Secondary Magazine
Issue 63

National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 63 of the Secondary Magazine.

Contents

Contributors to this issue include: Alison Clark-Wilson, Mary Pardoe, Richard Perring, Peter Ransom and Jim Thorpe.

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National Centre
for Excellence in the
Teaching of Mathematics

From the editor

Welcome to this issue of the NCETM Secondary Magazine. As usual we have a varied selection of articles. Focus on… perfect shuffles suggests ideas for the classroom, whereas Letters between teachers, a genuine reaction to an observed classroom experience, is a more reflective piece.

Although the structure of the magazine may not always be exactly the same, we aim to provide a familiar and useful combination of pieces in every issue.

'Surprise' is a wonderful notion: if we can achieve surprise in the classroom we may be rewarded with gasps of wonder, stares of astonishment… How do you employ surprise in the mathematics classroom? I hope you enjoy the piece On surprises in this issue, and perhaps come across a few surprises yourself.

The University of Chichester has been responsible for much important mathematics education over the years, and Alison Clark-Wilson's interview may particularly interest those of you who have gained inspiration in that institution across the way from Butlins.

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It's in the News!
How much?!

The fortnightly *It's in the News!* resources explore a range of mathematical themes in a topical context. The resource is not intended to be a set of instructions but rather a framework which you can personalise to fit your classroom and your learners.

How much does it cost a parent to kit out one child for PE lessons? According to a recent article in *The Observer* it is an average of £130, with boys costing even more, averaging £155. This has led to calls for schools to ditch logos and other labels from their PE kits. But surely this can't be right?! An average of £130 seems remarkably high doesn't it?! What's all the money being spent on? This fortnight's *It's in the News!* gives students the opportunity to plan how they would investigate the reported average cost and, in doing so, gives a context for practising the first phase of the data handling cycle.

This resource is not year group specific and so will need to be read through and possibly adapted before use. The way in which you choose to use the resource will enable your learners to access some of the Key Processes from the Key Stage 3 Programme of Study. The *It's in the News!* resources also provide the opportunity for students to work on many of the Functional Maths skills.

Download this *It's in the News!* resource - in PowerPoint format

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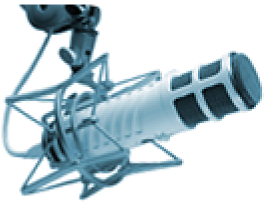

National Centre
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The Interview

Name: Alison Clark-Wilson

About you: after initially qualifying as a Chemical Engineer, a sudden need to 'get a well-paid job with school holidays' when I found myself widowed with three children under the age of ten led me to a two-year secondary mathematics PGCE course at West Sussex Institute for Higher Education. I taught in Portsmouth and Worthing before being lured to The Mathematics Centre, University of Chichester, in January 2001, as the Centre's Coordinator. Nine years on, I am Head of the Centre, which is now located in a prestigious listed building – still in sunny Bognor Regis, of course!

My work is diverse in that, in any working week I can find myself working with teachers in mathematics classrooms, representing the NCETM South East Regional Team, or the Mathematical Association, at different events, teaching on our own MA in Mathematics Education course, or working on curriculum and teacher development projects with local authorities and industry partners. At the heart of all of these activities is improving both the mathematical experiences of learners in school settings and the public face of mathematics!

The most recent use of mathematics in your job was…
developing a mathematics task and the supporting resources for an online professional development video conference being organised by the North East London Maths Association and London Grid for Learning. The task isn't new – not many are! – pupils and teachers from KS1 to KS4 will be exploring the properties of the different two-dimensional shapes they can construct by joining the dots on a 3 by 3 Geoboard. The online PD with the teachers aims to share 'Inspirational approaches to Assessment for Learning in mathematics' – which will all come from the participating teachers of course!

Some mathematics that amazed you is…
Geometry is undoubtedly my untapped area of subject knowledge – and I really enjoy using dynamic geometry software to revisit previously learnt mathematics and encounter new ideas. I am much inspired by bringing the static images in the classic text A Book of Curves by E H Lockwood to life! Along the way I have learned about inversion techniques to produce a dynamic Arbelos – look it up!!!

Why mathematics?
I think, like many, I began because 'I was good at it' – but I hadn't considered the immense satisfaction I would experience as I begun to work with young people and colleagues. I firmly believe that, if we view mathematics as a knowledge base that helps us both to make sense of the world around us and communicate these interpretations to others, all of us are capable of understanding mathematics. Consequently, most of my work is focused on encouraging all those with whom I work to reconsider their own perceptions of mathematics and, in doing so, make the subject more accessible and achievable for others. Yes, of course we are hampered by (archaic) assessment systems and political interference – but we have to stay true to our personal values!

A significant mathematics-related incident in your life was…
At the age of 12 in my mathematics lesson I discovered Euler's theorem - I just didn't know that he had got there first!

The best book you have ever read is…
I am not sure it is my best book ever but I really enjoyed *Uncle Petros and Goldbach's Conjecture: A Novel of Mathematical Obsession* by Apostolos Doxiadis – it provides a really sensitive insight into a family's perception of the maths prodigy in their family. It has made it onto the reading list of our MA course too!

Who inspired you?
I was introduced to the world of mathematics teaching by a 'dream team' of Adrian Oldknow, Afzal Ahmed, Adrian Pinel, Carol Plater and the much-missed Warwick Evans – all of whom helped to shape the mathematics teacher I became.

When I joined The Mathematics Centre at Bognor in 2001, it was under Afzal's mentorship. Afzal has a deep, insightful knowledge of the craft of mathematics teaching and his ability to take the most complex situations (mathematical and pedagogical) and, by posing a few simple questions, enable those involved to find their own solutions has been truly inspirational to me – it is all in *Better Mathematics* (HMSO 1987) – where the notion of a 'rich mathematical activity' was first suggested!

If you weren't doing this job you would:
...spend far more time with my family!!!
...have a lower carbon footprint!
...get more sleep!

On surprises

Being surprised is a human reaction that is very important in learning and doing mathematics.

Wittgenstein wrote that mathematicians enable us to 'see the value of a mathematical train of thought in its bringing to light something that surprises us'. We can give this to students as a justification for the hours that they spend learning mathematics only if we also give them opportunities to be surprised while they are doing mathematics.

Further, surprises help students learn. They motivate students and often prompt reflection and deep thinking. Seeing unexpected connections between parts of mathematics that students had thought were quite separate usually intrigues students, and sometimes helps them understand better the connected parts.

Coming across surprising events, phenomena or connections can be a great reward for taking risks. So shouldn't we support cautious students in taking risks when they are doing mathematics – until they find out for themselves that risk-taking is often very rewarding?

A way of doing this is to invite students to construct different-looking examples of something, some of which may be very unexpected by everyone else.

Some mathematical surprises are well known, such as cutting a Möbius strip down the middle and not getting two separate pieces, or the fact that you see exactly the same amount of yourself, no matter how close up or far away you are from a mirror, or that $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \ldots$. But we can think how to engage students so that many other things surprise them.

Some professional mathematicians say that there are just two kinds of surprises in mathematics – 'Who would have thought that would be the right *answer*?' and 'Who would have thought that would be the right *question*?' How can we enable students to experience those kinds of surprises?

There are some entries in Mathemapedia that can help us think about how coming across the unexpected encourages learning – for example, Cognitive Dissonance, Cognitive Conflict, and Anticipation. Also, you might read Awareness of the Nature and Values of the Educational System in the Mathematics Matters Lesson Accounts.

Mathematics Teaching 200 was a special issue about surprise, and includes an article, 'Surprise and Inspiration' by Anne Watson and John Mason. And an issue of Plus magazine included a piece about the element of surprise in mathematics - 1089 and all that - by David Acheson.

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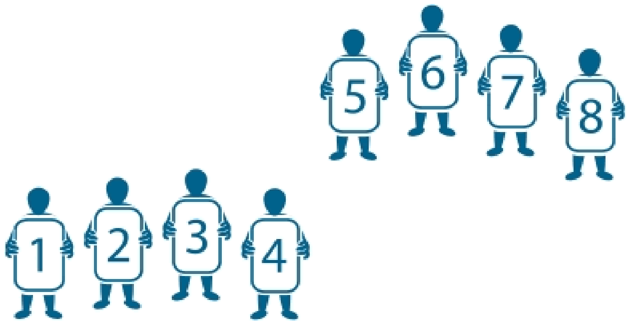
Focus on…perfect shuffles

There are some magic tricks that use pretty elaborate mathematics...and…magicians can perfectly shuffle a deck of cards.
Persi Diaconis, Professor of Mathematics at Stanford University, August 2003

To perform a perfect riffle shuffle, known as a *faro* or *weave* shuffle, you split an ordered arrangement of objects into two halves, or into two parts as nearly equal as possible, and then alternatively interleave, in order, the objects from the two parts.
Suppose the 'objects' are students standing in a row in the order of numbered cards that they are holding:

Let's see what happens when they perfectly shuffle themselves!

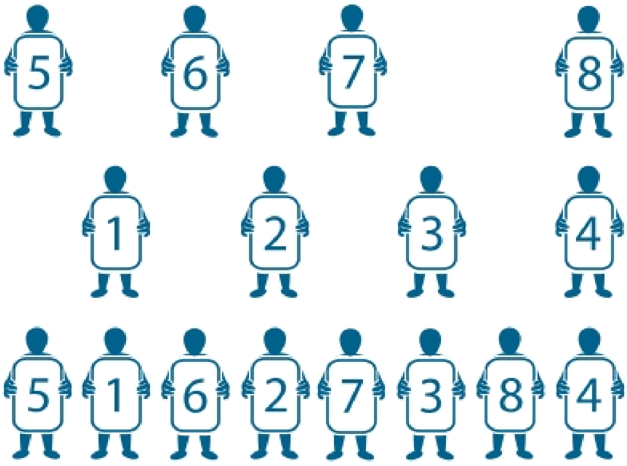
Because eight is an even number the students can move apart into two rows each containing the same number of students:



Now they can decide to do an *out-shuffle*, or to do an *in-shuffle*.

If they interleave themselves so that after the shuffle the 'first' and 'last' students, holding respectively 1 and 8, are still 'first' and 'last', they do an out-shuffle. But if the first student becomes second while the last student becomes second from last, they do an *in-shuffle*. An *Out-shuffle*:

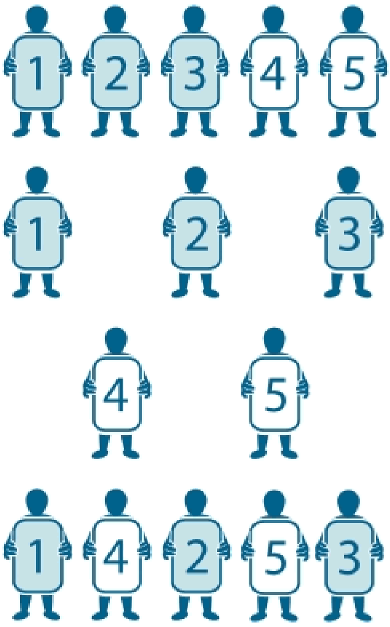
In-shuffle:



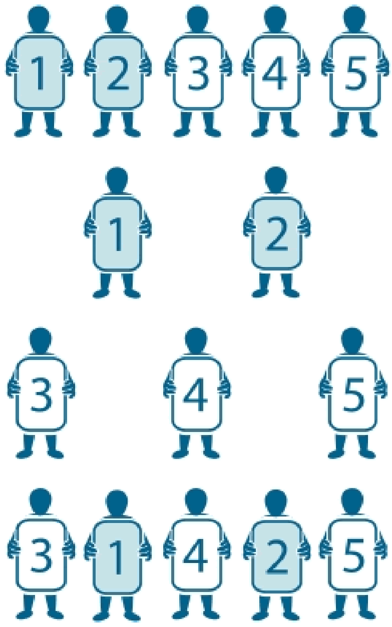
Suppose the number of students is *odd*, say that there are five students. Because the five students cannot move apart into two rows each containing the same number of students, they have to decide whether to split apart *before* or *after* the centre of the row.

If they split *after* the centre, they will then have to do an *out-shuffle*. But if they split *before* the centre they can then only do an *in-shuffle*:

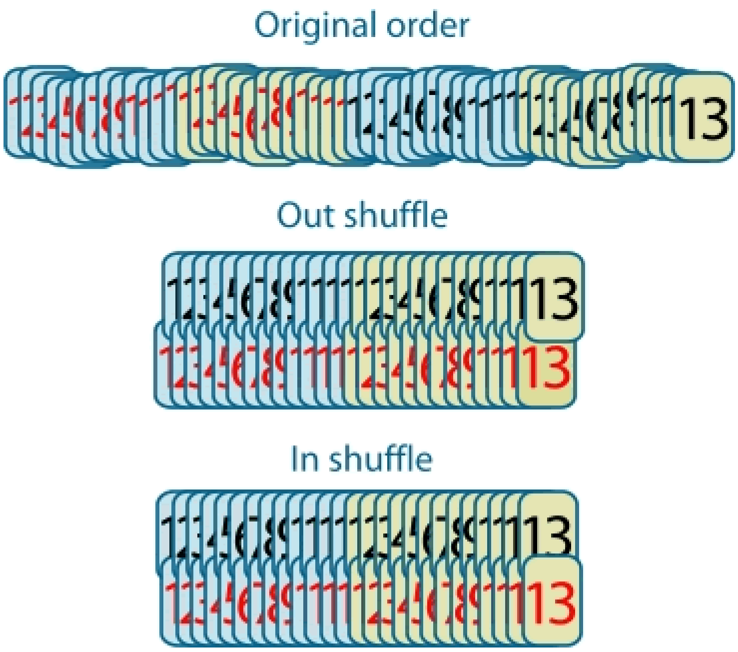
Out-shuffle



In-shuffle



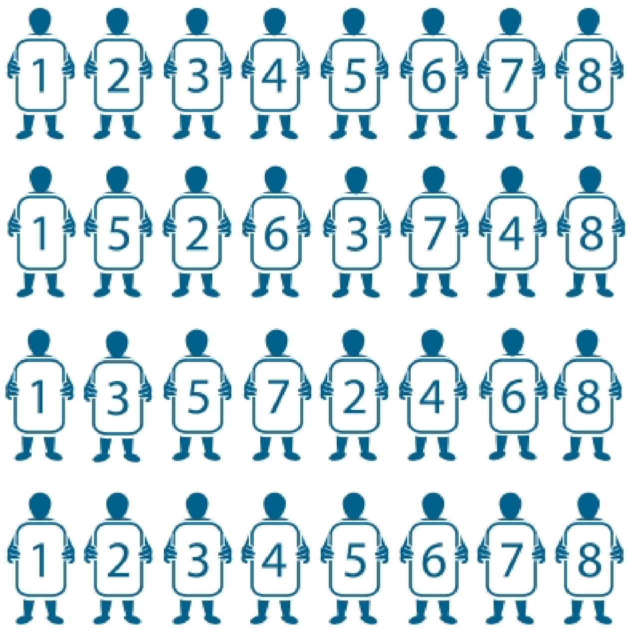
Of course the objects that are being shuffled are often playing cards. Because the number of cards in a normal pack is an even number, 52, the pack may be cut into two piles each containing 26 cards, and then *out-shuffled* or *in-shuffled*.



YouTube has a videoclip where you can learn how to do a perfect shuffle of a pack of playing cards.

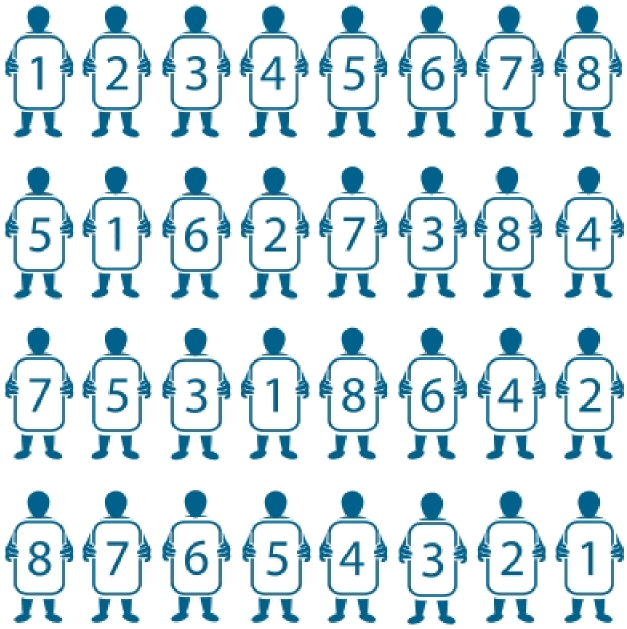
It is natural to ask what will happen if we repeatedly perfectly shuffle an ordered arrangement of objects. Will we ever get back to the original order?

Surprises are in store! If the eight students do *out-shuffles* repeatedly, this is what happens:



After only three *out-shuffles* the original order is restored!

Will the students also be back in their original order after three *in-shuffles*?



The students have done three *in-shuffles*, but they are NOT back in their original order!

So they carry on:

Three more shuffles have restored the original order!

If you experiment with other numbers of students in the row your findings may surprise you. For example, with 13 students in the row the original order is restored after 12 *out-shuffles* or 12 *in-shuffles*. But with three more students, that is with 16 students in the row, it takes only four *out-shuffles* or eight *in-shuffles*!

You will see that with any odd number of students you first return to the original order after the same number of *in-shuffles* as *out-shuffles*. With even numbers of students this is not the case.

If you and your students persevere with this exploration you may eventually alight upon some extraordinary generalisations, three of which can be summarized as follows:

To restore the original order,

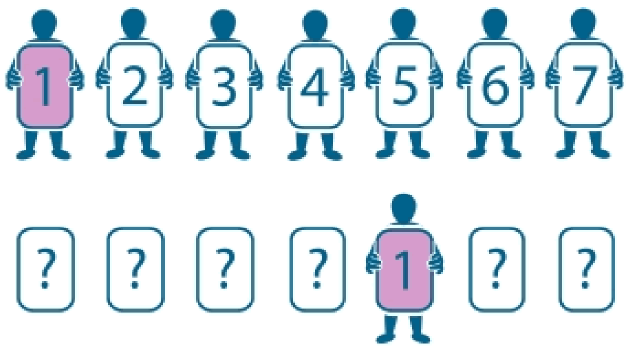
- when the number of objects is *odd*,
  - the number of *shuffles* of the same kind required is:
    - the power to which you must raise the number 2 to make a number that has a remainder of 1 when divided by the **number of objects**.
- when the number of objects is *even*,
  - the number of *out-shuffles* required is:
    - the power to which you must raise the number 2 to make a number that has a remainder of 1 when divided by ***one less than*** **the number of objects**;
  - the number of *in-shuffles* required is:
    - the power to which you must raise the number 2 to make a number that has a remainder of 1 when divided by ***one more than*** **the number of objects**.

Try out these generalisations!

Other naturally occurring questions about perfect shuffles, that magicians and mathematicians have asked themselves, have also lead to very surprising links in mathematics. For example, an unexpected connection emerges when you explore answers to the following question:

> Given any number of objects, is it possible, by repeatedly doing perfect shuffles, to get the first object (the top card) to *any* particular position?

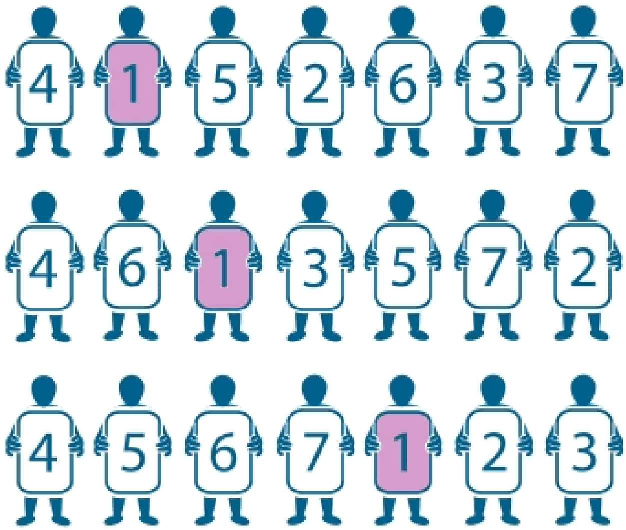
For example, suppose that there are seven students. Can we get the first student, holding '1', to the fifth position?



Alex Elmsley, a Cambridge University graduate with one of the most inventive brains in magic, was exploring ways of getting the top card to a particular position in the pack. He happened to start jotting down in his notes 'I' to represent an *in-shuffle* and 'O' to represent an *out-shuffle*.

Then he noticed something quite amazing! He saw that, by writing **as a binary number** the number of the position immediately before the position that he wanted to reach, the required shuffles were just presented to him 'on a plate'!!

Let’s explain. We are trying to get the first student to position 5. The number of the position immediately before position 5 is 4, and 4 written as a binary number is 100. If we do the sequence of shuffles that Alex jotted down as IOO, that is an *in-shuffle*, then an *out-shuffle*, then another *out-shuffle*…



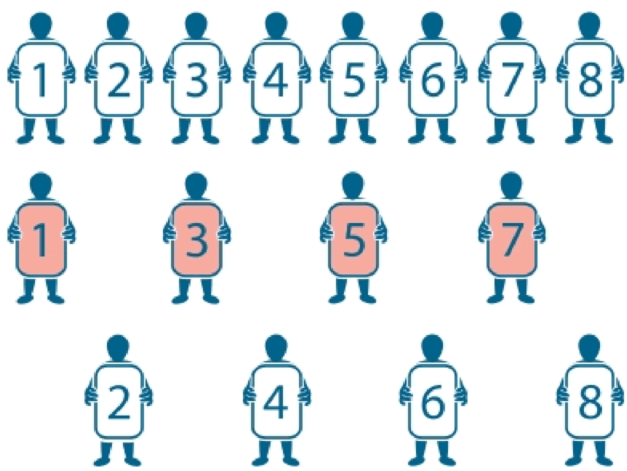
...we've done it!

Try out this amazing finding on other examples!

Magicians often check the results of a sequence of perfect shuffles (which, with playing cards, is usually difficult and clumsy to do) by undoing the sequence. They do, backwards, the sequence of reverse shuffles. They call the reverse of an *in-shuffle* an *in-sort*, and the reverse of an *out-shuffle* an *out-sort*. *Sorts* of playing cards are much easier to do than *shuffles*.

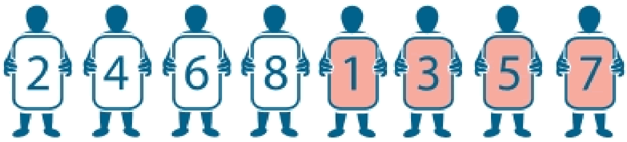
This is how the eight students do a reverse *shuffle*, or a *sort*.

Alternate students move away from the other students to form a new row:



Then the half-rows move together to form a new row of all eight students. If they form the new row so that the student who was originally first (the student holding '1') is again first, they will have done an *out-sort*:

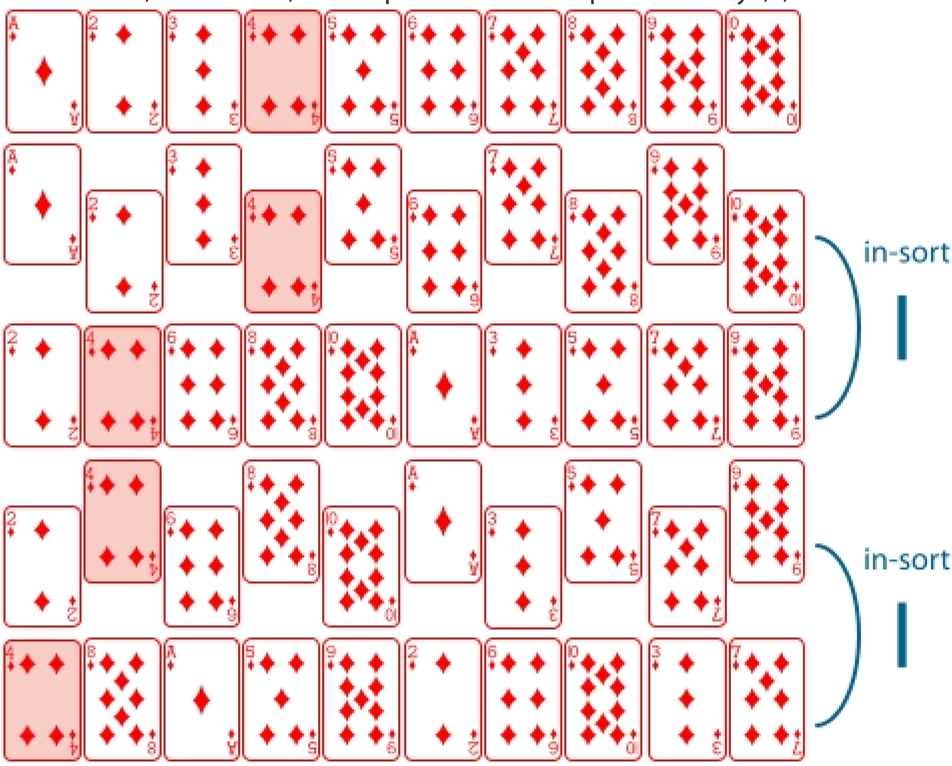
But if the half-rows join together so that the student who is now first is the student who was originally second, they will have done an *in-sort*:



Some conjuring tricks depend on the magician being able to get a playing card that is at a particular position in a stack of any size up to the top of the stack. Because this problem is the reverse of the problem of getting the first card to a particular position, it is sensible to explore it using sorts rather than shuffles.

Alex's Elmsley's amazing discovery about the unlikely link with binary numbers again gives magicians a simple procedure to follow in any such situation. For example, suppose that you have a stack consisting of the first ten diamonds arranged in size order, and that you want to get the four of diamonds to the top.

The card that you want to move is at position 4. Subtract 1 from 4, to get 3, then write the decimal number 3 in binary as '11'. Now do, backwards, the sequence of *sorts* represented by I, I, which is *in-sort*, *in-sort*…

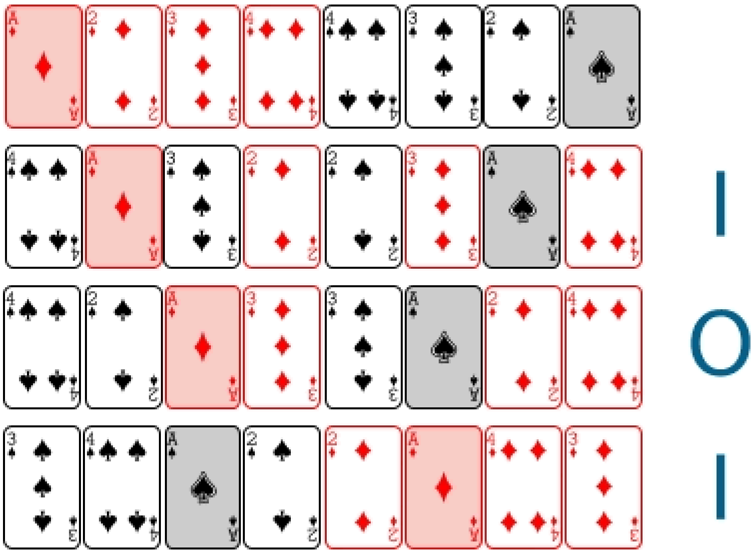


...the four of diamonds is at the top of the stack!

Generally, just subtract 1 from the number giving the present position of the card, and write the result as a binary number. Then follow, backwards, the sequence indicated by the binary number, doing perfect sorts rather than perfect *shuffles*.

Here is one more surprising phenomenon. When you move the *top* card of a deck consisting of an even number of cards, to the nth position from the *top*, the *bottom* card of the deck automatically goes to the nth position from the *bottom*!

For example, when you follow the binary number procedure to get the top card in an eight-card deck to the sixth position from the top, the bottom card automatically goes to the sixth position from the bottom!



And the cards that were originally at the top and bottom are at symmetrical positions at every stage!

You might like to investigate whether the same symmetry exists whenever you move a card at a particular position up to the top.

You can explore these card moving procedures effortlessly using this [Faro shuffle simulator]() from natedog.com.

This article can be downloaded as [a separate PDF document]().

Letters between teachers

A senior teacher writes to a student teacher about ways of getting adolescents to work as a group, not just in a group. (First we set the scene and then you will read the letter that was sent as a response to the scenario described below).

Setting the scene

On this occasion, the letter is from a senior to a junior teacher, to misappropriate a C.S.Lewis title. More particularly, these are reflections on an Open University student teacher's lesson, written by the student's OU PGCE tutor. The original text has been altered in the interest of accessibility to those not present at the session. The session is worthy of remark because the students, by no means a bunch of mathematics aficionados, undertook an experiential activity working in groups, which was something of a departure from their more familiar exposition-and-example followed by exercises from the textbook; also something of a departure for the student teacher.

This risky venture included three adults in the classroom, the OU tutor, the PGCE student and in place of the usual mathematics teacher, an MFL teacher. Risky? I mean the risk of nothing much happening – perhaps because of the unfamiliarity of having to get started by reading instructions from worksheets rather than by listening in silence to the teacher, the risk of student talk meandering off task..., the liveliness of a class with the lid off – energy not damped down by the routine of repetitive exercises, and the management of unaccustomed behaviour.

The student teacher prepared activities and measurement tasks concerned with circles with the aim of identifying the invariance of the ratio of circumference to diameter. Learners worked in groups, and with some prompting worked **as** groups with only minimal teacher leading, but much teacher intervention.

The relevance on this occasion of learners working in groups was that it enables individual action to be turned into knowledge through discussion – of results deriving from the same activity modified by learner choice of measurement. Activities such as these which are similar enough to invite comparison, but different enough to produce contrast, add spice to discussion, and possibly the enjoyment of surprise.

It seemed important, however, to start with a safe common activity in order to encourage engagement in a process that was not to be under verbal control of the student teacher.

> **Draw a circle radius 5cm with a pair of compasses;**
>
> **cut out the circle;**
>
> **fold it in half.**
>
> **What do you notice about the fold line? Discuss with your partner.**

Later activities were more open, and made increasing demands on teacher intervention.

> **How might you measure the circumference of a circle?**
> **Discuss with your partner.**
>
> **Can you find several ways?**
> **Make a list of as many as you can.**
>
> **Which of your ways do you think the most accurate?**

The letter

Dear student teacher,

You observed that the group at one table were convinced that chords that did not pass through the centres of circles could be lines of symmetry. Amy was the student at the other table who was sure that a diameter was the only chord that was also a symmetry line, but only you, as teacher, were aware of the divergent opinions. Amy had not the confidence to assert her own opinion to the other table, but was present when you voiced it for her: "Amy thinks…". This facilitated a disagreement between peers instead of you weighing in as expert. Of course, at times that is what as a teacher you do, but not if the aim is to generate student discussion, uninhibited by teacher opinion. On such occasions you may become a mediator in mathematical disputes, helping students to refine their arguments.

To us, as teachers, the class is a group of learners, to the class themselves they're a bunch of social teenagers; and you're the person who wants to stop them being social. That difference is a conflict of interests: if, in contrast, you can draw on their sociality as a resource for learning [as you did yesterday] you may accordingly reduce the conflict of interests, as well as promote learning which is more independent of the teacher.

When that happens, as it can more often than not over a period of time, your presence in the classroom can be more relaxed as you are no longer the sole agent of control: the students' social interaction includes involvement in working at mathematics through experiment, reasoning and discussion, a constant flux of questions and ideas generated by the students themselves. Motivation is then intrinsic to the activity, not merely an imposed orderliness.

The twin issues of time and availability are paramount in the classroom: this is where resource design can be useful in making copies of yourself. So make worksheets, for example, to replace your talk: write as you talk rather than as you might write.

It's often better to get learners to work from action rather than opinion, so that:

instead of asking, *how many symmetry lines has a circle?*
prompt with, *make a different diameter fold;*
then, *and another;*
then, *another;*
and now ask, *how many diameter folds altogether could you make?*
and finally, *what does that tell you about how many symmetry lines has a circle?*

You need to consider how you want the answer to a question such as "What does that tell you about how many symmetry lines has a circle?" to feature in the lesson. Do you want it written down, individually, or recorded by the group, or perhaps considered again in a final plenary, or left blowing in the wind of conversation? You may want a piece of work checked at a particular stage, so might put "Come and show me your work" on the worksheet. Alternatively you might put "Compare your answer with someone else's". "Discuss with your partner", used above, is another injunction that is useful at times; alternatives may give a different flavour, for example, "See what someone else thinks".

There was more noise and talk than you'd want for a whole class instructional lesson, but the acid test was the quality of the posters that emerged at the end, and the willingness of a representative from each group to express a finding of that group. If I said that I overheard one student say "This was my best lesson ever", you might think I made it up, but I didn't.

The session provided an opportunity for you to consider issues about coping with classroom diversity, and how to get adolescents to work **as** a group, not just **in** a group.

Yours, a senior teacher

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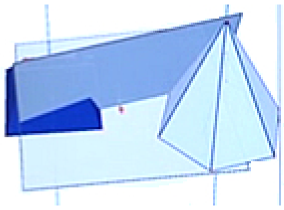

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5 things to do this fortnight

- The 10th Annual Institute of Mathematics Pedagogy, led by John Mason, Malcolm Swan and Anne Watson, will take place in Oxfordshire from 3-6 August. You might consider joining the group of teachers, teacher-educators, researchers and curriculum developers who will work together intensively and creatively with issues and ideas to develop thinking and practice.

- Have you discovered Cabri 3D interactive? You could watch this video in which a mathematics teacher uses 3-D geometry software to explore 3-D shapes with Y9 students, and discusses issues with Adrian Oldknow.

- Are you, or might you like to be, among the mathematics teachers who are experimenting with creative teaching approaches using music and dance? You could start by reading the Ofsted report – Learning: creative approaches that raise standards – that was published in January, and Anne Watson's paper Dance and mathematics: power of novelty in the teaching of mathematics. You could also watch an American video of teachers learning to use dance to teach mathematics, and listen to some mathematics songs.

- Your students who are considering studying mathematics at university may be interested to hear about the Maths Summer School that will take place at the University of London Union from 25-27 July. Students will explore exciting topics during seminars and lectures, and engage in problem-solving activities.

- Meet the Mathematicians meetings (MTM08 – MTM11) are special day-events for Year 12 and 13 students, organised by the British Applied Mathematics Colloquium (BAMQ). MTM09 included an interesting talk by Professor Jon Keating of the University of Bristol - *Some Thoughts on the Unreasonable Effectiveness of Mathematics*. If you or your students missed this fascinating lecture about surprising connections in mathematics, you can now watch a video presentation of it on YouTube in seven parts:

    - part 1
    - part 2
    - part 3
    - part 4
    - part 5
    - part 6
    - part 7

  You can find more articles and information focusing particularly on A-level and Further Mathematics in the FE Magazine.

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Diary of a subject leader

Issues in the life of an anonymous Subject Leader

A great weekend break set me up for the last week before half-term. Of course, one never switches off totally, and my searching of the second-hand bookshop at Claydon House, a National Trust property, revealed an excellent book about mazes, Labyrinth, by Adrian Fisher. This book has lots of useful material that I may use to stimulate creative team-work by students in lessons about directions and shape.

I also managed to pick up a crystal sphere there – one never knows when such things might come in useful. I'm thinking sunshine and lenses, and wondering whether, if we put it in the sun, it will focus the sun's rays so that a path of the sun's passage through the day will be burnt out, thus allowing us to work out how many hours of sunshine we have. The Campbell-Stokes sunshine recorder is based on such a principle, and the mathematics of this intrigues me.

Faculty meeting on Monday – all the time spent on sorting out classes for next year's Y10. We look at how students performed on the SATs papers – allowing for any extenuating circumstances – and identify students who need to be kept apart. Although no method is infallible, over the years the test data has helped us set targets for students, and improve our GCSE results. Our parents appreciate knowing how their children have performed in comparison with others, so, although we do not give exact positions, we indicate whether they are in the top, middle or bottom third of the class.

I need to get some euros for our half-term break to France, so some functional mathematics comes into play. The opening offer is €1.110 to the pound (we wanted to exchange £500), so we go round to a few other places haggling for the best rate, ending up where we started and agreeing €1.138 to the pound. To me that's €14 better than the opening offer, so it was worth the trouble.

The rest of the week passes swiftly and it's not long before we are enjoying the wet weather in France for a few days before it improves. Anyway, I manage to get a copy of the French mathematics magazine, Tangente, so that keeps me occupied in the evenings, since the TV is so dire. This magazine is an excellent source of unusual problems and articles. The puzzles and enigmas I find useful in providing inspiration for mathematical problems for the UK Maths Challenges.

We also had the opportunity to visit the inside of the Lille Citadelle, Vauban's masterpiece of symmetrical pentagonal fortification. As this is still used as a military base, places need to be booked in advance and no pictures can be taken of the inside. Here, however, is one I took of the main entrance!

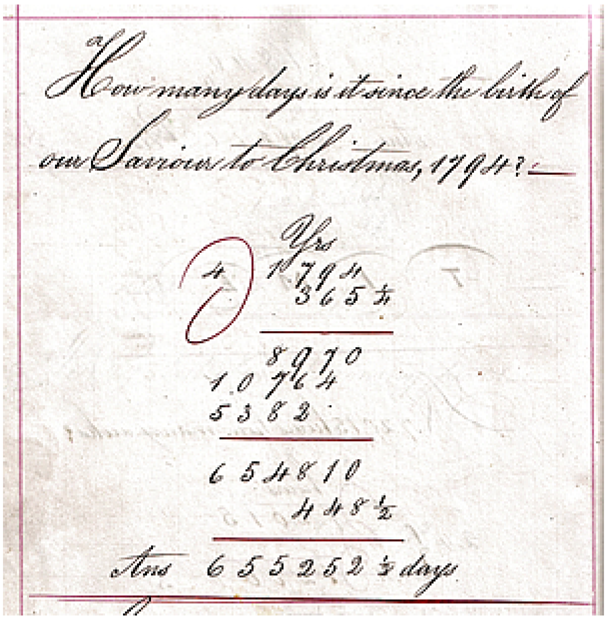
Once back home, I send off my contributions for the next year's Junior and Intermediate Mathematical Challenges, before sorting out my booster lesson for Y11 on Monday morning, just before the first GCSE paper. Nearly all my class turn up and we focus on the Foundation paper. Just before they go, someone asks about splitting an amount up in a given ratio, so I swiftly show them how to split £40 in the ration 2:3. Spooky! Those exact numbers appear in the paper and I wonder whether they will remember what I did.

On Tuesday I'm in earlier than normal to meet a student and parent about concerns over lack of work at the moment. It gets sorted. After school I'm relieved that nobody turns up for a revision session so I can get away slightly earlier than usual.

The Heads of Faculty meeting on Wednesday lasts far longer than usual due to discussion over Enrichment Week at the end of term, PLTS, faculty SEF, and next year's Faculty Improvement Plan. My shoulders are more bent at the end of the meeting, but it will all get done! In fact, I talk over the SEF with my second on Friday, and we sort it out for discussion at the faculty meeting next week.

At last it's Friday and the second mathematics paper is as accessible as the first. Our students are smiling. I hope they will be smiling again when the results appear! It's been a tough week, with only two non-contacts until today when I have four non-contacts, but two of those have been used in preparing pencil cases for the incoming new Y7s in September. We have a Parents' Evening for them in July, and to make sure that they have all the equipment they will need, we prepare and sell full pencil cases at a cost far below what they would have to pay in the shops. However, preparing 200 full pencil cases takes time!

My eBay purchase of an old mathematics exercise book arrives: what magnificent presentation – a thing of beauty! I look forward to showing students the penmanship of J T Warrington of the Eldon Academy, and the pride that he or she took in presenting his or her mathematics of 1868. Here's a page to appreciate – and find the erroneous digit!



How many days is it since the birth of our Saviour to Christmas, 1794?

Yrs
4 | 1794
365¼

8970
10764
5382

654810
448½

Ans 655252½ days