Secondary Magazine 135

National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 135 of the Secondary and FE Magazine

In this, the last edition before schools close for summer, we offer some things to mull over on the beach. If survival of the last few days is your main preoccupation this week, set some maths quizzes and sneak a read of the Secondary Magazine. Feel free to let us know any thoughts about this magazine by email to info@ncetm.org.uk or on Twitter @NCETMsecondary.

Contents

Heads Up
Here you will find a checklist of some of the recent, or still current, mathematical events featured in the news, by the media or on the internet: if you want a "heads up" on what to read, watch or do in the next couple of weeks or so, it's here. If you ever think that our heads haven't been up high enough and we seem to have missed something that's coming soon, do let us know: email info@ncetm.org.uk, or via Twitter, @NCETMsecondary.

Classroom View: Triggers and the Unexpected
How do you manage those unexpected, unplanned moments in the classroom – the moments where the lesson plan might as well have never happened? Here we consider three types of those moments: first - when a student raises something unexpected; second - when you, the teacher, have a moment of unexpected mathematical insight; and third - when the tools/resources (let's face it, usually the technology!) lets you down. Here we are encouraged to focus less on the terrors of the unexpected and more upon the opportunities they provide.

Sixteen Plus: Demystifying *x*
Algebra. One mention and a collective "sigh" fills the room. We know this to be true with many classes, but with post-16 GCSE retakers, the dread is more entrenched. Here we describe use of a simple resource that students have found naturally leads them to observe and express algebraic relationships without the sense of intimidation.

From the Library: Everyone can be a Mathematician
In this article, reproduced with permission from the Association of Teachers of Mathematics, Tom Francome describes the features in his department's approach to teaching mathematics - these are unusual but being adopted increasingly by departments across the country. Students are taught in all-attainment groups and each mathematical theme on the curriculum lasts for half a term. Tom describes the strategies and pedagogy behind making this work so that all students have the opportunity for genuine mathematical thinking.

It Stands to Reason: Cogs, Gears and Fractions
Reflecting on the concrete and pictorial representations often available to children working on fractions in primary school, we consider concrete and pictorial ways of representing fractions that might appeal more to secondary students. We look at the systems created using different sized gears and how this might support an understanding of fractions.

Qualifications and Curriculum: Conversations with the Exam Boards
We report on an interview that two Maths Hub leads recently had with representatives from the three main exam boards. They approached the meeting with a handful of the most frequently asked (by teachers) questions and are preparing to make the exam boards' answers publicly available soon.

Heads Up

Although the focus of the prominent announcement on 12 July from the Schools Minister Nick Gibb was on increased funding for the teaching for mastery programme in primary schools, there was also an item of interest for secondary teachers. Details were released of the review, to be led by Professor Sir Adrian Smith, into the 'feasibility of compulsory maths study for all pupils up to 18'. More information about that at the bottom of this DfE press release.

Can sending texts to parents of secondary schools pupils improve their children's maths attainment? The results of this trial from the Education Endowment Foundation (EEF) suggests it can.

To help provide information about the recruitment situation for mathematics teachers in secondary schools, please consider completing the Mathematical Association (MA)'s 60 Second Survey on Recruitment and Retention in Secondary Schools.

In his latest blog, NCETM Director Charlie Stripp reflects upon his recent experience teaching GCSE-resit students at his local sixth form/FE college, and his thoughts about post-16 resit provision. He also regrets letting on that he's an Arsenal supporter!

The Advisory Committee on Mathematics Education (ACME), the independent committee that develops advice on mathematics education policy in England, has recently published a new report: Problem solving in mathematics: realising the vision through better assessment.

If you're not on a beach on 24 August, and still have room in your head for a morsel of maths thinking, you might like to sign up for a free webinar on using the A level resources developed by Underground Maths, the body that started life in 2012 as the Cambridge Mathematics Education Project (CMEP).

Ofqual's new chief regulator, Sally Collier made her first major speech at the end of June. In it she pledged to improve exam marking and acknowledged the need for Ofqual to adapt to an era where students discuss exam questions on social media. Read the TES's report of her speech here.

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Classroom View
Triggers and the unexpected

"(F)ew subjects other than mathematics….hold out the possibility of such terrors in wait for the teacher" state Rowland and colleagues in an article discussing the management of the unexpected in the maths classroom (Rowland *et al*, 2015). Do you get those moments of puzzlement when a student just does not seem to get what you expected to be obvious, or that feeling of impending panic when you just can't think how the next line of a solution works, when one's "capacity to reason is momentarily frozen"? In a volume of articles titled, "Mathematics Teaching: Tales of the Unexpected" (Research in Mathematics Education, July 2015) a number of authors address the triggers for those moments, explore ideas for coping with them and consider the benefits that can accrue.

Rowland *et al* identify three triggers: responding to students' ideas, teacher insight, (un)availability of tools and resources. In an example of the first, Year 3 pupils are asked to "split" an oblong into two and then into four. Predictably most draw horizontal and vertical lines. One boy, Elliot, uses diagonal lines. The teacher asks the class "Has Elliot split his board into quarters?" (Figure 1). Given the four pieces are not all congruent, how might one proceed with a group of 7 and 8 year olds?

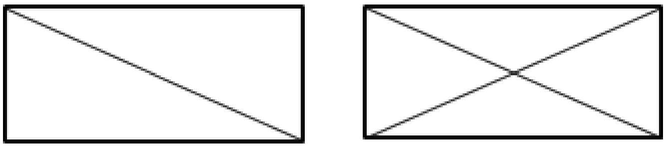

Figure 1

As an example of the second trigger, teacher insight: when leading students to "see" how the number of factors of a positive integer n can be calculated from the powers of its prime decomposition, one of the authors had introduced the lesson with the example $n = 72$. On writing

$$72=2^3 \times 3^2$$

on the board, he immediately "realised that this was not such a good example, since both 2 and 3 play dual roles in the decomposition," as indices and primes. He hastily changed the value of n to 6125. For the third trigger concerning tools and resources, no doubt all of us have had to extemporise given the failure of some piece of technology at a critical part of a lesson. In contrast, the authors give an example of a teacher taking opportune advantage of a large number grid left in the classroom to enhance their explanation – but then running into difficulties when the language and mode of thought supported by this representation turn out to be inconsistent with the approach already given.

These types of trigger were identified from the authors' research. The regularity of these occurrences leads them to the conclusion that these "contingent moments", i.e. events triggered from the nature of classroom mathematical learning and teaching, are to be expected. Empirical research identifies the significant "amount of discrepancy that exists between the teacher plan and the classroom reality" and furthermore "(t)eaching must be opportunistic because it cannot control its own effects." Teachers have to "think on their feet", to make "inflight" judgements. The authors suggest that through training and experience we do learn to predict the typical responses of students to certain stimuli, and we are able to foresee misconceptions and predictable errors. But in addition, by recognising that the classroom will have these "contingent moments", and by being a "reflective practitioner" these "unexpected moments that unsettle and disturb the teacher" can help deepen both our subject and pedagogical knowledge.

This "awareness that something was amiss" itself is a trigger for professional development "because such awareness always has within it a seed of learning and professional improvement."

"(T)eaching is a kind of improvised performance within a loosely structured framework of routines." Perhaps rather than focussing on the terrors of the unexpected idea it is more helpful to consider the value of each of these inevitable contingent moments in our improvised performance in the same terms Antony Gormley described the value of art: its worth "lies in its ability to stimulate thoughts we lost, or thoughts that would otherwise not exist at all."

Tim Rowland, Anne Thwaites and Libby Jared, "Triggers of contingency in mathematics teaching", *Research in Mathematics Education*, 2015, Volume 17, no.2 pages 74 – 91.

Antony Gormley in the Financial Times, 2/10/2015 via *Bedales Arts Magazine* 2015/16

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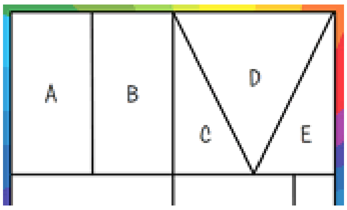


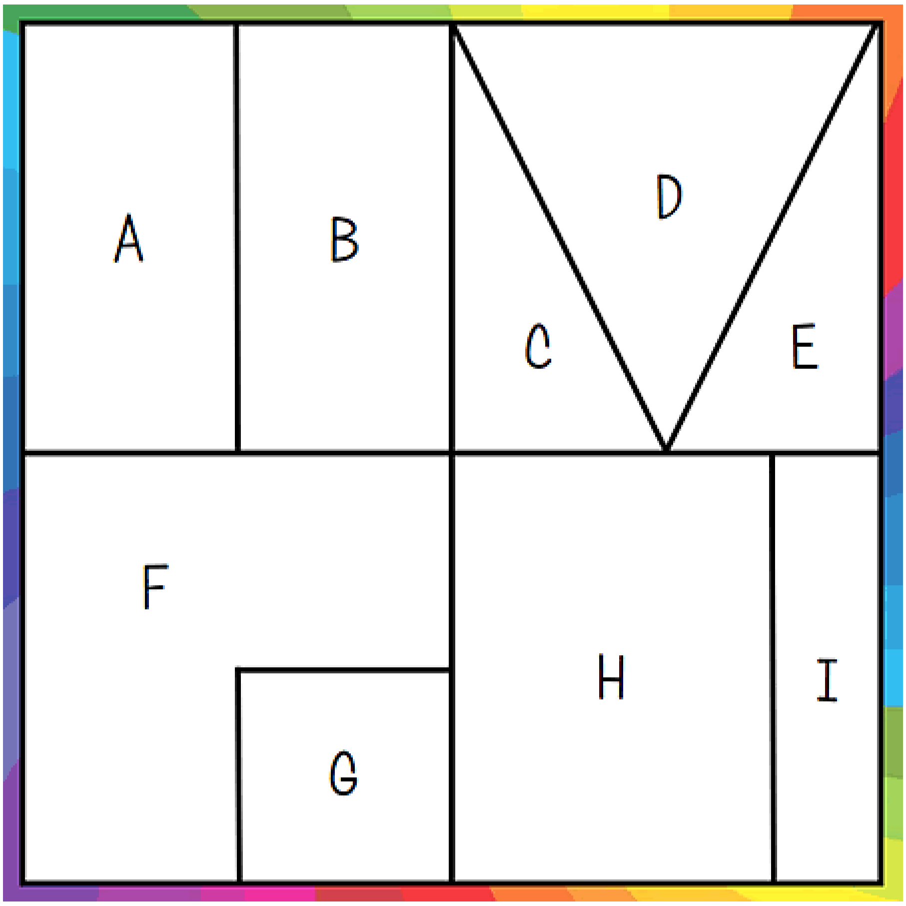
Sixteen Plus
Demystifying *x*

Algebra. One mention and a collective "sigh" fills the room. It's very difficult to change students' minds and attitudes when it comes to algebra. A lot of that is down to the fact that the resit student often feels that perfecting algebra is just beyond their grasp. It's hard to undo the damage that has been done by having such a negative outlook.

The ideal solution would be to start again – use a 'Men in Black' neuralyzer to wipe the slate clean. But we can't. It's also very tricky to go back to basics in a resit class – a group of 16-19 year olds may not be receptive to using Cuisenaire rods to represent systems of equations.

The following activity has been used to great effect in a "Taster Day" session, introducing prospective GCSE resit students to a different way of thinking. It would equally be effective in an introduction session at the start of the academic year.

It all stems from this diagram:



The session begins with students being asked to "Say what they see" – there are no wrong answers, and this builds students' confidence. Squares, rectangles, quadrilaterals, hexagons, compound shapes, angles, lines, letters, … The list will go on as long as you want it to.

Next, work on fractions. Talk about the whole square being a whole one, and then discuss what fraction of the shape each letter could represent. Students quickly see shapes with the same fraction, and then develop this to find fractions of the trickier shapes. In one class, there were at least three different ways that students figured out F and G.

Once all fractions are discovered, the notion that the square has a side length of 16cm is introduced. Using the fractions, and this new measurement, students are encouraged to find the AREA of each shape in the diagram. Students make connections between the fractions and the areas – layers of understanding, meaning that progress is actually tangible!

Then comes the algebra. Students are familiar with this diagram; they understand how the shapes relate to each other. Can they make statements using the letters that are true?

The most readily seen is

Area A = Area B. Area A and Area B are the same fraction and have the same area. The relationships between the shapes mean that students then progress onto saying that:

Area A + Area B = Area F + Area G, and:

Area A + Area B = 64.

More and more equations flow. The students have successfully used algebraic notation, and they have been successful because of the connections they have made with other areas of maths. This activity could even be extended to simultaneous equations.

It's these connections that are all important. We may not be able to press the reset button to re-teach students algebra. But, we can show them how it links to the areas of maths that they are more comfortable with. Through these links, students' algebraic skills will bloom.

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From the Library

In the article, Everyone can be a mathematician, reproduced with permission from the Association of Teachers of Mathematics, Tom Francome describes the unusual features in his department's approach to teaching mathematics. Students are taught in all–attainment groups and each mathematical theme on the curriculum lasts for half a term. Tom describes the strategies and pedagogy behind making this work so that all students have the opportunity for genuine mathematical thinking.

- Read Everyone can be a mathematician

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It Stands to Reason

Comments, requests, advice and resources offered by contributors to online mathematics education Twitter-chats, such as **#mathscpdchat** and **#mathschat**, suggest that many teachers are frequently looking for 'new' ways of trying to help students solve problems involving fractions.

One of the most inspirational items on the NCETM website is a series of 17 video chapters in which Caroline Ainsworth explains to Pete Griffin how she set about helping her pupils truly master calculations involving various kinds of number. In Chapter K we see Caroline's pupils thinking about fractions by selecting, comparing and reasoning about Cuisenaire rods. Caroline uses some rods to 'show' pupils just one fraction; the pupils then set about using rods to 'make' and explore other fractions for themselves. It is inspiring to see these young pupils engaging so productively in an empirical phase of learning about fractions – and interesting to hear Caroline and Pete's reflections, including their references to the work of Madeleine Goutard.

It is likely that many students are still struggling in Key Stages 3 and 4 to reason with, and about fractions because they did not have (as Caroline's pupils did) adequate opportunities to explore fractions empirically when they were younger. So at this later stage what kinds of representation (other than rods, which they might possibly and unwisely regard as rather 'babyish') could make up for this? What representations are likely to engage older students and provide experiences and situations in which they are likely to make observations that help them understand fractions more deeply?

This question prompted me to think about the principle of 'spurious purposes' in teaching mathematics; when you want pupils to understand a particular mathematical idea you get pupils interested in doing something else that is not ostensibly that piece of mathematics. While they are involved in the interesting 'something else' they 'by-the-way' develop understanding of the mathematics that you want them to learn.

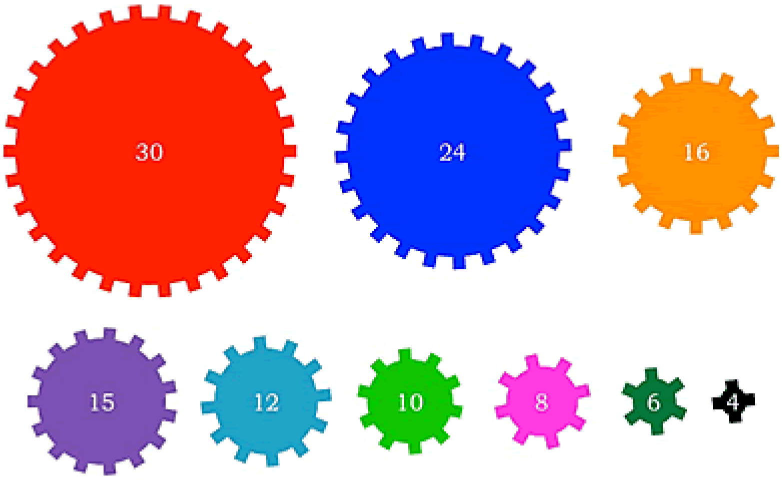
Example task

Changing gear with fractions

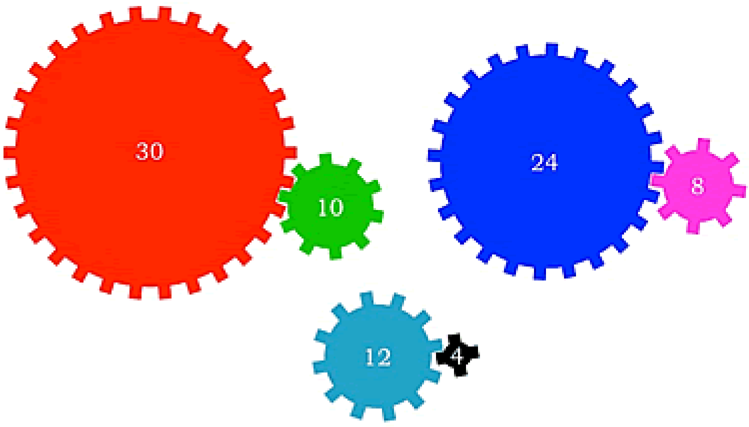
A possible introduction

If your pupils know nothing about gears you could start by showing them at least the first three minutes of this film, which introduces the basic ideas, and this video of some gear animations. They could themselves experience the effect of dragging the mouse on this Rotating Gears applet.

When pupils start designing for themselves, and exploring, gear systems they could at first select the gears from a particular, limited set that you have represented for them on a sheet, such as this set of gears with 30, 24, 16, 15, 12, 10, 8, 6 and 4 teeth …



A possible first task might be to find (for a particular value of *n*) systems of two gears in which one gear rotates n times faster than the other. For example, in each of these three two-gear systems which gear turns fastest? How many times faster than the other gear does it turn?



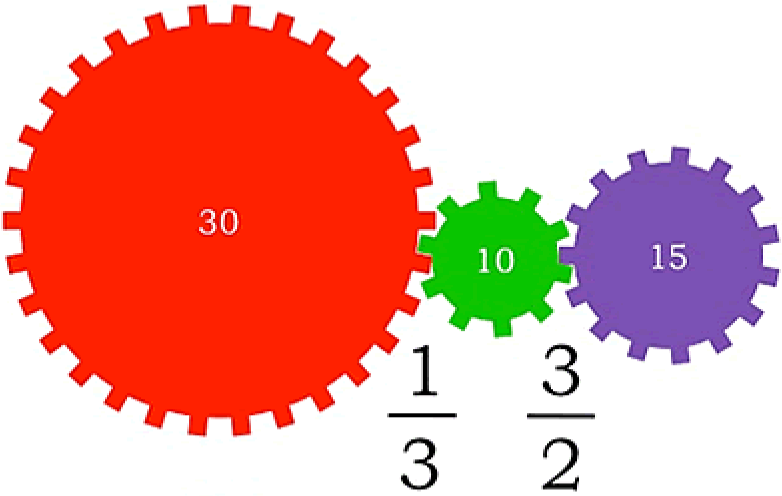
Can pupils describe what they think would be the consequences of using these three two-gear systems in some practical situations, such as on a bicycle?

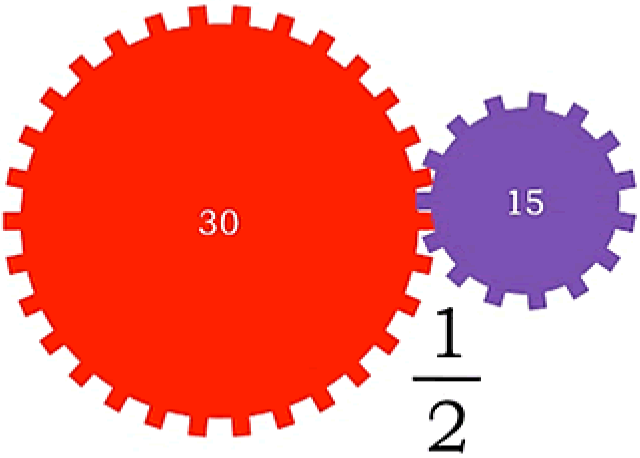
These three systems are, in a particular, clear way, 'equivalent'. You could introduce the idea of a gear-fraction …

$$\frac{\text{number of teeth on the right-hand gear}}{\text{number of teeth on the left-hand gear}}$$

… and ask pupils to explain ***how*** these three systems are equivalent. What other pairs of gears will be equivalent to these? Why? What is the vital connection between the numbers of teeth on the two gears in ANY pair of gears that is 'equivalent' to these three pairs?

Pupils usually enjoy making more complicated gear systems, and looking to see what happens. For example they might ask themselves what the gear-fractions reveal in a system of more than two gears? For example, when they compare the gear-fractions in these two systems …

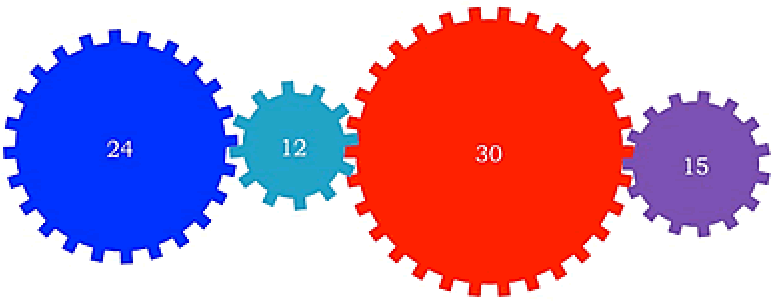




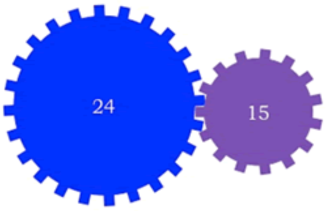
… they may notice that …

$$\frac{1}{3} \times \frac{3}{2} = \frac{1}{2}$$

As a result, they might ask themselves what is a sensible way to combine the gear fractions? In this four-gear system ...



… what is the list of gear-fractions when written in order going from left to right? How could you combine those gear-fractions? (Remember that in a gear-fraction the number of teeth on the right-hand gear is given as a fraction OF the number of teeth on the left-hand gear). How is the four-gear system above related to this next two-gear system?

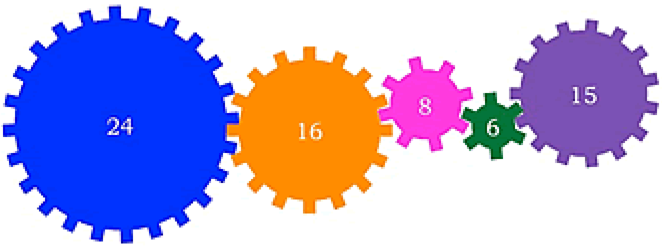


During their explorations of these two particular gear-systems pupils' reasoning is almost certain to include thinking deeply about these two facts …

$$\frac{1}{2} \times \frac{5}{2} \times \frac{1}{2} = \frac{5}{8}$$

$$\frac{15}{24} = \frac{5}{8}$$

… and if they create other gear systems with the same first and last gears as above, such as this …



… they will encounter other significant facts, such as this …

$$\frac{2}{3} \times \frac{1}{2} \times \frac{3}{4} \times \frac{5}{2} = \frac{5}{8}$$

There are lots of questions to ask, and there is plenty of explaining to do!

As with most activities based on the principle of 'spurious purposes', pupils are likely to encounter a variety of mathematical ideas while exploring gear systems. For example, if they want to represent gears by drawing circles that are in proportion to the number of gear teeth, they will have to draw on their understanding of similarity, and some knowledge about circles – they will have to ask themselves, and answer, questions such as '*As I have represented a gear with 12 teeth by drawing a circle of radius 2 cm, what should be the radius of a circle that represents a gear with 15 teeth?*' And links between the ideas of fraction and ratio are unavoidable!

You can find previous *It Stands to Reason* features here

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Qualifications and Curriculum
Conversations with the Exam Boards

A change as big as the new GCSE comes with much uncertainty for heads of department, school leaders, teachers and anyone else involved in the delicate task of preparing students to optimise their grade and be ready to face whatever the exam papers throw at them next summer. With the aim of improving lines of communication, two Maths Hub Leads, Matthew Linney and Dean Rowley (from the North Mids and Peaks and Norfolk and Suffolk Maths Hubs respectively), met last month, with representatives from the three main exam boards: Joanna Deko (Pearson [Edexcel]), Neil Ogden (OCR) and Andrew Taylor (AQA).

Matt and Dean's intentions were to put questions frequently raised by teachers to the exam boards. The answers given would then be compiled so that teachers can easily compare provision of the three main boards. We asked Matt about the meeting.

What is the broad, long term aim?

We want to provide other Maths Hub leads, subject leads and teachers with up to date information about the GCSE changes. The idea is to bring all the information together in one place under a list of FAQs, so that teachers can see, at a glance, each exam board's response and what they are doing with regards to certain aspects of the new GCSE.

What topics did the meeting cover?

Each exam board was asked to provide information on:

- teaching resources provided by them
- CPD offered to individuals in support of the new teaching
- making accurate predictions within the new grading system
- guidance on grade boundaries
- textbooks and other commercial resources approved/recommended
- any work they are doing with clusters of schools or groups of teachers
- links to specimen and practice papers
- impact on further maths and statistics GCSEs.

Particular focus was on the new problem-solving expectations and what the exam boards were providing in terms of guidance, resources, CPD and exemplification to support schools with teaching these skills.

What were the most interesting things you found out?

It was reassuring to find that all boards are thinking extensively about providing support for the problem solving element, through their own CPD networks and sessions.

We were pleased to learn about the quantity of resources offered by the exam boards. These will all be linked in the index we create so that teachers will have one place they can go and find, for example, problem solving resources from the three main boards. Largely the resources are free and don't require teachers to sign up to a particular board, though some will require a login account.

Was there very much difference in provision between boards?

No. The responses showed a lot of commonality of provision across the three boards.

What are the plans for disseminating this information to teachers? Where can I find the answers to the questions?

We plan to publish a reference matrix with each question and the responses given by each of the exam boards, with any links to relevant pages and resources on exam board websites. The matrix will be available through Maths Hubs and the NCETM Secondary Magazine. It may form the basis of a new NCETM microsite concerned with providing teachers with a one-stop-shop for all they need to know about Qualifications and Curriculum.

Can you give us a taste of what to expect...?

Yes, sure. Here's an example of one question we asked:

Do you have a website dedicated to CPD and what CPD do you currently run for the new GCSE?

AQA said:

Free online training is available with "GCSE Mathematics: Getting Started". Bespoke and free face-to-face support is available from AQA Maths Advocates. For example, to help teachers with the reasoning and problem solving element at KS4. There is a PD tab available and there are both face-to-face and webinar training sessions. Some of these are free and have previously included teaching and learning, pedagogy, KS1/2, aspiring HODs and subject knowledge enhancement.

Pearson (Edexcel) said:

Both free and paid for CPD can be accessed via Training from Pearson where you can find a wide variety of online and face to face events. If you missed or couldn't attend one of our face to face launch events or online content sessions you can still access presentations and listen to the recordings available. Introduced in 2013, The Mathematics Collaborative Networks are free local teaching networks introduced to help support, train and share best practice with maths teachers and heads of department across the country. Network meetings run throughout the year on a termly basis in over 40 hub centres, you can find out more here.

OCR said:

CPD for OCR qualifications can be found here. Past course documents are also available. GCSE (9-1) Maths, offers planning approaches for the qualification (for delivery over 1 or 2 years), problem solving, new foundation and higher tier content, co-teaching GCSE (9-1) Maths with the Additional Maths FSMQ, webinars and past paper reviews.

What next? Will exam boards and Maths Hubs maintain these lines of communication?

Now that we have a vehicle for communication, we will continue to communicate with the exam boards – the initial meeting was beginning to build that relationship. Meetings will happen as and when there is new information to disseminate. The boards are addressing uncertainty around grades this term with mock assessments taking place currently, to help with mapping old grades to new grades, and for helping teachers understand what, for example, a grade 9 question will look like.

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