a journey through practical mathematics*, at Rewley House, University of Oxford on 21-22 June. The focus is on the history of counting and calculation.

If the link between mathematics and cookery interests you then have a look at these 'Sconic Sections'.

Image credit
Page header by DidWee (adapted), some rights reserved

Tales from the classroom: non-pupil days

One of our non-pupil days each year is "moderation day". It is nearly always the last day of the spring term. It's an institution with roots well before any of our current SLT were at the school. While it appears a bit generous, it is a very much appreciated breather and catch-up, and its existence almost certainly "buys" us fringe benefits elsewhere. Certainly as a maths teacher it feels like the only time ever when we seem not to have to be taking the lead. Just about every Inset day or meeting requires us to either take a lead, or affects us at least as much as all other departments - and usually, because of mathematics being core, a little more than most. It doesn't really bother me. I quite like the pressure and focus, and just get on with it. When other subject leaders complain about an extra revision session, or another mock just for maths and English, I used to offer a swap of role. I've yet to be taken up on that. I now just raise my eyebrows and move on.

We had a very "loose" plan for our "moderation" day in maths. Unfortunately for me the first hour and a half were taken up responding to parents who were taking the opportunity of a day without "teaching" to come "up the school" to, well, "get things off their chest". We had planned to spend the first hour or so finalising our plans to increase our problem solving at KS3. When I arrived nearly two hours late it was yet to happen, my team having kindly deciding that I might like specific input. They had, however, started "break" and were well into it each sharing and marking the buns they had baked (they had also kindly left me three). I know that in other departments in my school that I line-manage, I would at this point be feeling exasperated at the wasted time, and probably be asking what had been done in the past two hours. But, with my own team, I know better. In any case, they got in before me with about five messages to call x, find y, and reply to email from z. I reluctantly promised my return, this time within five minutes.

I surprised them all by returning in three minutes (I'd had a moment of insight and decided only to respond to z - it was my head teacher after all). By now the buns were down to two, yet a hot tea had appeared. As I began to sip, a data sheet appeared on the board. It was a question-by-question analysis of all higher students that had sat a mock the previous day, together with a priority list of topics for each individual student, lists for each group, and highlighted students where we could swap groups to allow us to maximise our coverage of weakness by rejigging groups at different times. As the first of the buns hit my lips, I was asked, "Could we just have two minutes to finish this, we only have two more students to enter, then we have done everybody, except James from your foundation group because he sat the higher?" So, not only was the higher analysis done, but so was the foundation. I was correct in keeping my calm and not becoming exasperated. They actually are much better without me!

I was now beginning to wonder if they had put some of those parents up to it, just to get me out the way. "What we need from you [me] is some help on how to get them better at this stuff. You've got more experience of this than the sum total of all of us put together".

OK [relief], they do still need me. I then spent some time talking about ideas to keep students motivated as we move into the twilight zone of exams, pre-exam briefings, thinking about how we have resources that students can dip in and out of, while allowing us to keep track of their progress over the all-important ten-week countdown. The head popped in to wish us a good holiday, and ask if our day was useful! And then we began looking at resources, discussing individual questions in resources, and how those resources would tease out misunderstanding, or reinforce good exam technique. Suddenly it was 3pm and the end of term. I quickly hurried my team out the door to start the holiday they so greatly deserve. With a warm heart and feeling good about where we were, I set about the pile of photocopying generated by a very productive "moderation" day, musing that with very little planning from me, we had had one of our most productive collaborative days. I should keep out of the way more often. I also wondered if as

teachers we had more of that collaborative time built into our working weeks, would our students not learn more, even if they saw less of us!

At this point the head came in wanting some info...

The author is a mathematics subject leader and assistant principal working in the South West