Secondary Magazine 105

Welcome to Issue 105 of the Secondary Magazine

In this issue of the Secondary Magazine the idea of developing mathematical understanding links several articles. From the editor considers opportunities for departments to work collaboratively to focus on developing understanding; there is the first of a new set of articles in the Key ideas in Teaching Mathematics series and there are some Geometry problems chosen to develop understanding. This might provide something to think about over the Christmas break? Very best wishes for 2014 from the NCETM.

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National Centre for Excellence in the Teaching of Mathematics

From the editor: developing understanding

The Ofsted report Mathematics: made to measure states:

> *While the best teaching developed pupils' conceptual understanding alongside their fluent recall of knowledge, and confidence in problem solving, too much teaching concentrated on the acquisition of disparate skills that enabled pupils to pass tests and examinations but did not equip them for the next stage of education, work and life.*

It also recommends that schools should

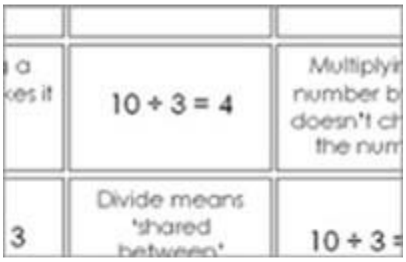
> *develop the expertise of staff in choosing teaching approaches and activities that foster pupils' deeper understanding, including through the use of practical resources.*

As teachers of mathematics, it is assumed that lessons are constructed to allow pupils to develop their understanding of mathematics and consequently make progress in their learning. How has your department responded to Mathematics: made to measure? How does your department consider the strategies and resources that you use to develop and deepen understanding of topics across the mathematics curriculum? You may find the following resources to be useful starting points.

The NCETM microsite What makes a good resource has a variety of resources that have been tried, tested and recommended by teachers. In Teaching negativity, a teacher talks about the difference between teaching negative numbers (two minuses make a plus etc) and teaching the concept of negativity using the context of happiness.

Simultaneous biscuits is another resource from the same microsite where a teacher describes how her pupils learn to solve simultaneous equations in the context of buying packets of biscuits. It is the aim of the teacher to generate some 'rules'

The Always, sometimes, never task allows pupils to explore their understanding of multiplication and division; in articulating the constraints implicit in the sometimes statements, students have an opportunity to check and develop their understanding.



What will you do now?

There are probably some topics that you know your pupils find difficult. You could pick a topic, say negative numbers perhaps, and make this a focus for the coming weeks. Here are some suggestions. You could:

- collect some rich activities that focus on negative numbers
- work on these activities as a department
- team-teach an activity
- observe another teacher working with their class
- talk to pupils about their understanding of this topic
- make a YouTube video about the topic.

Do tell us how you are developing pupil understanding in your teaching.

Key Ideas in Teaching Mathematics - Similarity, ratio and trigonometry in KS3

In this and subsequent issues, the Secondary Magazine will feature a set of six articles, written by Keith Jones, Dave Pratt and Anne Watson, the authors of the recent publication Key Ideas in Teaching Mathematics. While not replicating the text of this publication, the articles will follow the themes of the chapters and are intended to stimulate thought and discussion, as mathematics teachers begin to consider the implications of the changes to the National Curriculum. This article, which refers to the new curriculum, intended for first teaching in autumn 2014, is the first in the series and focuses on Similarity, ratio and trigonometry in Key Stage 3. Future articles will feature Geometric Reasoning, Statistical Reasoning, Place Value, Algebra, and Probabilistic Reasoning.

The geometry curriculum for Key Stage 3 requires that all children should be able to 'use ... trigonometric ratios in similar triangles to solve problems involving right angled triangles'. As with most statements in the programme of study this masks a number of interesting issues. At the very least, we need to know what is meant by 'solve problems'. The relevant overarching statement reads: 'solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.' Teaching students to solve routine problems with trigonometry is often approached in a procedural manner, sometimes using mnemonics. Typically, many students misapply these procedures by misidentifying right angled triangles, misidentifying the appropriate sides, and misapplying the inverse function to find angles. However, the inclusion of trigonometric ratios for all students by the end of KS3 may well lead to procedural teaching because of time and assessment pressures.

The research in this area focuses on the efficiency of particular methods in enabling students to solve problems without the usual errors, but most of these studies depend on particular tests of short-term outcomes rather than focusing on longer term development of understanding. Rather than focus on particular teaching methods, therefore, it seems more useful to analyse the conceptual threads which could support understanding trigonometry ratios at KS3 for a wide range of students, and which also contribute to strengthening understanding in other areas of mathematics.

In a survey of 29 teachers' preferred approaches to teaching trigonometry, most teachers started with collections of triangles with which students would make measurements, tabulate and compare results. Four different conceptual pathways underpinned these approaches:

1. similarity approach: the angle is fixed so that similar right angled triangles are being explored. The ratios of corresponding sides are found to be constant – some teachers avoid using the word 'ratio' and talk about division instead. An advantage of this approach is that it relates to work on similarity and enlargement. The trigonometric ratios can be seen to be specially related to right-angled triangles, which has given them historical and practical importance. A disadvantage of this approach is that it is limited to angles less than 90°.
2. functional approach: the angle varies and the ratio of sides also varies with a fixed hypotenuse (for sine and cosine) or a fixed base for tangent. This approach can be related to the use of graphing software, and contributes to students' experience in analysing and interpreting graphs. The tangent approach relates to gradient. A disadvantage is that the use of ratios to solve right-angled triangles appears to be an unrelated procedure.
3. multiplier approach: if a unit radius circle is used in the functional approach, sine and cosine are multipliers. This approach relates to students' understanding of scaling. The language associated with right-angled triangles then emphasises the multiplicative relation between sides, and can be helpful later when resolving vectors.

4. combination: for a range of right-angled triangles, angles and sides are measured and ratios calculated. Results are compared and conjectures can be made. This approach can trigger discussions about accuracy with measurements, and the status of conjectures from data, but a large number of results and needed because of the number of variables involved and some students may remain unconvinced.

A purely procedural approach is not included in these descriptions, and yet in one study such an approach was found to be more efficient in helping students to solve routine problems (finding missing sides and angles in given right-angled triangles), when compared to the unit circle approach. The decision that faces teachers of the new curriculum is whether to 'cut to the chase' and use a procedural approach in order to answer routine assessment questions, or whether to build up a conceptual understanding that is connected across mathematics and of lasting value. In all the above conceptual approaches, there has to be a stage of encapsulating the ideas into definitions of sine, cosine and tangent and some practice of use. This stage of formalising processes after conceptual exploration is typical of school mathematics at this stage.

To solve non-routine problems requires students to recognise when and how the use of trigonometric ratios is relevant. This requires experience of analysing a problem and coordinating their resources to tackle the problem. At KS3 only right-angled triangles are considered, so these have to be identified in the problem. In some problems they might have to introduce a right angled triangle in order to set up the situation they can then solve. They need experience in doing this. For example, many survey problems are solved by constructing a perpendicular to a given straight line in order to create right angled triangles. This process offers a meaningful context for some classic geometrical work.

Students then have to be familiar with how the sides relate to the angles so that they can use the appropriate ratios - this is also true in routine problems. They then have to decide how to use the ratios to find the unknown measures - this is also true in routine problems. This step may not be obvious, as a problem might include several right angled triangles in which, for example, the hypotenuse of one might be the opposite side to a particular angle in another. Students therefore have to make decisions about the sequence of work they have to do to solve a particular problem.

Trigonometry, therefore, can be seen as a context for coordinating a range of KS3 mathematics as well as developing problem-solving skills more generally. As with many hard-to-learn ideas, the alternative to a procedural approach is to coordinate the development of components between years and teachers. Measurement, division, ratio, similarity, construction, graph plotting and scaling all have a part to play in this development.

Keith Jones, Dave Pratt and Anne Watson

A resource for the classroom – ratio and proportional reasoning problems

This issue of the magazine has an article linked to the recent publication, Key Ideas in Teaching Mathematics. There is a website that accompanies the book which provides links to some relevant resources. This month's article is related to trigonometry: trigonometric ratios are now a requirement for all students in KS3. A full understanding of these includes knowledge of measurement, division, ratio, similarity, construction, graph plotting and scaling. Thus it can connect students' knowledge of number, proportion, and geometry. The resources for the classroom are a suite of problems which have been selected to develop reasoning in ratio and proportion. Some of these problems may be familiar whilst others may be new to you; all have been chosen to develop and deepen understanding.

The website states:

> *The fundamental concept behind ratio and proportional reasoning is the multiplicative relationship in which quantities, whether discrete or continuous, are compared using scalar multipliers. For students who may only have a 'repeated addition' sense of multiplication, understanding ratio wherever it turns up can be hard. Understanding these ideas can be 'make or break' learners of mathematics, so there is a huge body of research about how people might learn it better. Proportionality appears everywhere in the curriculum and in other contexts, but explicit teaching often treats it in a few standard unconnected ways. When the concept appears in measures, trigonometry, or gradients, it is often treated implicitly.*

The individual problems are:

- Zin Obelisk
- Ratios and dilutions
- Product wars
- Balancing 2

What will you do now?

You could:

- select a problem and try it out with a particular class
- select a problem and work with a colleague to consider how you can use the problem to develop understanding for a group of pupils
- include some of these problems in your scheme of work
- consider how these problems develop the powerful aspects of the curriculum and the links between them.

Do tell us what you find out...

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Focus on…Every Child Counts

In this article, the third in the series, Andy Tynemouth from Edge Hill University and National Adviser for Every Child Counts *discusses the problems of supporting learners who struggle to make sense of mathematics*

In the preceding two articles I have set the scene for the present one. The first article was published in Issue 103 and the second in Issue 104. These articles form a series of which this represents the penultimate. Thus far I have written about the theory that underpins the practice of using a Learning Map, explained its context and development within the intervention programme Numbers Count (NC)[1] as part of Every Child Counts (ECC) from Edge Hill University. Much of what I write here depends on an understanding of what comes before so if you have not yet read the first two articles then now would be the perfect time to do it. On the other hand you could read this, pique your interest and then go back later: although that might be putting the horse behind the cart.

Last time I promised you that in this article I would outline three reasons that I see the Learning Map working so well. I will deal with the first two in this article and the third in the next issue. I will also explore some more of our Learning Map practice using pictures and videos as well. In the next issue we'll also look to future developments as the Learning Map has already led to interesting new approaches.

The three reasons that I believe the Learning Map has been so successful are:

- it makes the learner's prior and current learning clear to them;
- it supports the development of a 'growth mindset'
- it provides a platform to maintain a positive and effective educational dialogue between the teacher and the learner

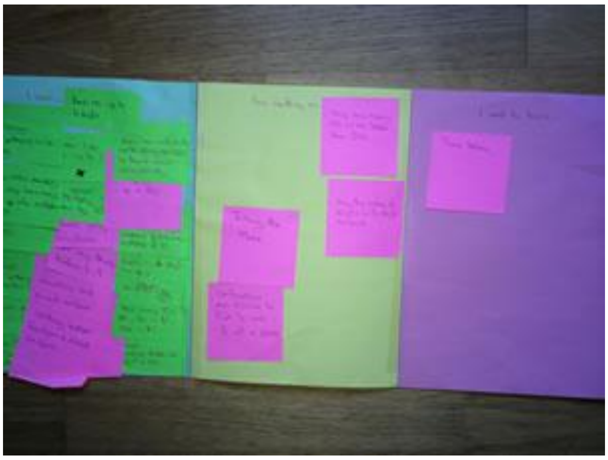
Let's look at these in order.

The first reason that the Learning Map has such a powerful impact is that it empowers the learner in a way that is very real to them. When we conceived the idea of the Learning Map it was clear to us that in order for it to have a positive impact it was important to heavily emphasise the 'I cans' and limit the as yet unachieved targets. The teacher would review the Diagnostic Assessment (see previous articles) with the learner, writing on post-its all the resources that the learner had at their disposal. Statements of their resources might read *'Count on and back in tens from any number'* or *'Use Dienes to show the composition of 4 digit numbers'*. The teacher would ensure that there were many more 'I cans' than anything else. There might be as many as 20 'I cans', 6 'what I'm working ons' and 3 'what I'd like to learns'.

Upon completing the Learning Map the teacher would usually ask the learner how they felt about their Learning Map. Almost without exception this prompts remarks like 'I never realised I knew that much stuff' or 'I can't believe that's all for me'. The picture below illustrates what a Learning Map might look like following its construction by the teacher and the learner:

The first thing you will notice is the balance of statements. Far more are green, in the *'I can do'* column, than are amber or red, in the *'I find tricky'* or *'I want to work on'* columns. This balance gives the learner a real sense of their own ability and reduces the amount of apparently unavailable mathematics. It is important to acknowledge that the inclusion of statements is negotiated between the teacher and learner. NC teachers report that despite extensive Diagnostic Assessment (see the first article) they still learn more about their learners during construction of the Learning Map. Something about it provides a different context where both parties 'sit back' and objectively discuss the learner's resources.

Furthermore the post-its are mobile so the learner can move the statements across from right to left as they achieve their targets. The picture below shows a Learning Map at the end of the programme:



Many of the reds have migrated from *'I want to learn'* to *'I can'*. These migrations take place over time and in dialogue between the learner and the teacher. Sometimes the dialogue would be initiated by one, sometimes by the other, and at different times during the lesson. Frequently the teacher would start the lesson either by reviewing previous learning or by highlighting the existing resources, already in the *'I can'*, that will support the learner in the coming lesson. At other times something that has happened or been said during the lesson would spark a review. In this video you will see two clips back to back of J and C working with me during the KS3 trials. In the first clip you see the Learning Map being used to set the scene. I use the Learning Map to help J to see the connection between her existing resources (prior learning) and what we will be looking at next (current learning – learning which is currently taking place in the zone of proximal development: to be further unpacked in the next article). In the second you will see C and I reviewing the learning he has done since we have been together. C's ability to accurately answer the question 'how many 4s in 40?' came as a real revelation to me (perhaps to both of us). In both cases the

Learning Map allowed us to see the route from prior learning to current learning. This also allows us to project towards future learning – the *'what I want to learn'* column.

The second reason is to do with the impact that this ability to map, or track, their own learning has on the learner's self-image. As I have said previously, many of the lowest attaining in mathematics have developed a very negative view of themselves as learners. Some, even as young as six, have already started to develop what we call 'the fear' or what might be more commonly known as 'mathematics anxiety' ('math anxiety' to our American cousins). Whatever we call it, the impact of suffering from it is severe: attainment is capped, leaving the learner vulnerable to all the social ills that I mentioned in the footnotes of the last article. The mechanism is simple and terrible and can start anywhere in the cycle. Roughly, it goes like this: *'I am (or see myself as) poor at mathematics; I become afraid of failing at mathematics; I do not engage with mathematics; I become poor at mathematics;'* and repeat. In short, it's a vicious circle, and breaking this circle is the single most important thing to do for someone locked into it. Sufferers, heartbreakingly even the littlest ones, describe themselves as 'unable' to do mathematics, as people who 'can't learn mathematics', who are simply 'just not good at it' – as if that meant anything[2]. I vehemently disagree with their prognosis, if not to their face. It seems to me that they are developing what Carol Dweck describes as a 'fixed mindset'[3]. That is to say that they don't believe that they can increase their mathematical capacities, they just get things 'wrong' because they aren't good enough.

Much of the blame for the existence of this vicious circle, and the mindset it produces, may well be fairly attributed to a combination of how we conceive of mathematics, and therefore how we teach (or, more optimistically, how we have taught) it. If we are to take the view that mathematics is some sort of fixed body of knowledge[4] – out there to be discovered, as it were – then it seems reasonable to assume that you can tell another what it is and how it works and they will either know it or they won't. This is what is commonly known as the 'transmissionist' paradigm. I.e. a fixed body of knowledge is transmitted from a learned teacher to a passive and receptive learner. Implicit in this view is a powerful notion of 'right' and 'wrong'. If there is a fixed body of knowledge, which can be transmitted from the teacher to the learner, then the learner will either get it 'right' or they will get it 'wrong'. This is (perhaps a little too commonly) expressed in the classroom as praise for the 'right' answer, no praise for the 'wrong' answer.

As all teachers know, learners are very good at discerning where their teacher's attention lies. In most circumstances it is to the learner's advantage to know where the teacher's attention lies. It helps them to attend to the most productive aspect of the experience. In this case the teacher's attention, as far as the learner is concerned, is whether they have got the answer 'right' or got it 'wrong'. In fact the teacher is transmitting, not a fixed body of knowledge, but a virus that can prevent a learner from being able to engage with mathematics, however you conceive it. The fear of getting mathematics wrong exercises a strong influence on those whose attention is focused on 'right' and 'wrong'. Of course, not all learners will develop the disease – but many will.

For these reasons ECC professional development emphasises the importance of placing the emphasis on praise, scaffolding and support focusing on the effort, thinking and process of solving mathematical problems rather than on their outcome. The Learning Map has proved to be a sturdy tool for levering learners' thinking away from an outcome orientation ('right' or 'wrong') and towards a process orientation (effort, thinking and problem solving). We believe that this is because the learner is constantly reviewing their resources in the light of the current and future learning. What we would call a formative rather than summative process. The Learning Map is not about 'right' or 'wrong' but about possibilities, and the chance to talk about the process, not just the outcomes.

Another reason is that the migration of the learner's statements is the product of dialogue, as opposed to monologue (see footnotes from the second article). NC teachers tend to adopt the view that they are seeking out meaning with their learners, not that they already know all the meanings and transmit them to their learners. As we will see in the next article, these are fundamental to a view of good practice.

Footnotes

[1] NC is one of the Every Child Counts suite of programmes developed and rolled out nationally by Edge Hill University. It is a teacher-led, one to one mathematics intervention aimed at the lowest 6% of attainers. These are learners for whom nothing else will work. Originally conceived and designed to target Year 2 its success has led to its development to address the needs of learners up to, and including, Year 8. For more information please see the Every Child Counts website.

[2] Apparently it does to 30% of the population. In a survey of the adult population, commissioned by National Numeracy and conducted by YouGov, they found that '30% think that maths ability is more innate than learned'.

[3] Carol Dweck (2000), a very estimable American cousin and highly regarded psychologist, suggests that learners can develop two very different mindsets or self-theories. Those with a fixed mindset believe ability to be fixed or innate and will tend not to engage in tasks that they are not likely to succeed with easily. Those with an incremental mindset believe that through their efforts they can progress, even with something that they struggled with. Crucially, her extensive studies seem to show that it is possible to alter these mindsets through intervention. Perhaps as interesting is that this may be true for learners beyond secondary school age.

[4] In his 'Philosophy of Mathematics Education' Paul Ernest argues that there are a number of views of the ontological status of mathematics. The two that I find best represent the dichotomy that we are faced with in this context are:

- Platonism - is a belief that mathematics is a static unified body of certain knowledge. It is there to be discovered and according to Ernest leads to teaching that focuses on the development of conceptual understanding and unified knowledge
- Conventialism - holds that mathematical meaning is based on social and linguistic convention. It is a result of human activity, a dynamic process.

I associate the former with a view of mathematics more likely to encourage a teacher to emphasise outcome ('right' and 'wrong') and the latter more likely to encourage the dialogic view briefly discussed in the last article and espoused by Anna Sfard (2010).

References

Dweck, C. (2000) *Self-theories: their role in motivation, personality, and development*. Brunner/Mazel Hove

Ernest, P. (1991) *The Philosophy of Mathematics Education* Abingdon: RoutledgeFalmer

Sfard, A. (2010) *Thinking as Communicating* Cambridge: CUP.

5 things to do

Charlie Stripp, Director of the NCETM, has started a series of occasional blogs, on topical matters of mathematics education. The blogs, with a light-hearted nod to their mathematical content, are brought together under the title Charlie's Angles. The latest entry is The Joy of Cancelling!

If you can get to London, you may like to attend one of the free Gresham College lectures such as Notation, patterns and new discoveries on 23 January 2014 at 6pm, or Probability and its limits on Tuesday 18 February 2014 at 1pm. Alternatively you can watch previous lectures online, such as Butterflies, chaos and fractals.

You could read the research from another Nuffield Foundation project Key Understandings in mathematics learning, conducted by Professor Terezinha Nunes, Professor Peter Bryant and Professor Anne Watson. A suggested starting point might be the chapter linked to our geometric theme in this issue, Understanding space and its representation in mathematics.

Have a look at the new website that provides diagnostic questions for mathematics and think how you might use these to probe students' understanding.

This might make you laugh.

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Tales from the classroom: who is watching you?

'm in a fortunate position in my school in that I get to lead an assembly every Friday morning for our Year 11 students. We don't really cover any of the normal spiritual or moral aspects in these Friday morning assemblies. We just aim to raise motivation, engagement and of course students' self-esteem. We have worked really hard, particularly in our core subjects to raise standards, and have seen ourselves rise from the bottom quartile within our county to now be securely in the top quartile. Four years ago it was a real struggle, but as students have seen older student progress, results rise, they too have both wanted a piece of that, and realised they can be a part of the process too. Our success appears to breed further success. Consequently a main theme of our assemblies is celebrating success, so they tend to be lively and enjoyable occasions. A great way to start a Friday.

The vagaries of our timetable mean that directly after the assembly I have one of my Year 11 groups. All of them have expected targets of C, and they all attained that last summer. We are now striving for an A, (dare I say A* for a few?) and on the whole we are getting there. Half of them I've taught for what seems an eternity, and the rest I picked up at the beginning of this year. This is an unusual group for me. My normal staple is the disaffected level 4c, or level 3 on entry. I spend a year pumping them up to believing they can achieve a C, and most normally do.

What is very strange to me about this "post-assembly" lesson is that it never really seems to start, yet by the end of it we seem to have achieved quite a bit. I always arrive last having caught up with any winners from the assembly with hand-shakes or photos for the achievement noticeboard. Most of my class will be sat down with their books out having a chat, sometimes about maths, often about the assembly, and frequently about me.

Last Friday I arrived at my room to find students talking about a video they had seen. Concerned it may have been inappropriate I discreetly listened in. I was pleasantly surprised to find it was one that I had recorded for them to help them with their revision on surds. Most definitely a video-nasty! It seems the repeated pasting of links into emails, directions to my YouTube channel when marking books, and little slips with the URL handed out at parents' evenings was making an impact - even if only for four students.

I attempted to get started on Histograms, but the distraction of the "What video? etc." was making better inroads. With false modesty I'd thought I would exploit the moment and show the start of the video. It was part of their homework, so a gentle reminder would not go amiss. We discussed histograms whilst the waited for the school network to allow me on. So, 15 minutes later we were ready to roll with the video.

There are about sixty videos that I have recorded over the past year or so. Most are about tackling topics in mathematics in preparation for an exam, but there are some tutorials for my staff to help with their data analysis, so it took a while to search for the one we wanted - and this is when it all became a bit of a joke.

Jack started a round of applause as he spotted one video had 104 views! Zaz fanned the flames with:

> "Yeh that's five times more than the one on Composite Area - Wow sir, you are really hitting the big stuff!"

My only repartee was

> "Well Zaz, at least you can multiply by five! I've taught you that if nothing else."

It was all very good natured and we were quite enjoying ourselves, although I was attempting to maintain a modicum of authority and suggest they really ought to watch them because they are helpful. Then Josh, normally a very quiet lad, hollered “STOP!”

Everybody fell silent - I only wish I could have the same effect! (I’m putting Josh’s success down to its rarity value.) Then followed a bit of Josh and Sir ping-pong:

> “What Josh?”
> “Sir look”
> “I am Josh - at you”
> “Not at me Sir, at the screen”

(I look at my computer screen)

> “Not that screen Sir, the whiteboard Sir”
> “Josh - they are the same - that’s how a whiteboard works…”
> “No Sir look there, its three thousand nine hundred and ninety six”.
> “What is Josh?”
> “The number of views for that video”

Well Josh was correct. One of my videos now has over 4000 views. I was flabbergasted. Jack however, was not…

“That’s 'cos it's being used for torture”

All very amusing.

Why write about it? Well, every one of those students went home and watched that video. And that is my reason for noticing it. Every student went home to watch it - just because lots of other people had watched it. I have no idea why this particular video has such a disproportionate number of views, other than this idea that success breeds success. Or perhaps interest spurs interest…

Recording the videos is a pain. I battle constantly with the technology. There is such frustration with old computers crashing, interruptions, the whiteboard and mic not connecting properly. Yet, I do it because even for the videos that have just made it into double figures for the handful of students that have watched them, they have made a real difference. I doubt such a difference is being made for the other 4000 viewers. It has, however, opened my eyes to the world in which my students live. I’m not a technophobe, but neither am I sleepless before the latest release. Yet I hadn’t really, and still don’t, communicate in this virtual world in a way that properly motivates my own students into watching the videos I record especially for them. Yes my videos are useful to my students that bother to watch. And surprisingly and unexpectedly I also have a genuinely worldwide audience. Yet unfortunately I still struggle to reach those closest to me. It is both reassuring and alarming to realise that success will breed success, yet success is itself so fickle. Grab it while you can…

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