MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.4 Composition of numbers: 6–10

Teacher guide | Year 1

Teaching point 1:

The numbers six to nine are composed of 'five and a bit'. Ten is composed of five and five.

Teaching point 2:

Six, seven, eight and nine lie between five and ten on a number line.

Teaching point 3:

Numbers that can be made out of groups of two are even numbers; numbers that can't be made out of groups of two are odd numbers. Even numbers can be partitioned into two odd parts or two even parts; odd numbers can be partitioned into one odd part and one even part.

Teaching point 4:

Each of the numbers six to ten can be partitioned in different ways. The numbers six to ten can be partitioned in a systematic way.

Teaching point 5:

Each of the numbers six to ten can be partitioned into two parts; if we know one part, we can find the other part.

Overview of learning

In this segment children will:

- explore how the numbers six to ten can be thought of as a family of numbers made up of 'five and a bit'
- become familiar with both the cardinal and ordinal value of the numbers six to ten
- explore the definition of odd and even numbers for the first time, and counting in twos
- build on their experience of partitioning to develop fluency in partitioning the numbers six to ten, and solve associated missing part problems.

Teaching points 2, *4* and *5* in this segment explore the numbers six to ten in the same way that the numbers to five were considered in segment *1.3 Composition of numbers: 0–5*. However, some new concepts are also introduced here:

- *Teaching point 1* (on the 'five and a bit' nature of the numbers) is specific to the numbers six to ten.
- *Teaching point 3* introduces the concept of odd and even numbers for the first time, now that we have a larger number set (one to ten) to work with.

This segment begins by considering the numbers six to ten as 'five and a bit', mainly by representing them on tens frames (used 'fives-wise') and fingers – since children can now subitise numbers to five, they should be able to quickly recognise the 'and a bit' parts of these numbers. This approach will help children to develop a deeper understanding of the composition of these numbers, as well as allowing them to apply previous knowledge.

Children will become familiar both with the cardinal value of the numbers six to ten (*'How many?'*), and their position in the linear number system (their ordinal value). This is the first time that children will come across two-digit numbers (given the inclusion of ten). Teaching should focus on the cardinality and ordinality of ten, rather than the idea that '10' represents 'one ten and no ones'. Place value in two-digit numbers will be explored in depth in segments *1.9 Composition of numbers: 20–100* and *1.10 Composition of numbers: 11–19*.

In the course of this segment, children will also explore odd and even numbers for the first time, and will learn that even numbers can be made out of groups of two, whilst odd numbers cannot. They will learn to identify odd and even numbers within ten, a skill which they will build on when they start working with two-digit numbers. Children will also develop fluency in counting both in even *and* odd numbers; this forms the basis of adding two, which will be covered, along with other additive facts, in segment *1.7 Addition and subtraction: strategies within 10*.

Once children have mastered the identification of odd and even numbers they will explore rules for partitioning odd and even numbers:

- Even numbers can be partitioned into two odd parts or two even parts.
- Odd numbers can be partitioned into one odd and one even part.

These can then be used to facilitate partitioning and identification of missing parts in *Teaching points 4* and *5*, where teaching goes beyond the 'five and a bit' structure to explore all of the different ways of partitioning each of the numbers six to ten into two parts. As with segment *1.3*, the focus should be on fluency in partitioning these numbers. The full set of teaching steps are not provided below – instead, *Teaching points 4* and *5* provide an overview and we recommend referring back to the teacher guide for segment *1.3* for more detailed guidance.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks

Teaching point 1:

The numbers six to nine are composed of 'five and a bit'. Ten is composed of five and five.

Steps in learning

	Guidance	Representations
1:1	In segment *1.3 Composition of numbers: 0–5*, before beginning detailed work on the numbers one to five, we reviewed counting to ten, including: • chanting/singing number-based rhymes and songs • counting forwards and backwards within ten. Here you can take the opportunity to review again, to ensure that children have a sense of sequence and synchronicity in counting from zero to ten, and have secured the stable-order principle. Refer to segment *1.3*, step *1:1* for useful representations. Again, it is important to link the following four aspects of number: • number names (for example, 'six') • numerals (for example, 6) • quantity (cardinal) value (for example, a set of six objects) • place in the linear number system (for example, seeing that the number six 'sits' between five and seven). The final aspect here will be developed further in *Teaching point 2.*	
1:2	Present children with a range of contexts and concrete objects within the number set six to ten. Ask children to count the objects and record the numeral to show the quantity in each example. Continue to embed the cardinality principle by repeatedly using the stem sentence: ***'One, two, three… There are ___ objects.'***	

Provide variation by also asking children to:

- record the quantities using number names
- draw pictures to represent given numerals or number words.

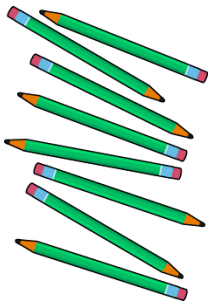
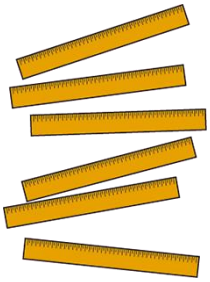
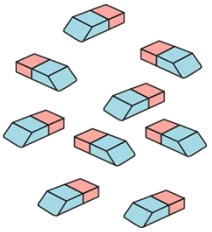
Ensure that children have access to the number names to refer to for spelling (for example, in the form of a number track) – children particularly struggle with the spelling of 'eight'.

By the careful choice of examples, you can also continue to highlight the concepts of order irrelevance, conservation and abstraction. For more information see segment *1.1 Comparison of quantities and measures*, step *2:4*.

'How many pens are there?'

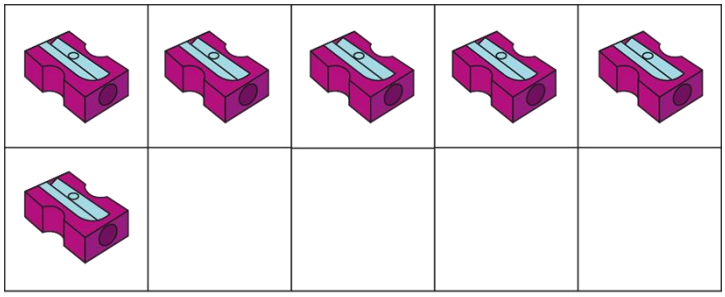
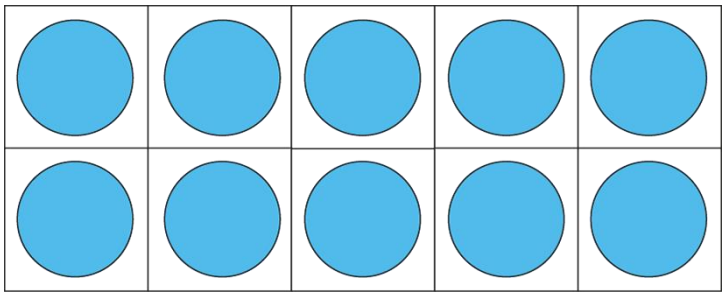
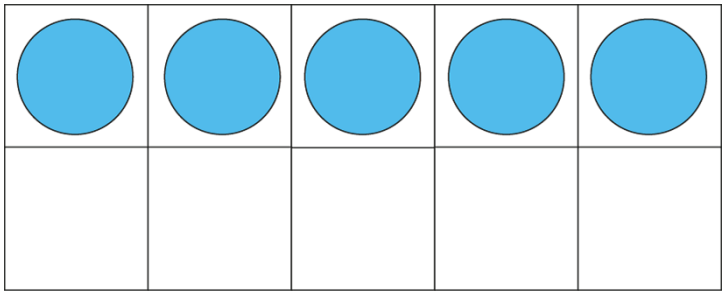
'One, two, three, four, five, six, seven. There are seven pens.'

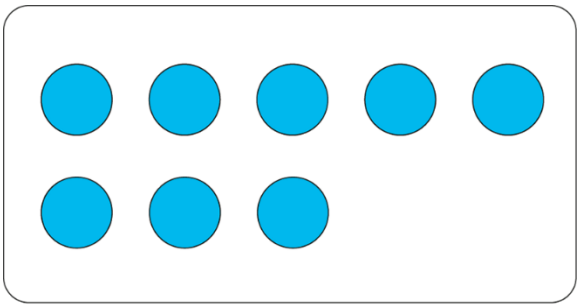
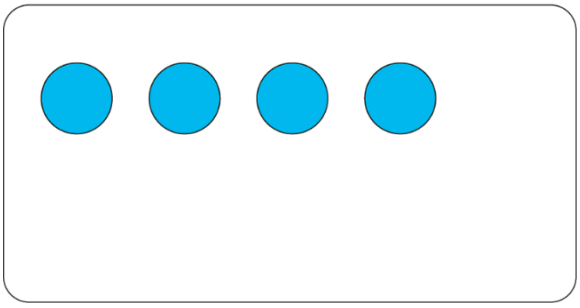
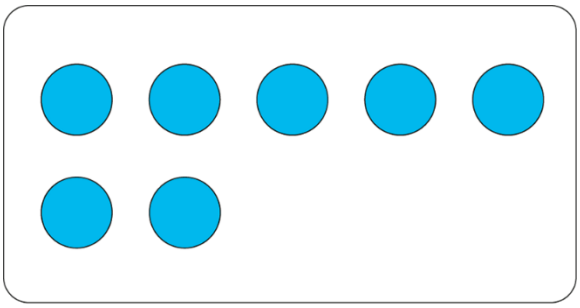
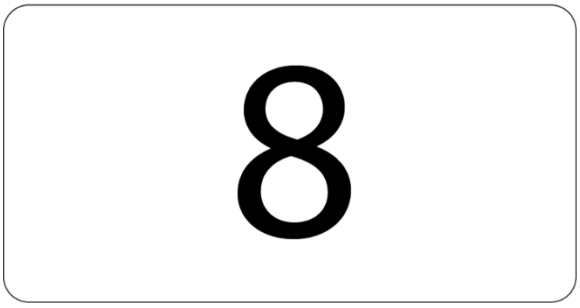
'Fill in the missing number names and numerals.'

		
eight		
8		

'Use drawings to show these numbers.'

ten	6	seven

1:3	Now, begin to represent the quantities in a systematic way to draw attention to the 'five and a bit' structure. Either working digitally with pictures, or with concrete resources and laminated tens frames, ask the children to move individual items in a given set into a tens frame. Make sure you use the tens frames in the horizontal 'fives-wise' layout as shown here. Use the following stem sentence: ***'___ is five and ___ more.'*** Progress to the use of generalised representations, such as counters, on the tens frames. Provide children with practice by: • asking them to complete a tens frame • asking children to match a given number card to the corresponding tens frame arrangement. The number ten has a particular significance in our number system. Throughout this and the subsequent steps, pay particular attention to ensuring that children easily recognise that two fives make ten.	Pictorial:  6 *'Six is five and one more.'* Generalised representations: *'Find the matching number card.'*  *'Ten is five and five more.'* *'Complete to show nine.'*  *'Nine is five and four more.'*
1:4	To provide further practice and embed a sense of the relative size of the numbers, present children with two numbers and ask them to say which is larger/smaller. Present the numbers with either: • both as generalised representations	

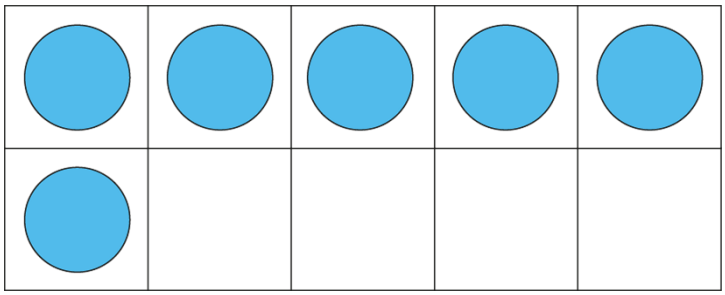
	• one as a generalised representation and one as the numeral. This builds on what children learnt in segment *1.1 Comparison of quantities and measures*. You could extend by presenting missing symbol (>, <, =) problems.	*'Which is larger?'*   *'Which is smaller?'*  
1:5	Now use the part–part–whole representation (cherry or bar model) to represent this 'five and a bit' structure. At this stage, link the part–part–whole model to a visual	

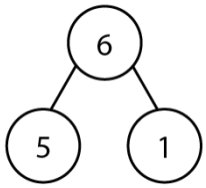
representation, such as the tens frames as used in the previous steps; children will start to work without the visual scaffold in step *1:9*.

For now, ask children to complete or draw a part–part–whole diagram for a given tens frame, or vice versa.

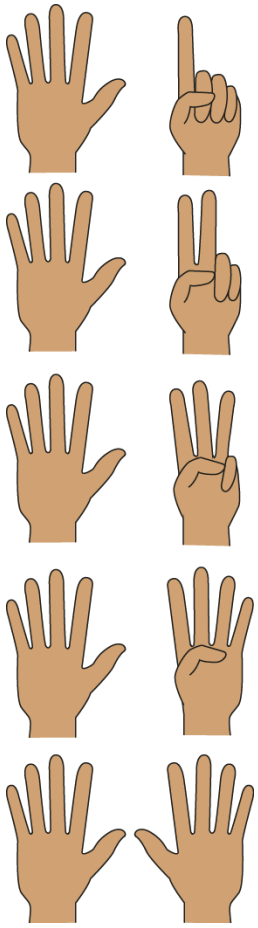
Encourage children to describe the parts and wholes (as introduced in segment *1.2 Introducing 'whole' and 'parts': part–part–whole*), using the stem sentence: ***'___ is the whole; ___ is a part; ___ is a part.'***

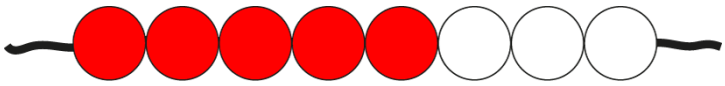
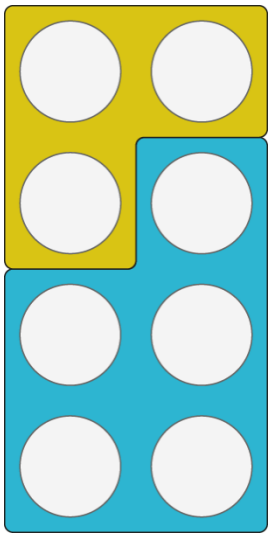
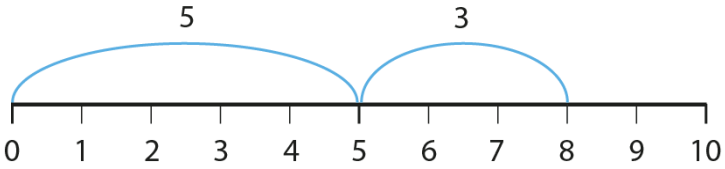
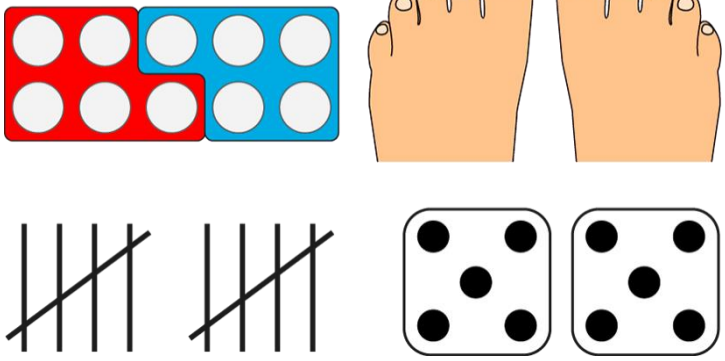
Note that it will be useful to have a set of 'five and a bit' part–part–wholes for the numbers six to ten on display in the classroom throughout the rest of your work on this segment. Children will be gaining confidence in linking, for example, five objects and three objects to make eight objects, but linking the numerals 5 and 3 to the numeral 8 involves a further level of abstraction. Encourage children to refer to the part–part–whole models when helpful.

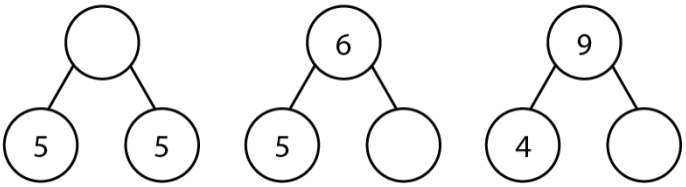
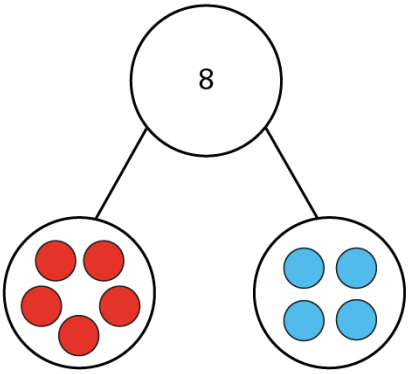
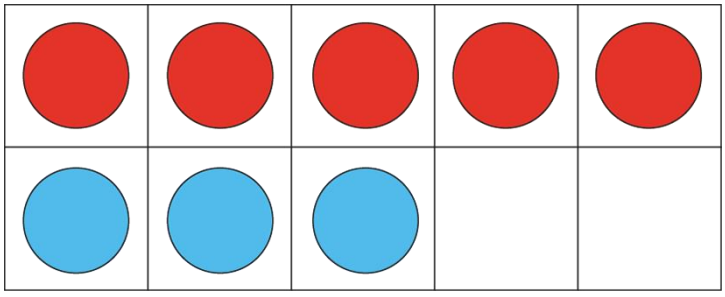
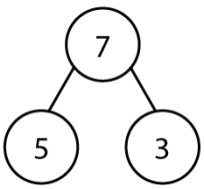




- 'Six is five and one more.'
- 'Six is the whole; five is a part; one is a part.'

1:6	Fingers are an excellent model for exposing the 'five and a bit' structure. Do not assume that all children will be able to represent numbers using their fingers as indicated – spend some time teaching children how to do this using the 'Grow it, show it, throw it' progression: • 'Grow it' – to begin with, children count out a given number, one finger at a time. • 'Show it' – once children have mastered 'grow it', give them a number to show; they open the correct number of fingers in front of them to show the number (without counting out one finger at a time). • 'Throw it' – finally, for a given number, children have to open the correct number of fingers behind their own backs before bringing their hands in front of them.	
1:7	Vary the practice by asking children to: • represent a number that is: • spoken • written as a numeral • written as a number name. • say, or write, the number of fingers that you or another child shows to them • represent a given part–part–whole diagram using their fingers • complete/draw a part–part–whole diagram to match a number represented using fingers (demonstrated or image). Throughout, encourage children to subitise the 'and a bit' quantity – counting can be used as a checking strategy. Children should continue to name the parts and the whole.	

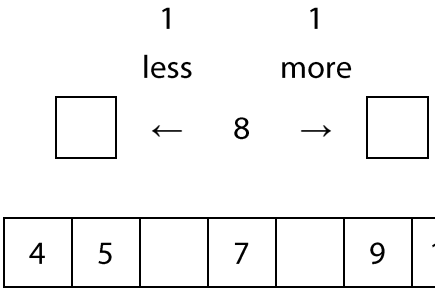
1:8	Provide further varied practice, continuing to expose the 'five and a bit' structure of the numbers six to ten. For example: • provide a range of representations and ask children to *'Circle the ones that show eight.'* • provide a representation and ask children to show it using: • a tens frame and counters • a part–part–whole diagram. Useful representations include: • dominoes • pairs of dice • tally marks • bead strings (arranged into 'five and a bit') • multilink sticks (with five represented in one colour, and the 'and a bit' represented in another) • base-ten number boards • number lines • money – using five pence and one pence coins only (do not use two pence coins as some children will not be sufficinelty familiar with coin values to remain focused on the teaching point) • part–part–whole (cherry and bar models) • real life contexts, such as: • candles on a cake – for example, five lit and three unlit • children on a seesaw – for example, five on one side and three on the other. Again, pay particular attention to ten as five and five.	Bead string:  Base-ten number boards:  Number line:  Ten as five and five: 
1:9	So far, children have been working with quantity values. Finally, move on to examples where the numbers are presented in numeral form only.	

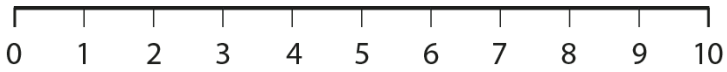
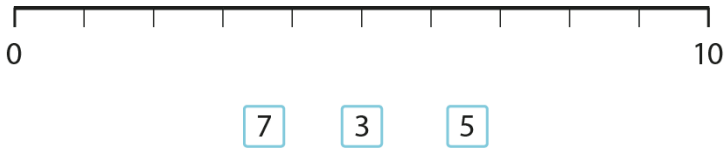
	Present part–part–whole diagrams, or story-based problems, with missing parts, restricting examples to cases where one of the parts is five. The children may initially want to refer to the part–part–whole models completed and displayed in step *1:5*, but the ultimate aim is for children to consolidate and become fluent in partitioning the numbers six to ten into 'five and a bit'.	Part–part–whole questions: *'Fill in the missing numbers.'*  Story-based problems: • *'I clap five times. How many more claps before I've clapped seven times?'* • *'I have eight house points to award. I've awarded five. How many do I have left?'*
1:10	To challenge children and assess their depth of understanding, you could end the teaching point with 'true or false' dòng nǎo jīn questions, such as those presented here. To develop children's reasoning skills you could use a problem where each person might be wrong, for example: *'Nathan says he has two counters.* *Sasha says she has five counters.* *Jo says they have eight counters all together.* *One person has made a mistake – who?'* Work with the children until they see that each person might be wrong, and see if they can resolve the problem in three different ways.	'True or false' problems: • 'The two parts represent the whole.'  • 'The part–part–whole model matches the tens frame.'   • 'I clap five times. If I clap three more times, I will have clapped nine times.'

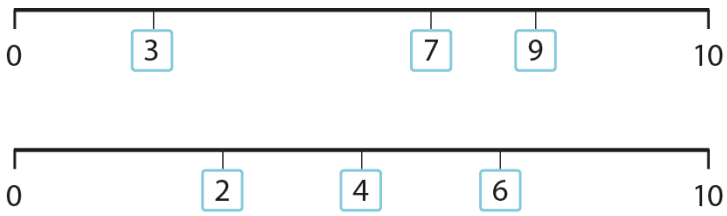
Teaching point 2:

Six, seven, eight and nine lie between five and ten on a number line.

Steps in learning

	Guidance	Representations
2:1	For this teaching point, you will need to work with all of the numbers from one to ten, but highlight at certain points where the numbers six to nine 'sit' in this group. Begin by ordering a set of 1–10 number cards with the class to create a 1–10 number track. For now, do not include zero, since then the number one card would be the second card, not the first one; zero will be considered in step *2:3* when we move to a number line. Work on counting, forwards and backwards, until children are confident in knowing 'one more' and 'one less' for these numbers. Providing the ordered number cards or a number track for support where necessary, present the children with missing number sequences (counting in ones only) to complete. More ideas for varied practice with 'one more' and 'one less' can be found in segment *1.3 Composition of numbers: 0–5*, steps *6:4* and *6:5*.	*'Fill in the missing numbers.'* 
2:2	Now return to the number cards; with one and ten already placed, challenge the children to think about where a given card should be positioned relative to those. Three, for example, will sit nearer (but not next to) one, whilst eight will sit nearer (but not next to) ten. This activity requires the development of proportional thinking, for example, '*eight is here relative to the 1* and *the 10*'. This is a significant step forward from children's usual tendency to look at the	

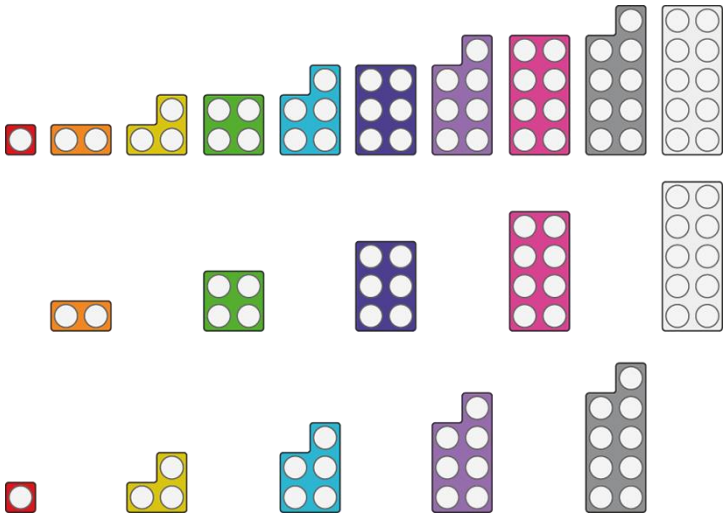
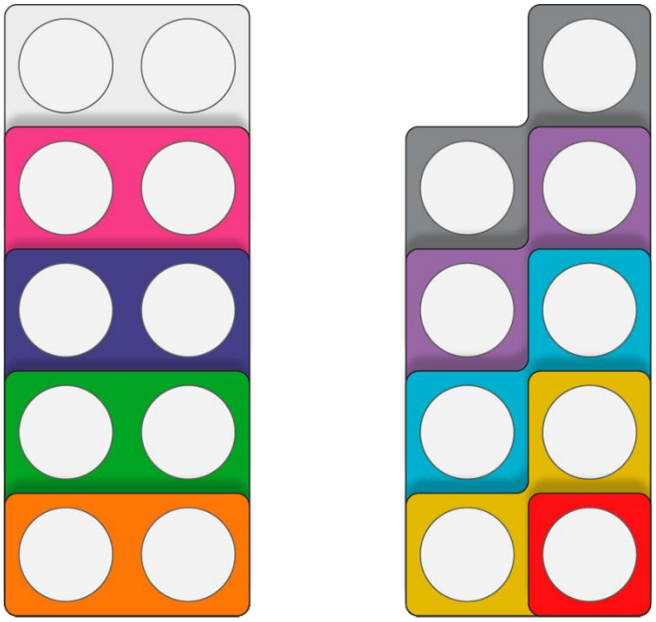
	1 and 'count up' in imaginary steps from there (an additive form of working).	
2:3	Now move on to counting forward and back on a number line; unlike the number cards, zero is now included – on a number line it is the intervals (jumps) that are counted and not the numbers. As in the previous step, discuss the position of numbers relative to *both* zero and ten to further embed proportional thinking, for example: • *'Five sits in the middle of zero and ten.'* • *'Eight is nearer to ten than to zero.'* • *'Is four nearer to zero or nearer to ten?'* At this stage rely on visual perception (for example, visually four is closer to zero than it is to ten on the number line), rather than calculation (for example four is six steps away from ten, but only four away from zero).	
2:4	Progress to the use of 'unmarked' number lines – challenge children to estimate the position of given numbers. Teach the children to first mark on the midpoint (five) before trying to place a given number. Continue to watch out for children counting up from zero in imaginary steps to find the correct position; instead keep the focus on the position of numbers relative to *both ends* of the number line. For the numbers six to nine, encourage children to use their understanding of the 'five and a bit' structure of each number, placing each number at a position between five and ten based on the value of the 'bit'. After each estimation, reveal the full set of numbers on the number line to support checking. There is no need for children to be completely accurate with their placements, rather they should be	*'Estimate where each of the numbers lie on the number line.'*  *'Estimate where each of the numbers lie on the number line.'* 

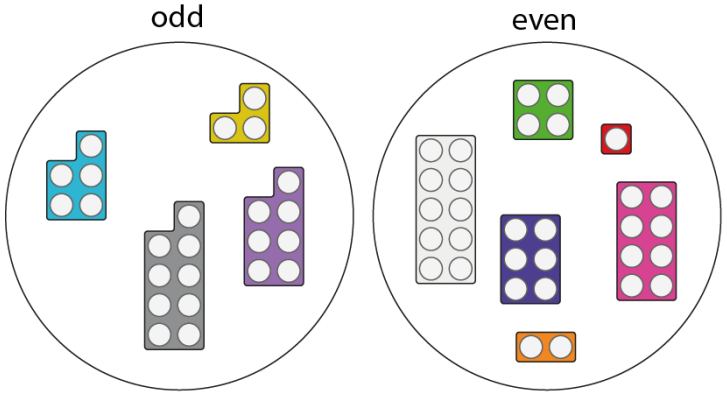
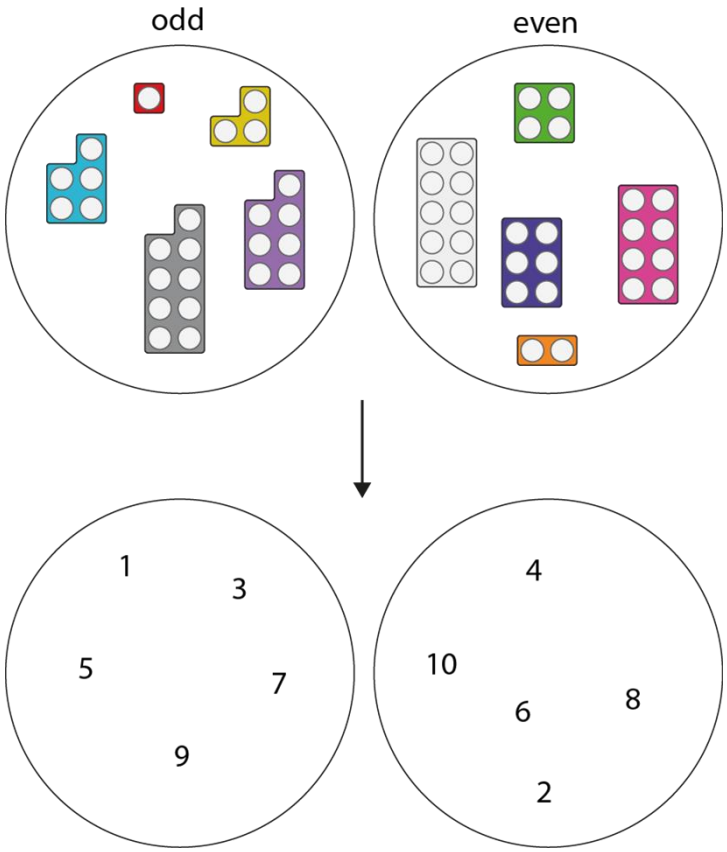
	making reasonable judgements that demonstrate they are developing proportional thinking.	
2:5	To promote and assess depth of understanding use dòng nǎo jīn 'spot the mistake' problems.	*'Spot the mistakes.'* 
2:6	Finally, you can build on the use of comparative language covered in segment *1.1 Comparison of quantities and measures.* As children work with the number line representation, they can use the phrases 'greater than', 'less than' and 'equal to', and the related symbols, when expressing their explanations. Provide further practice using these symbols, in the form of missing symbol or number problems, in particular focusing on the numbers six to ten as well as their size relative to the numbers zero to five.	Using comparative language and symbols: *'Why is this incorrect?'*  *'4 is shown in the middle of the number line, but four is less than five.'* $4 < 5$ Missing number and symbol problems: • *'Fill in the missing symbols.'* 3 ◯ 0 7 ◯ 9 • *'Fill in the missing symbols and numbers.'* • *'Explain why you made your choices.'* 7 ◯ □ □ ◯ 8 Dòng nǎo jīn: *'Use the numbers 6, 7, 8, 9 and 10 to complete the set of expressions. You can use a number more than once.'* □ > □ □ < □ □ = □

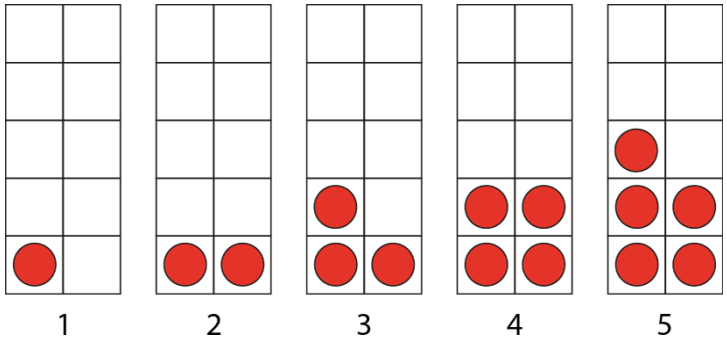
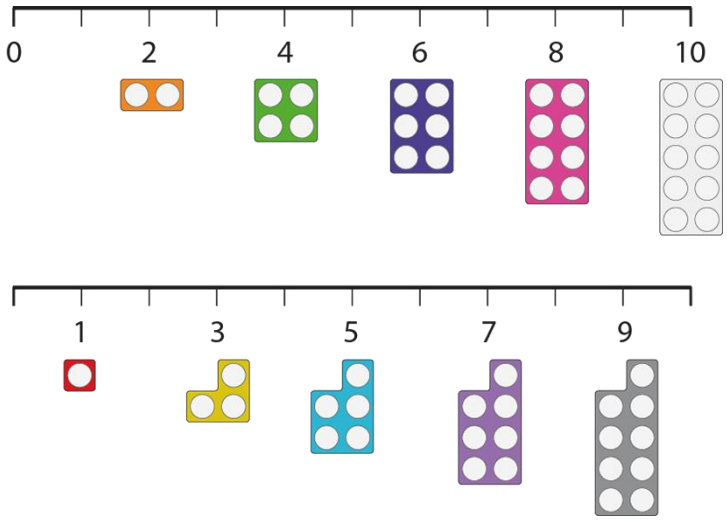
Teaching point 3:

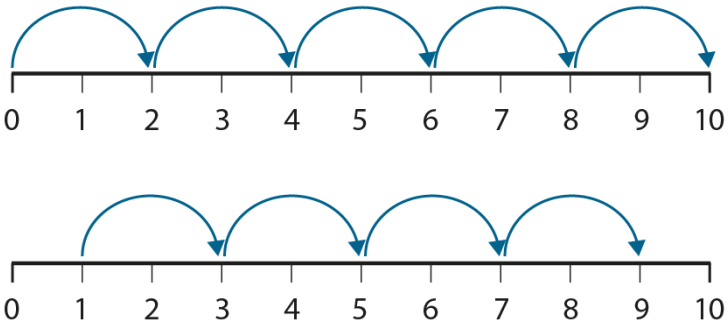
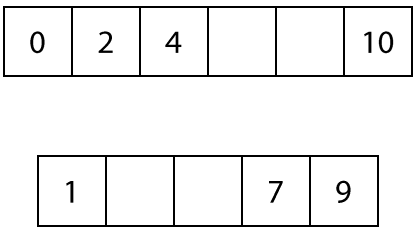
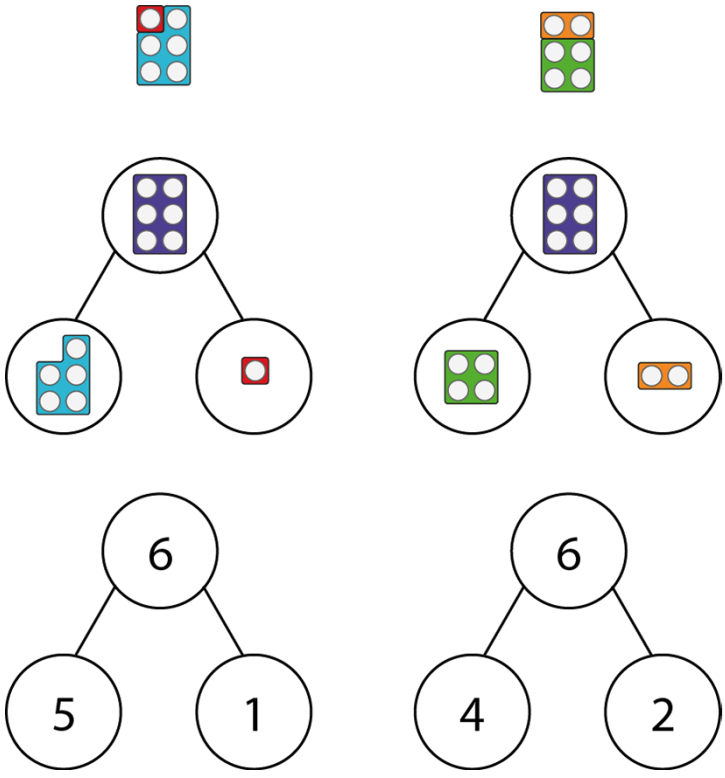
Numbers that can be made out of groups of two are even numbers; numbers that can't be made out of groups of two are odd numbers. Even numbers can be partitioned into two odd parts or two even parts; odd numbers can be partitioned into one odd part and one even part.

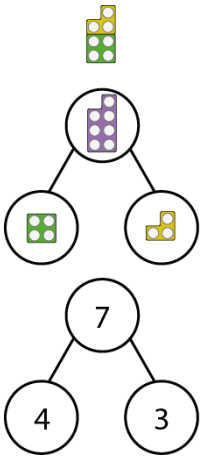
Steps in learning

	Guidance	Representations
3:1	Base-ten number boards or cards with dots are a useful resource for this teaching point. After allowing children time to freely explore the shapes, introduce some parameters. Ask children to put the shapes into numerical order, then discuss and group the shapes according to whether they are made of pairs or not (and instead have one dot on its own). Then introduce the language of odd and even, applied to the relevant shapes/groups. To further explore the concept, arrange the even and odd shapes in stacks and ask children: • *'What's the same?'* • *'What's different?'* Encourage use of the stem sentences: • ***'___ is made of (a) pair(s); it is an even number.'*** • ***'___ is not made of pairs; it is an odd number.'***	Ordering base-ten number boards:  Stacking base-ten number boards: 

3:2	Allow children to apply their developing understanding by sorting base-ten number boards into odds and evens; they should use the language of the stem sentence to justify their sorting. Use a dòng nǎo jīn question: provide sorted numerals and shapes with mistakes for children to spot, explain and correct.	*'Benita sorts these shapes into odd and even numbers. Is she correct? Why/why not?'* 
3:3	Progress to using numerals. Sort the base-ten number boards into odds and evens, and then replace each shape with its numeral one-by-one. Give children practice identifying whether numbers are odd or even based on numeral representations only.	

3:4	Now move to the use of tens frames to reveal the language of the teaching point in the following generalised statements: • ***'Numbers that can be made out of groups of two are even numbers.'*** • ***'Numbers than can't be made out of groups of two are odd numbers.'*** Arrange the tens frames vertically and complete them row by row – i.e. 'twos-wise' – to expose the odd and even structure. Give children the opportunity to engage with the process of making each number to ten, following the row-by-row rule, so that they can see that the generalised statements are true. Normally a 'five and a bit' structure (filling up the first row or column first) is applied to a tens frame. However the 'twos-wise' filling is applied here to expose the structure of odd and even numbers.	
3:5	At this stage, it is important for children to develop fluency with the patterns by practising counting in odds and evens. You can use base-ten number boards alongside number lines, as shown, to draw children's attention to the 'skip counting' concept. Then remove the shapes and progress to using only the number line, highlighting the 'jumps' of two. With all numbers marked on the number line, highlight the fact that: • odd and even numbers alternate • between two even numbers there is an odd number • between two odd numbers there is an even number.	'Skip counting' – number lines with base-ten number boards: 

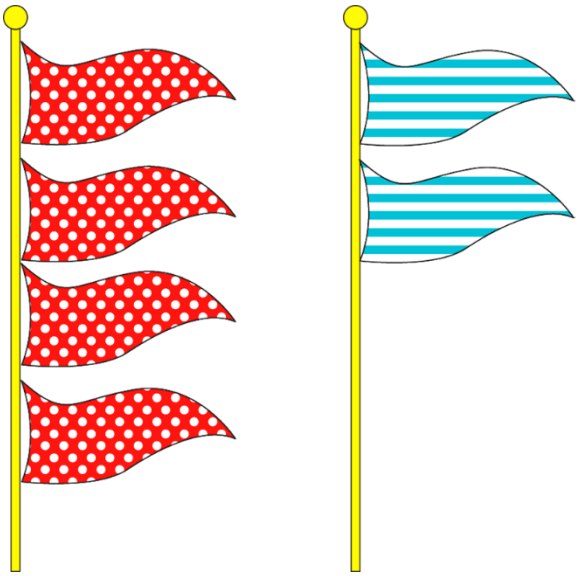
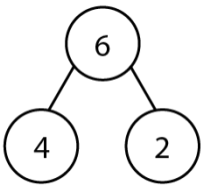
		'Skip counting' – highlighting 'jumps' of two: 
3:6	For further fluency practice, provide odd and even number sequences for children to complete.	*'Fill in the missing numbers.'* 
3:7	When children are fluent in identifying odd and even numbers using both number names and numerals, progress to an investigation of the odd/even patterns when partitioning a number into two. Allow children to explore one even number to ten at a time, investigating some of the different ways that each number can be partitioned into two parts. You could provide double-sided counters or base-ten number boards for support – the latter will provide a greater scaffold for the identification of odd and even patterns. For each partitioning: • record using a part–part–whole diagram (cherry or bar model) • ask children to identify whether each number (both parts and the whole) is even or odd. There is no need to be systematic at this stage – the focus should be on exploration. As the pattern is gradually revealed, work towards use of the generalised statement:	

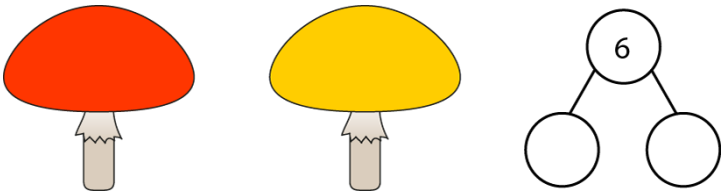
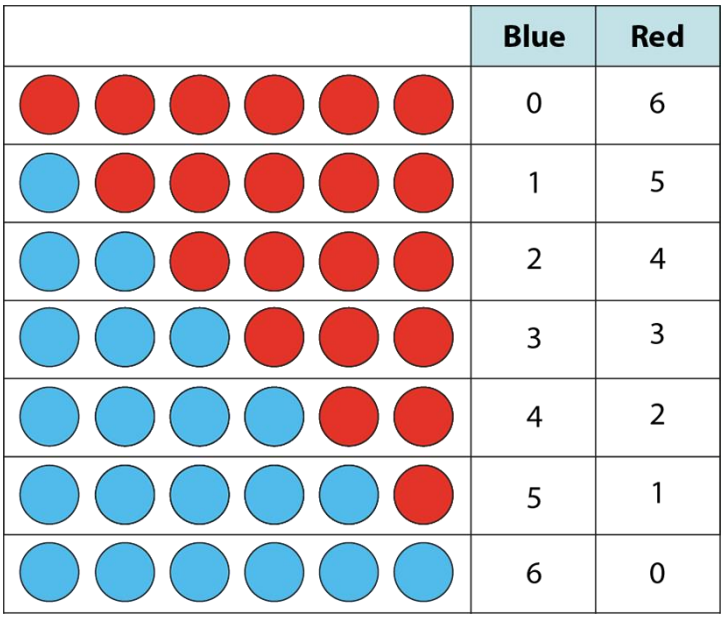
	'Even numbers can be partitioned into two odd parts or two even parts.' Challenge children using a dòng nǎo jīn question: *'Why can't an even number be partitioned into an odd part and an even part?'*	
3:8	Now repeat the process for the partitioning of odd numbers, working towards the generalised statement: ***'Odd numbers can be partitioned into one odd part and one even part.'*** Again, challenge children using a dòng nǎo jīn question: *'Why can't an odd number be partitioned into two odd parts?'*	

Teaching point 4:

Each of the numbers six to ten can be partitioned in different ways. The numbers six to ten can be partitioned in a systematic way.

Steps in learning

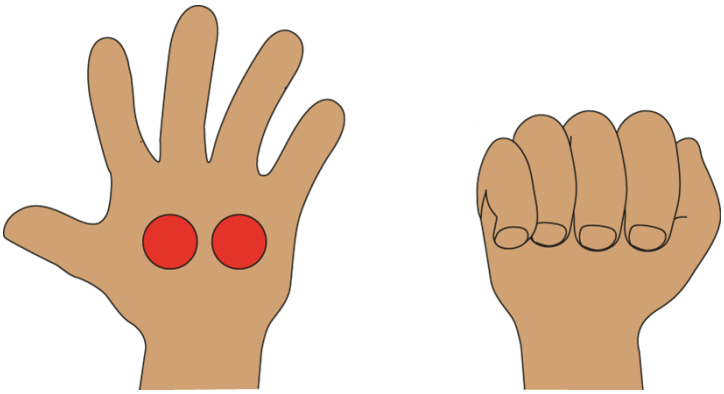
	Guidance	Representations
4:1	This teaching point should follow the same progression used in *Teaching points 3* and *4* of segment *1.3 Composition of numbers: 0–5*. Here we provide some sample representations for the number six; these should be used in conjunction with segment *1.3*, and then extended to the numbers seven to ten. Children can use what they learnt about rules for partitioning odd and even numbers (*Teaching point 3*) to help them. The focus should be on children developing fluency in partitioning the numbers.	
4:2	Allow children to explore the different ways to partition six into two parts, moving beyond the earlier focus on the 'five and a bit' structure (*Teaching point 1*). Follow these steps in progression, varying the representations provided: • Use concrete or pictorial representations where the context suggests a particular way of partitioning (for example, flags with different patterns). • Represent the partitioning on a part–part–whole diagram. • Provide contexts where the items are identical (for example, dots on mushrooms), or where there is more than one way of partitioning (for example, faces partitioned according to different features). As with segment *1.3 Composition of numbers: 0–5*: • remember to include cases where one of the parts is zero	*'What numbers can you see?'*  *'I can see six flags. Four are spotty and two are stripy.'*  *'Six is the whole; four is a part; two is a part.'*

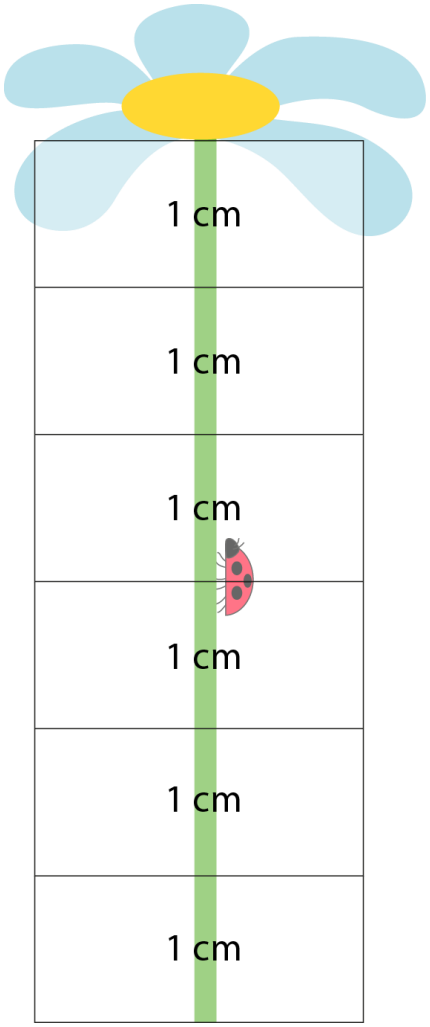
	• include contexts that link to measures • progress to partitioning into more than two parts • challenge children to spot mistakes. Children should now be very familiar with the stem sentences: • ***'___ is the whole; ___ is a part; ___ is a part.'*** • ***'___ is a part; ___ is a part; ___ is the whole.'***	*'There are a total of six spots on the mushrooms. Find different ways to split them between the two mushrooms.'* 
4:3	Now move on to the use of double-sided counters, encouraging children to work systematically to find all possible ways to partition six into two parts. More guidance is provided in *Teaching point 4* of segment *1.3 Composition of numbers: 0–5*, however, now that children have encountered the bar model you could use this to represent the partitioning.	*'Turn one counter over at a time and name the parts that make the whole number.'* 

Teaching point 5:

Each of the numbers six to ten can be partitioned into two parts; if we know one part, we can find the other part.

Steps in learning

	Guidance	Representations
5:1	Now progress to application of the learning from *Teaching Point 4* – children should solve missing part problems for wholes in the range six to ten. Refer to segment *1.3 Composition of numbers: 0–5*, *Teaching point 5* for further guidance. Begin with concrete/pictorial contexts and remember to include cases where: • the known part is zero • the missing part is zero • the context relates to measures/the relative value of numbers • the parts are not visual, for example: • *'I am going to clap four times.'* • Clap once. • *'How many more times do I need to clap?'* Again, children can apply what they learnt in *Teaching point 3* to help them reason, using the following generalised statements: • ***'If the whole is odd and one part is even, the other part must be odd.'*** • ***'If the whole is odd and one part is odd, the other part must be even.'*** • ***'If the whole is even and one part is odd, the other part must be odd.'*** • ***'If the whole is even and one part is even, the other part must be even.'***	Concrete: *'I have six counters. There are two counters in my open hand. How many counters are there in my closed hand?'*  *'The whole is even and one part is even, so the other part must be even; there are four counters in the closed hand.'*

		Measure context: *'A flower is six centimetres tall and the ladybird is three centimetres up the flower. How much further does the ladybird need to go to get to the top?'* 
5:2	Repeat steps *4:2–5:1*, focusing on partitioning seven, then eight and so on. As you get towards the larger numbers there will be an increasing number of combinations, and it may take longer to move through the teaching points. Ensure that you plan a variety of contexts and use a range of representations. Throughout, continue to keep the focus on fluency in partitioning the numbers, ultimately working towards children being able to partition quickly and confidently without relying on visual images.	