MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.29 Using equivalence and the compensation property to calculate

Teacher guide | Year 5

Teaching point 1:

If one addend is increased and the other is decreased by the same amount, the sum stays the same. (same sum)

Teaching point 2:

If one addend is increased (or decreased) and the other is kept the same, the sum increases (or decreases) by the same amount.

Teaching point 3:

If the minuend and subtrahend are changed by the same amount, the difference stays the same. (same difference)

Teaching point 4:

If the minuend is increased (or decreased) and the subtrahend is kept the same, the difference increases (or decreases) by the same amount.

Teaching point 5:

If the minuend is kept the same and the subtrahend is increased (or decreased), the difference decreases (or increases) by the same amount.

Teaching point 6:

The value of the expressions on each side of an equals symbol must be the same; addition and subtraction are inverse operations. We can use this knowledge to balance equations and solve problems.

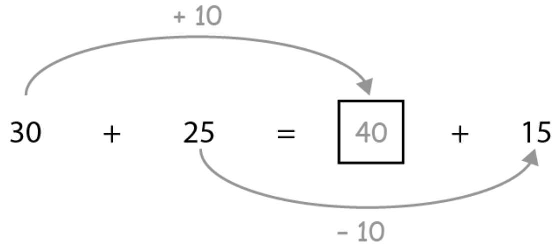
Autumn 2018 pilot

Overview of learning

In this segment children will:

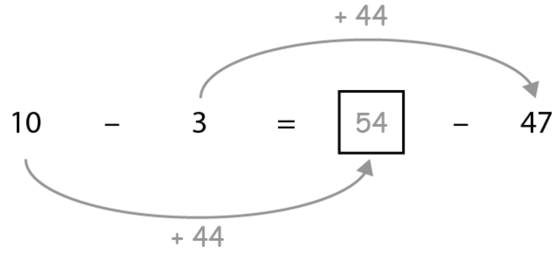
- deepen their understanding of the structures of addition and subtraction
- use their understanding of these structures to explore the compensation property for addition and subtraction through the rules described in *Teaching points 1–5*
- apply the compensation property for addition and subtraction as a calculation strategy and, where applicable (for same sum and same difference), to balance equations
- deepen their understanding of equivalence and the equals sign
- apply their understanding of equivalence and inverse operations to balance equations in situations where application of the compensation property is less useful or not appropriate.

This segment begins with the idea of 'same sum' with which children should already be familiar. In segment *1.19 Securing mental strategies: calculation up to 999,* children used this approach as a strategy to simplify mental calculations, where it was referred to as 'redistributing'. In this segment (*Teaching point 1*), children will revisit this strategy, extend its use as a calculation strategy to larger numbers and decimal fractions, and use it to balance equations, for example:

- 7.1 + 1.9 = 7 + 2 = 9
  (calculation strategy)
- 

  (balance equation)

Similarly, in *Teaching point 3*, children will build on what they learnt about 'same difference' in segment *1.12 Subtraction as difference*, using it both as a calculation strategy and to balance equations, for example:

- 53 – 39 = 54 – 40 = 14
  (calculation strategy)
- 

  (balance equation)

As well as using the approach to aid mental calculation, children will also learn that same difference can be applied to avoid exchange through zeros when calculating using the column subtraction algorithm; for example, *20,000 – 1,658* can be transformed to *19,999 – 1,657* before applying the column algorithm.

In *Teaching points 2, 4* and *5*, children will explore the effect on the sum/difference of changing the value of one number in an addition/subtraction problem, and will apply their understanding to using known, or given, facts to solve related calculations, for example:

- increasing/decreasing only one addend:
  - 19.18 + 0.8**2** = 20 so 19.18 + 0.8**1** = 19.99
  - 36 + 47 = 8**3** so 36 + 49 = 8**5**

- increasing/decreasing only the minuend:
  - 6**2**,865 – 41,294 = 21,571 so 6**3**,865 – 41,294 = 22,571
  - 52.13 – 3.76 = **4**8.37 so 22.13 – 3.76 = **1**8.37

- increasing/decreasing only the subtrahend:
  - 4,975 – **7**0 = 4,905 so 4,975 – **6**0 = 4,915
  - 2,568 – 367 = 2,20**1** so 2,568 – 368 = 2,20**0**

The final compensation property in this list is likely to be the one that children find the most challenging. Teachers should ensure that the idea is initially explored using examples with very simple values and clear pictorial representations as exemplified in *Teaching point 5*; teachers should also look out for errors in which the difference is adjusted in the 'wrong direction'.

Throughout learning on the compensation properties (*Teaching points 1–5*), children need to recognise that the strategies are not transferable between operations; what works for addition does not work for subtraction, and vice versa, as they are dealing with different structures of calculation in each case. The strategies learnt for each operation, however, *can* be applied to the different structures for that operation (for example, 'same difference' can be applied to contexts that include the reduction, partitioning and difference structures of subtraction). Teachers should ensure that practice includes a variety of addition and subtraction structures.

In this segment, each compensation property is explored separately. However, in each case, there is emphasis on children gaining a deep awareness of the underlying structure rather than learning the 'rule' by rote. The aim is for children to have a strong understanding of the relationships between the values of the addends and the sum, and between the values of the minuend, subtrahend and difference. It is important for children to be provided with 'mixed' practice at the end of this segment, during which they have the opportunity to apply learning from all teaching points, including making sensible and efficient choices when calculating and balancing equations.

As children explore and apply the compensation properties of addition and subtraction, they will also gain a deeper understanding of equivalence and the meaning of the equals symbol. This is developed further in *Teaching point 6*, in which they will encounter equations that cannot easily be balanced using one of the compensation properties and are instead balanced using the knowledge that addition and subtraction are inverse operations. Again, it is important for children to be presented with a range of equation-balancing problems so that they gain experience in identifying efficient strategies.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

If one addend is increased and the other is decreased by the same amount, the sum stays the same. (same sum)

Steps in learning

	Guidance	Representations

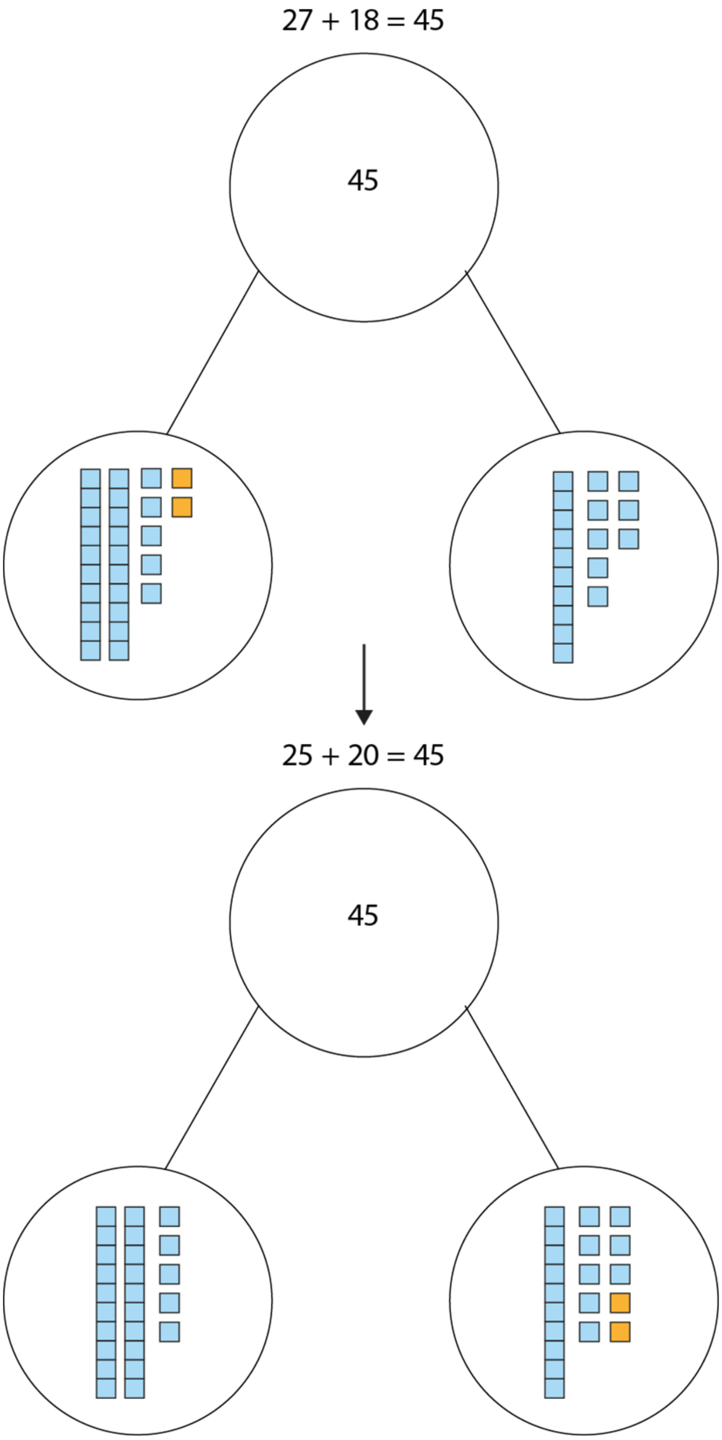
1:1

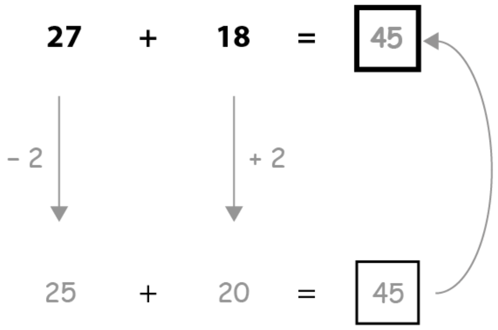
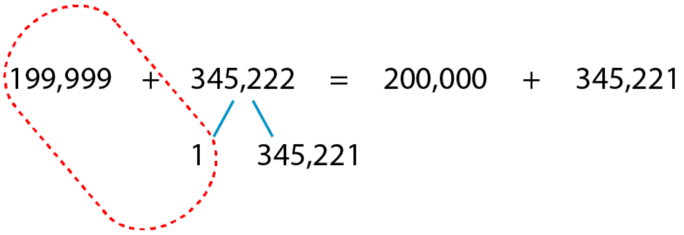
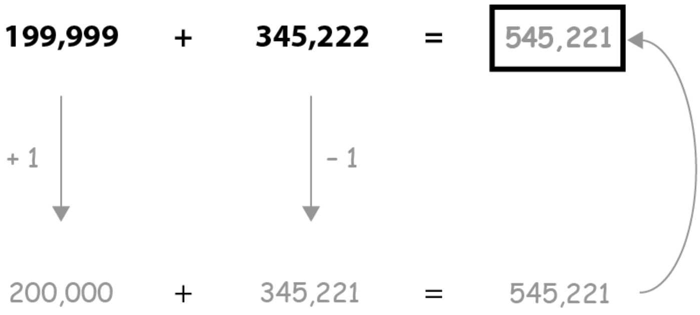
Children learnt about ‘same sum’ in segment *1.19 Securing mental strategies: calculation up to 999, Teaching point 2*. There it was used as a mental strategy to transform calculations, making them easier to solve, and was referred to as ‘redistributing’. In this teaching point, children will use the following generalisation, learnt in segment *1.19*, to transform calculations (extending to larger numbers and decimal fractions) as well as to balance equations: ***‘If one addend is increased by an amount and the other addend is decreased by the same amount, the sum remains the same.’***

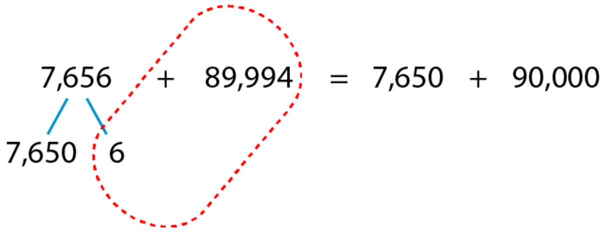
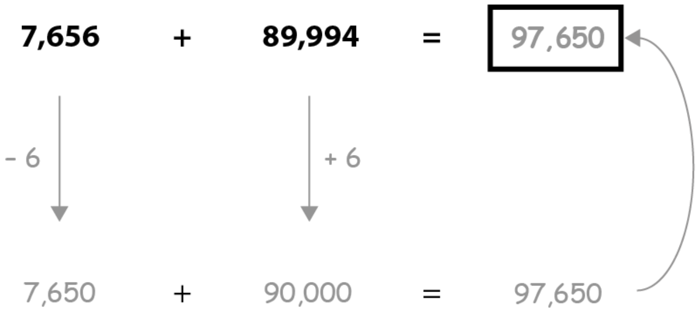
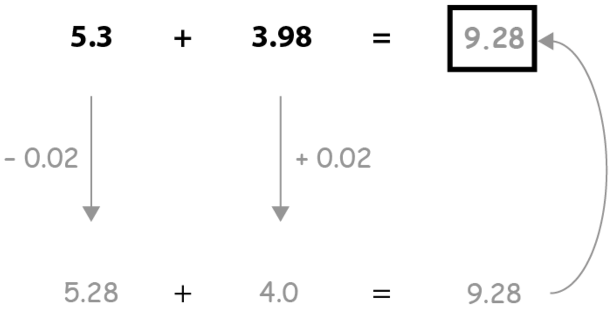
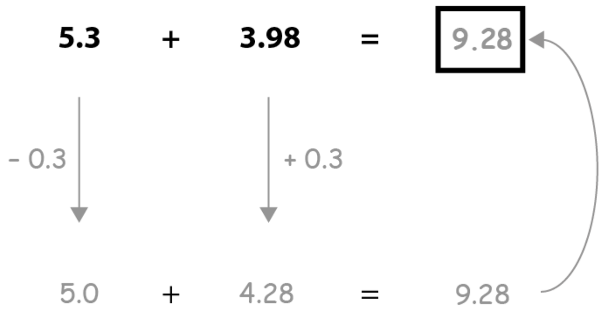
Begin by ensuring that children fully understand how the sum remains the same when the addends are ‘redistributed’, in the context of simplifying calculations:

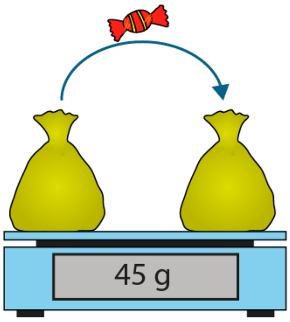
- Using representations from segment *1.19* (such as Dienes and equation pairs), review how we can partition the same two-digit number in a variety of ways. Draw attention to what stays the same and what changes, and remind children of the generalised statement above.
- Briefly review how this strategy was used to transform calculations to make them easier to solve, encouraging children to explain the reasoning, for example: *‘I’ve subtracted two from this addend, so I must add two to the other addend to keep the sum the same.’* You can move from more scaffolded to less

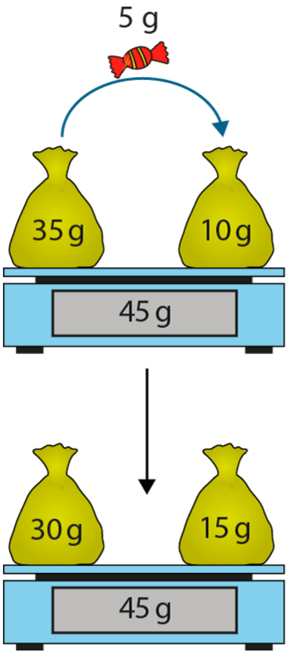
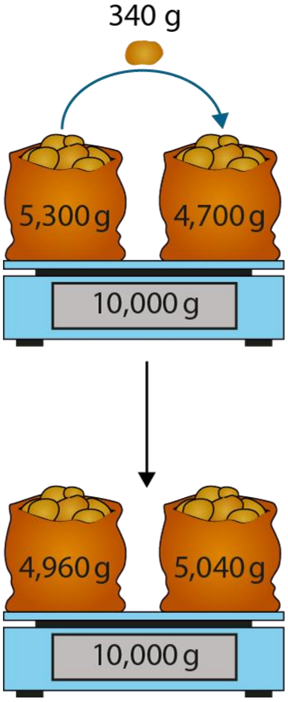
Part–part–whole model and Dienes:

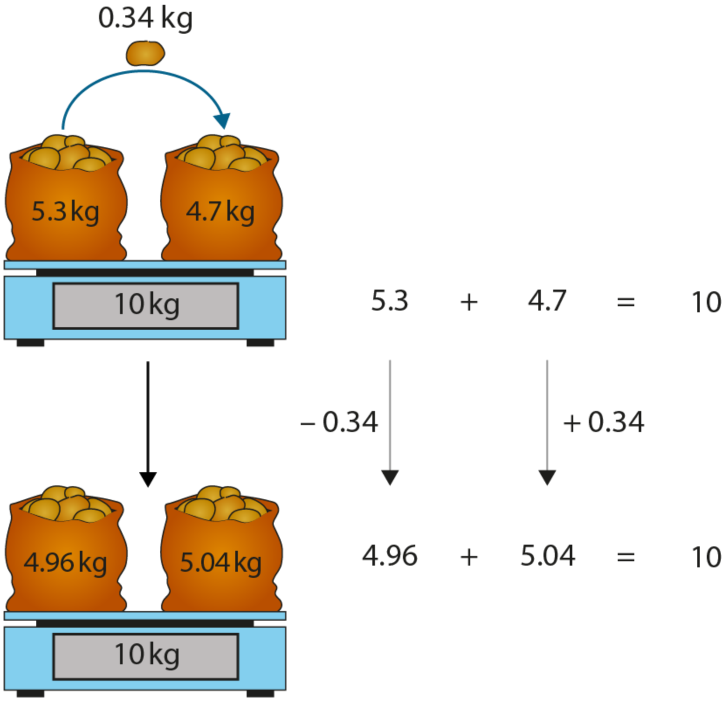
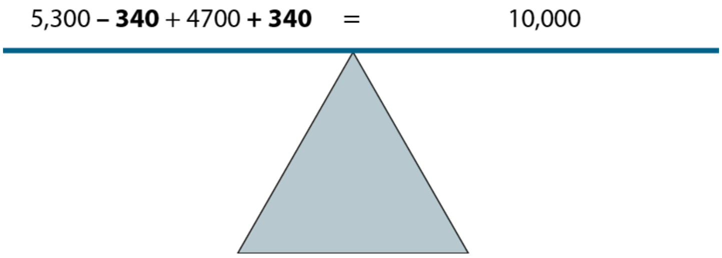


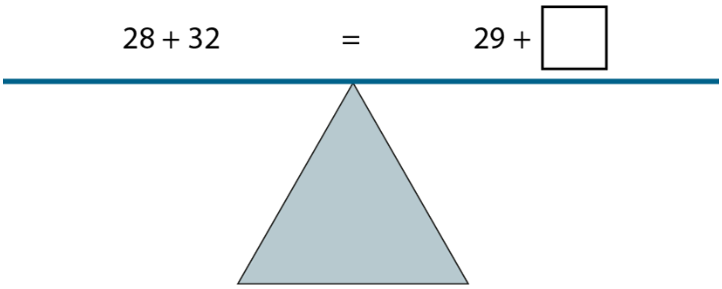
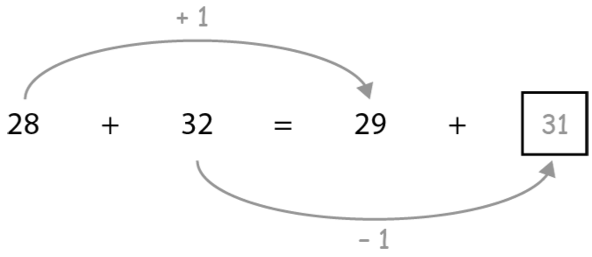
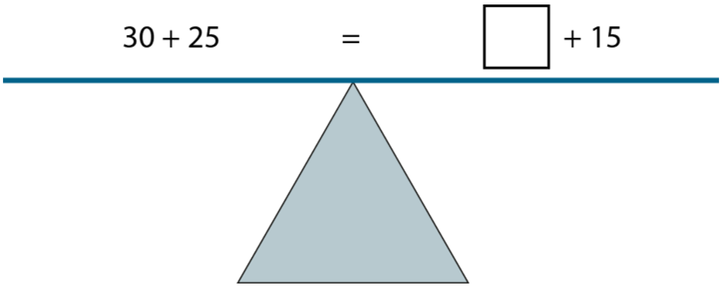
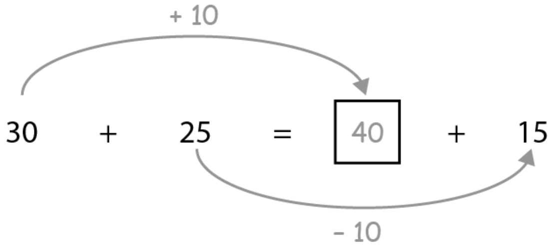
	scaffolded problems, as exemplified opposite, to make sure that children can confidently apply the strategy themselves.	Equation pairs/jottings:  Using 'same sum' (redistribution) as a strategy for simplifying calculations: **53 + 39 =** ☐ **27 + 18 =** ☐ 52 + 40 = ☐ 30 + 15 = ☐ **53 + 39 =** ☐ **27 + 18 =** ☐ 50 + 42 = ☐ 25 + 20 = ☐
1:2	Now extend the 'same sum' rule to simplify calculations with larger numbers. Draw attention to the fact that now we need to add or subtract amounts to make multiples of 10, 100 or 1,000 etc. to make the calculations easier to solve. Continue to use the generalisation from step *1:1* and encourage children to always describe what they need to add to, or subtract from, each addend to keep the sum the same. Include examples for which you might need to add to the second addend and therefore subtract from the first addend.	Adding to the first addend and subtracting from the second:  

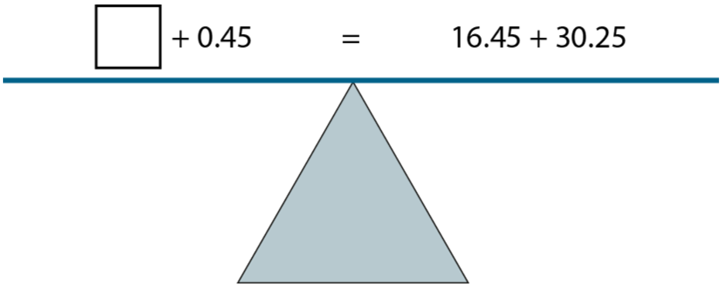
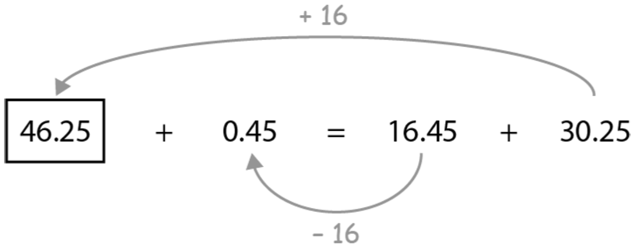
		Subtracting from the first addend and adding to the second:  
1:3	Similarly, extend the strategy to calculations with decimal fractions, discussing which redistributions make the arithmetic easier, for example: 4.5 + 2.9 = 5.0 + 2.4 or 4.5 + 2.9 = 4.4 + 3.0 55.26 + 45.24 = 55.25 + 45.25 or 55.26 + 45.24 = 50.25 + 50.25 Encourage children to look out for addends that can be increased or decreased to make a multiple of 1, 10, 100, 1,000 etc.	Example with two strategies:  

		Missing-number problems: *'Fill in the missing numbers.'* **7.1 + 1.9 =** ☐ **2.6 + 3.9 =** ☐ 7.0 + ☐ = ☐ 2.5 + ☐ = ☐ **2.91 + 3.24 =** ☐ **175.7 + 24.86 =** ☐ 3.0 + 3.15 = ☐ 176 + ☐ = ☐
1:4	You can use a dòng nǎo jīn question to assess and promote depth of understanding, for example: - *'Salvo says that the best way to solve these calculations is to use 'same sum':* *1,998 + 4,632* *1.21 + 3.78* *38,202 + 18,998* *Mia disagrees and says that it is quicker to use a written method. Who is right?'*	
1:5	Now explore how the 'same sum' rule can be used not only to simplify calculations, as in steps *1:1–1:4*, but also to balance equations. Begin by introducing a measures context, such as the one shown opposite, where one (unlabelled) quantity increases and the other (also unlabelled) quantity decreases by the same amount. Draw attention to the fact that the total quantity remains the same by asking children *'What changes?'* and *'What stays the same?'* Make sure you demonstrate that the total quantity remains the same, irrespective of which 'direction' the redistribution occurs in (i.e. for the example opposite, move a sweet from right to left, as well as from left to right).	Addends not quantified: 

	Now introduce values to the measures, choosing numbers such that the redistribution doesn't provide a simpler calculation but instead demonstrates, more generally, the equivalence before and after redistribution. Write equations to represent the situation and ask children to describe, in words, what is happening to the addends and the sum. Ensure that children can write and understand the equation representing equivalence before and after redistribution, for example: 35 + 10 = 30 + 15 35 + 10 = 40 + 5 etc.	Addends quantified:  35 + 10 = 45 − 5 + 5 30 + 15 = 45 so 35 + 10 = 30 + 15
1:6	Work through another example, as a class, extending the same idea to larger numbers and decimal fractions, until children can confidently write balanced equations. Encourage children to check that each side of the equation sums to the same value. Continue to use the generalised statement from step *1:1*, along with the following stem sentences: - ***'I've subtracted ___ from one addend, so I need to add ___ to the other addend to keep the sum the same.'*** - ***'I've added ___ to one addend, so I need to subtract ___ from the other addend to keep the sum the same.'***	Larger numbers:  5,300 + 4,700 = 10,000 − 340 + 340 4,960 + 5,040 = 10,000 so 5,300 + 4,700 = 4,960 + 5,040

		Decimal fractions:  so 5.3 + 4.7 = 4.96 + 5.04
1:7	You can represent the examples in 1:6 with balance scales, as shown opposite, to draw attention to the use of the equals symbol and the sum remaining the same. This will also help to prepare children for balancing equations in subsequent steps.	
1:8	Now explore how an understanding of the 'same sum' rule can be used to balance equations where there is an unknown. Present a context using two-digit numbers with a simple redistribution, for example: - *'There is the same number of children in Year 5 as there is in Year 6. In Year 5, there are 28 boys and 32 girls. In Year 6, there are 29 boys. How many girls are there in Year 6?'* Write expressions to represent the number of children in each year group. You can use balance scales, as shown	Redistributing one: *'There is the same number of children in Year 5 as there is in Year 6. In Year 5, there are 28 boys and 32 girls. In Year 6, there are 29 boys. How many girls are there in Year 6?'*

	opposite, to emphasise the equivalence of the two. Then, using the same stem sentences as before, encourage children to describe how the addends have changed: *'I've added one to one addend, so I need to subtract one from the other addend to keep the sum the same.'* Then repeat, using an example in which a value other than one is redistributed. Note that in the second example opposite, we know by how much the second addend has been reduced (in contrast to the first example, in which we knew how much the first addend had been increased). Throughout the rest of this teaching point, vary the position of the unknown and the value by which the addends are redistributed.	  *'There are 31 girls in Year 6.'* Redistributing ten: *'Sally plans to buy 30 bottles of cola and 25 bottles of juice for a party. When she gets to the shop, they only have 15 bottles of juice, but she wants to buy the same number of bottles of drink in total. If she buys all 15 bottles of juice, how many bottles of cola will she need to buy?'*   *'Sally will need to buy 40 bottles of cola.'*

1:9	Work through more examples as a class (both contextual and abstract), including larger numbers and decimal fractions, and varying both the position of the unknown and whether the known addend has been increased or reduced in value. It is important that attention is drawn to both addends either side of the equals symbol and that children don't just rely on the position of the addends in the calculation. You can do this by using examples where the position of the corresponding addend that has increased or decreased is *not* in the same position in both expressions, requiring children to apply the commutative law of addition, i.e.: 340 + ? = 350 + 720 here, the related addends are in the same positions on either side of the equals sign vs 340 + ? = 720 + 350 here, the related addends are in opposite positions on either side of the equals sign.	 *'I've subtracted 16 from 16.45, so I must add 16 to 30.25 to keep the sum the same.'* 
1:10	To complete this teaching point, provide practice in the form of: • missing-number problems, including sequences of related problems, as well as isolated problems • true/false-style questions; encourage children to explain their answers in full sentences (*'This equation is incorrect because…'* and *'This equation is correct because…'*) • real-life problems, including measures contexts, as shown here and on the next page: • *'Sam and Eva spent the same total amount of time doing activities at youth club. Sam spent 75 minutes painting and 35 minutes playing*	Missing-number problems: *'Fill in the missing numbers.'* 57 + 24 = 58 + ☐ 57 + 24 = ☐ + 22 ☐ + 24 = 47 + 34 57 + ☐ = 37 + 44

tennis. Eva spent 65 minutes doing pottery and the rest of the time playing football. How long did Eva spend playing football?'

Throughout, continue to ensure variation as described in steps *1:8* and *1:9* (varying the position of the unknown value by which the addends are redistributed, and order of the addends). Use numbers appropriate to the application of the 'same sum' rule, for example in the problem *57 + 24 = ? + 22*, children can spot the relationship between 24 and 22, whereas in the problem *427 + 274 = ? + 382*, neither 427 nor 274 is clearly easily relatable to 382.

0.57 + 0.24 = 0.58 + ☐

0.57 + ☐ = 0.77 + 0.04

0.57 + 0.24 = ☐ + 0.55

☐ + 45.75 = 65.75 + 124.128

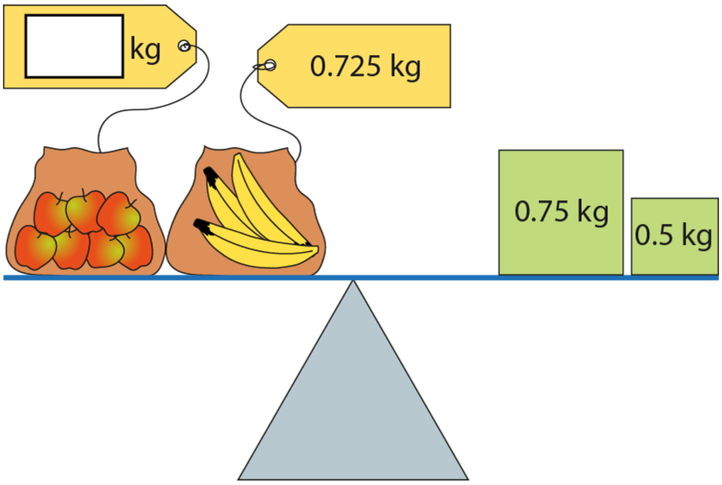
57,500 + 24,000 = 60,500 + ☐

True/false-style questions:

'Decide whether each equation is correct or incorrect.'

	✓ or ✗
101 + 99 = 100 + 100	
27 + 56 = 28 + 57	
248,000 + 564,000 = 148,000 + 464,000	
0.27 + 0.56 = 0.23 + 0.6	

Measures context:



Teaching point 2:

If one addend is increased (or decreased) and the other is kept the same, the sum increases (or decreases) by the same amount.

Steps in learning

Guidance

Representations

2:1

In segment *1.13 Addition and subtraction: two-digit and single-digit numbers*, children learnt to apply known facts within ten to addition of a single-digit number to a two-digit number, e.g.:

$3 + 6 = 9$

so

$23 + 6 = 29$

This teaching point uses and builds on this, drawing attention to the structure of the calculations to generalise how the sum changes when only one of the addends is changed. The calculations are intentionally kept simple so that attention can be drawn to the concept being introduced.

Begin by using the stories and representations from segment *1.13* (shown opposite). Compare the two related calculations and ask children:

- *'What's the same?'*
- *'What's different?'*

Draw attention to the fact that one of the addends has increased by 20 and the sum has increased by 20 as a result.

Now ask the children to solve a set of calculations, starting with a known fact and increasing one of the addends each time, for example:

$4 + 3$

$14 + 3$

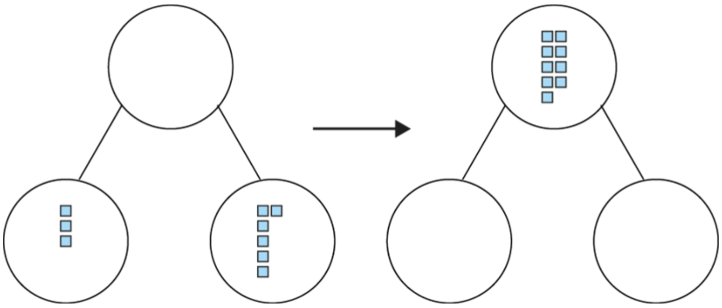
$24 + 3$

…

$94 + 3$

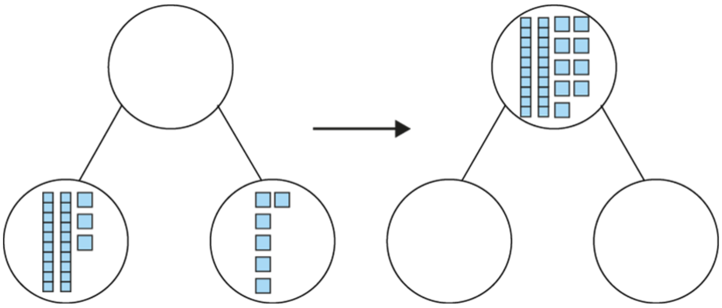
Related additon facts – Dienes and part-part-wholes:

'Diego walked for three minutes to get to his friend's house, and then walked for another six minutes to get to school. His journey took nine minutes altogether.'



$3 + 6 = 9$

'Dana walked for twenty-three minutes to get to her friend's house, and then walked for another six minutes to get to school. Her journey took twenty-nine minutes altogether.'



$23 + 6 = 29$

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Compare the completed calculations as a class and discuss similarities, differences and patterns. Focus on the connection between the known fact and each of the related calculations, drawing attention to the fact that when one of the addends is increased, the sum increases by the same amount:

4 + 3 = 7

4 + **10** + 3 = 7 + **10** = 17

4 + **20** + 3 = 7 + **20** = 27

…

4 + **90** + 3 = 7 + **90** = 97

Use the following stem sentence: ***'I've added ___ to one addend and kept the other addend the same, so I must add ___ to the sum.'***

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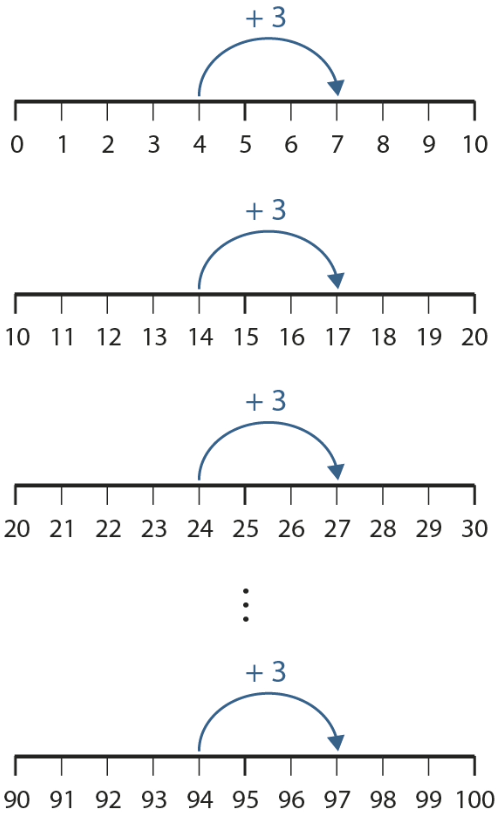
Related addition facts – number line:



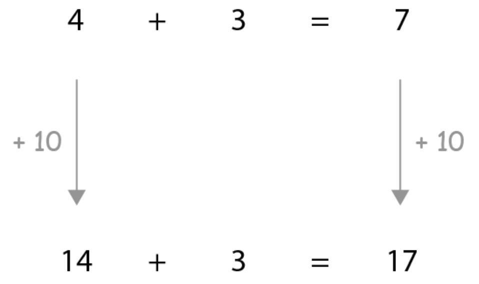
4 + 3 = 7

14 + 3 = 17

24 + 3 = 27

94 + 3 = 97

Comparing calculations:



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2:2

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Now explore an example where one of the addends decreases, e.g.:

96 + 4

86 + 4

76 + 4

66 + 4

As before, ask children to complete the calculations, and then discuss how the calculations are linked, and the patterns in the set.

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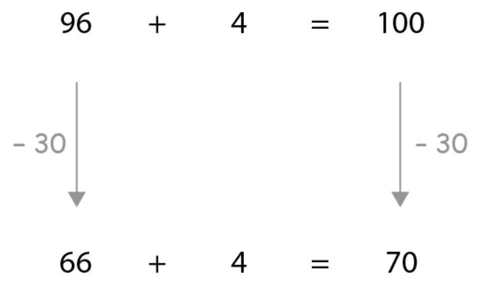

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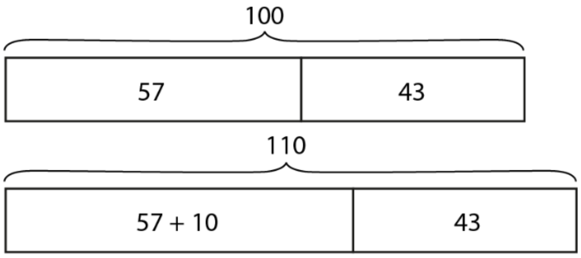
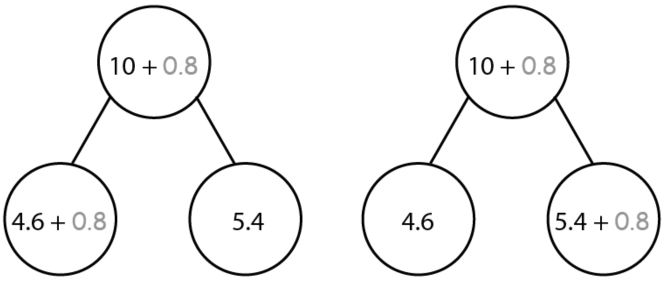
</table>

	Use the stem sentence: ***'I've subtracted ___ from one addend and kept the other addend the same, so I must subtract ___ from the sum.'*** Once children are comfortable with both patterns (increasing or decreasing one of the addends), generalise: ***'If one addend is changed by an amount and the other addend is kept the same, the sum changes by the same amount.'***	
2:3	Now apply the generalisation and stem sentences to solve problems such as the examples opposite. Note that the structure of the problems explicitly links to the generalisation: one addend is increased or decreased and the new sum must be found. Encourage children to identify how the addend and sum change, rather than recalculating.	• *'Together two children have saved £100. Yameena has saved £57 and Tom has saved £43. Yameena is given another £10. How much do they have now?'*  $$\begin{tikzcd} 57 \arrow[d, "+10"'] & + & 43 & = & 100 \arrow[d, "+10"] \\ 67 & + & 43 & = & 110 \end{tikzcd}$$ *'Tom and Yameena now have £110 altogether.'* • *'There were 30 children in a class: 16 boys and 14 girls. Then a boy left. How many children are there in the class now?'* $$\begin{tikzcd} 16 \arrow[d, "-1"'] & + & 14 & = & 30 \arrow[d, "-1"] \\ 15 & + & 14 & = & 29 \end{tikzcd}$$ *'There are now 29 children in the class.'*

2:4	Keeping with the structure described in step *2:3*, extend to problems with larger numbers and decimal fractions. Note that the power of this approach is that there is no need to work out the new value of the changed addend; this is exemplified in the jottings opposite where the 0.8 kg needs only to be added to the combined mass of the two cats.	*'The combined mass of two cats is 10 kg. The mass of one cat is 4.6 kg and the mass of the other cat is 5.4 kg. One of the cats gains 0.8 kg. What is their combined mass now?'*  $4.6 + 5.4 = 10 + 0.8 = 10.8$ $+ 0.8$ *'The combined mass of the two cats is now 10.8 kg.'*
2:5	Now explore how the generalisation can be used to solve isolated calculations mentally by relating them to known facts, rather than automatically always using a written method first. Encourage children to take time to properly examine each calculation they are presented with.	Using known facts and the compensation property to calculate mentally: • Example 1 $\begin{array}{ccccc} \mathbf{0.29} & + & \mathbf{28.71} & = & \boxed{29} \\ & & \uparrow{\scriptstyle +\,28} & & \uparrow{\scriptstyle +\,28} \\ 0.29 & + & 0.71 & = & 1 \end{array}$ $\mathbf{0.29 + 28.71 =} \boxed{}$ $0.29 + 0.71 = 1$ so $0.29 + \mathbf{28}.71 = 1 + \mathbf{28} = 29$

- Example 2

$$\begin{tikzcd} \mathbf{646{,}000} & + & \mathbf{54{,}000} & = & \boxed{700{,}000} \\ 46{,}000 \arrow[u, "+\ 600{,}000"] & + & 54{,}000 & = & 100{,}000 \arrow[u, "+\ 600{,}000"'] \end{tikzcd}$$

646,000 + 54,000 = ☐

46,000 + 54,000 = 100,000

so

646,000 + 54,000 = 100,000 **+ 600,000**

= 700,000

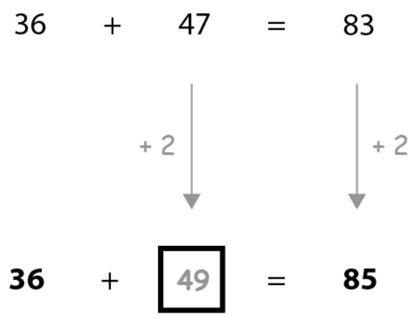
'Sort the calculations according to whether you would solve them mentally or using a written method.'

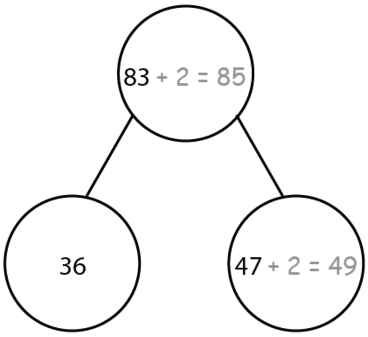
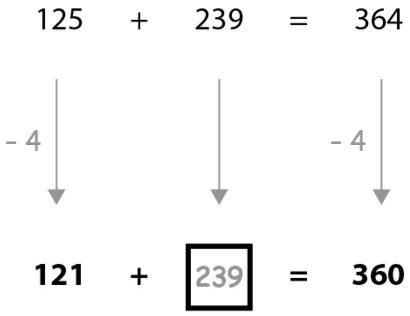
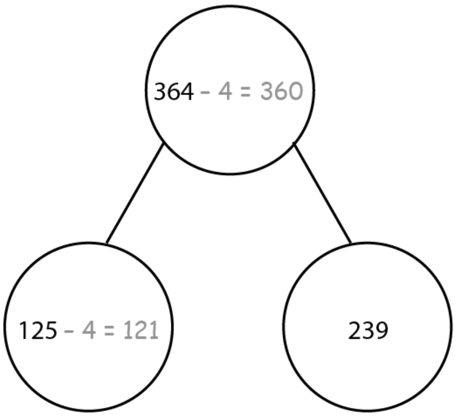
0.238 + 24.942	6712 + 2,764
529.25 + 0.75	9,270,000 + 85,000
3.555 + 4.445	6,400,000 + 600,000

Mental method	Written method

2:6	To develop children's fluency and understanding, present intelligent practice such as that shown opposite. Continue to ask children to explain the connection between the calculations using the stem sentences from steps *2:1* and *2:2*.	*'Complete the sentences.'* 19.18 + 0.82 > 19.18 + 0.81 because… 19.18 + 0.82 > 19.18 + 0.62 because… 14.18 + 0.82 < 19.18 + 0.82 because…

		• *'Fill in the missing numbers.'*
		19.18 + 0.82 = 20
		19.18 + 0.81 = ☐
		19.18 + 0.62 = ☐
		14.18 + 0.82 = ☐
		• *'Complete the sentences.'*
		  • 0.125 + 0.875 = 1
		  so
		  0.125 + 15.875 = 16
		  because…
		  • 40,000 + 60,000 = 100,000
		  so
		  40,000 + 3,060,000 = 3,100,000
		  because…
		• *'Fill in the missing numbers.'*
		39,000 + 34,000 = 73,000
		239,000 + 34,000 = ☐
		29,000 + 34,000 = ☐

2:7	Now shift the focus to problems with a missing addend, given a known change to the sum. First look at a problem in which the known addend remains the same (the unknown addend changes). Encourage children to reason about the value of the missing addend, using the following stem sentences: • ***'The sum has increased by ___; one addend has stayed the same, so the other addend must increase by ___.'***	Sum increases and known addend remains the same: 

	- ***'The sum has decreased by ___; one addend has stayed the same, so the other addend must decrease by ___.'*** Then look at a problem in which the known addend has changed (the unknown addend stays the same).	 Sum decreases and known addend decreases:  
2:8	Provide practice missing-addend problems where the change in the sum is known, extending to larger numbers and decimal fractions. Vary the position of the missing addend and whether the unknown addend has changed or not.	Missing-number problems: *'Fill in the missing numbers.'* 0.56 + 0.78 = 1.34 0.56 + ☐ = 2.35 25,560 + 23,400 = 48,960 ☐ + 23,500 = 49,060

2:9	To promote further depth of understanding present children with three-addend problems, including:
	• contexts where there is more than one way to find the 'new' sum, for example:
	  • *'At the beginning of the term, there are 192 pupils in Key Stage 2:*
	    Year 4: 68 children
	    Year 5: 65 children
	    Year 6: 59 children
	  During the term, one new child joins Year 5 and three children leave Year 4. How many pupils are there in Key Stage 2 at the end of the term? Explain how you worked it out. Do you think that was the most efficient way?'
	• true/false-style problems, such as the one shown opposite.
		'Which of these inequalities are correct and which are incorrect?'
		$20.35 + 20.45 + 20.55 > 20.35 + 20.35 + 20.55$
		$9{,}000{,}000 + 5{,}040{,}000 > 5{,}040{,}000 + 9{,}000{,}000$
		$19.278 + 64.258 < 64.258 + 19.277 + 0.001$
		'Can you explain why without calculating?'

Teaching point 3:

If the minuend and subtrahend are changed by the same amount, the difference stays the same. (same difference)

Steps in learning

Guidance

Representations

3:1

In segment *1.12 Subtraction as difference*, children were introduced to the difference structure of subtraction, including the idea of 'same difference' where certain pairs of numbers have the same difference. Children learnt that consecutive whole numbers have a difference of one and consecutive odd/consecutive even numbers have a difference of two. In this teaching point, understanding of same difference is developed and generalised, such that it can be used to support fluency and efficiency in mental calculation and to balance equations.

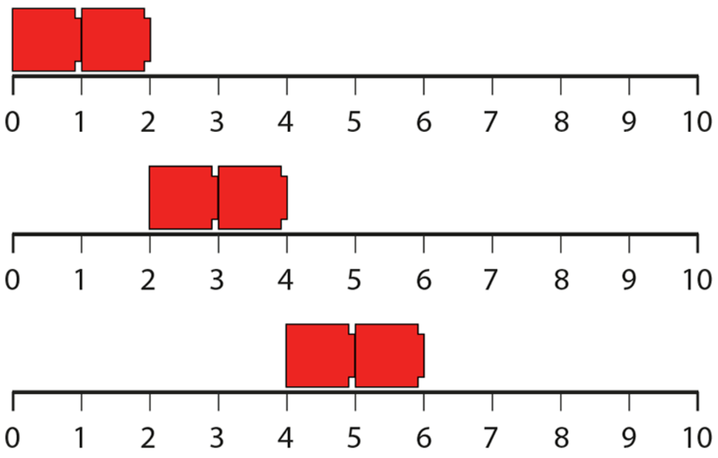
Begin this teaching point by introducing the idea of 'same difference' in a range of contexts such as:

- multi-link cubes on a number line (as in segment *1.12*)
- a line or Cuisenaire® rod relative to a ruler or number line.

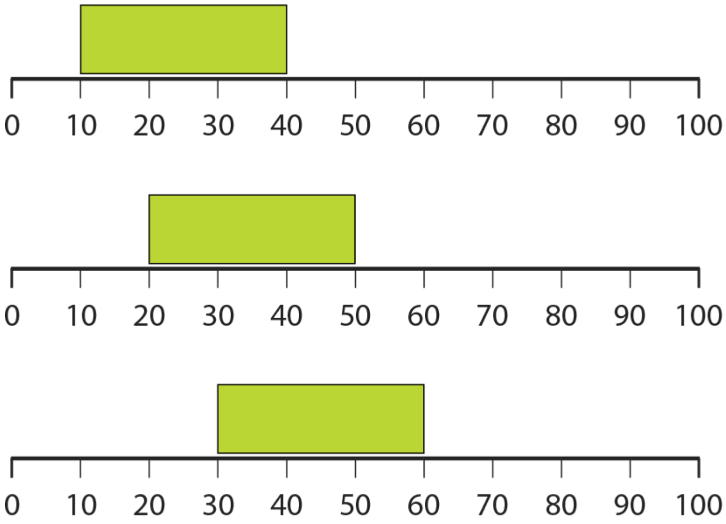
In each case, keep the difference the same value (length of multi-link cubes/line/Cuisenaire® rod) and increase/decrease the minuend and subtrahend by the same quantity by sliding the manipulatives/pictures relative to one another as shown opposite. Note that values are intentionally kept simple to draw attention to the 'same difference' structure.

Focus in on a drawn line relative to a ruler and ask children to begin to identify what is changing and what is

Multi-link cubes on a number line:

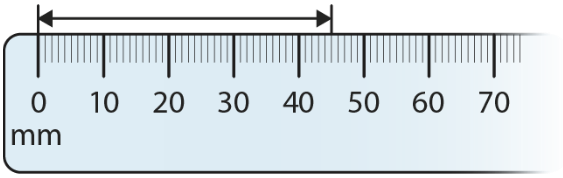


Cuisenaire® rod on a number line:

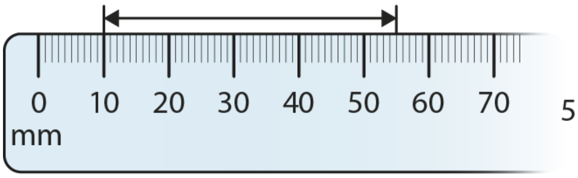


staying the same (the line stays the same length; the position of the ruler, and therefore the 'start' and 'end' values, changes). Then write equations to represent the different situations, as shown opposite, and again discuss similarities, differences and patterns.

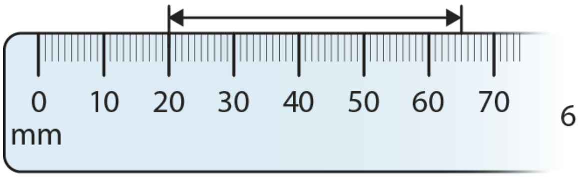
Fixed drawn line and ruler with equations:



45 – 0 = 45



55 – 10 = 45



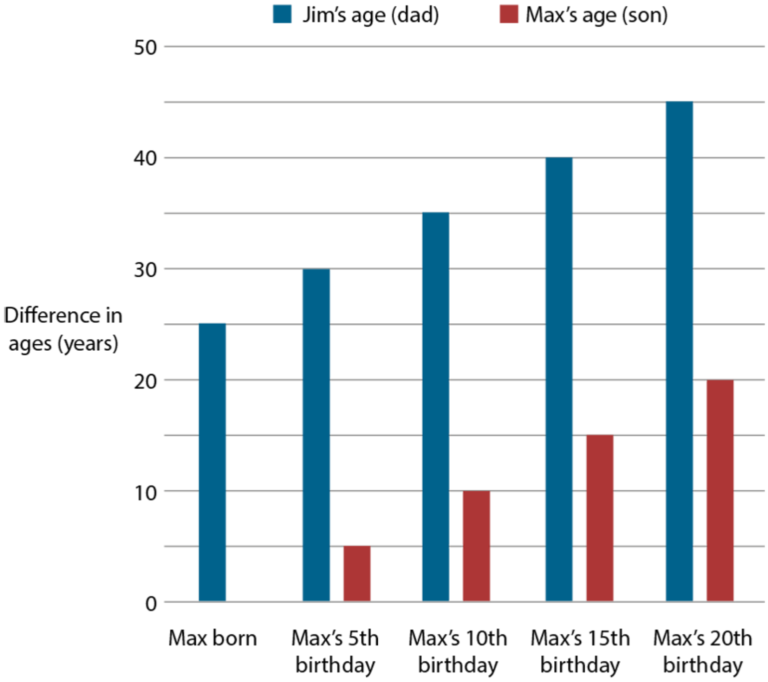
65 – 20 = 45

3:2 Work through some more examples, such as those shown below, illustrating same difference by recording equations. You can use bar charts to visually represent same difference.

Example 1:

- *'Jim was 25 years old when his son, Max, was born. What is the difference between their ages?'*
  (25 – 0 = 25)
- *'When Jim is 30, Max is 5. What is the difference between their ages?'*
  (30 – 5 = 25)
- *'When Jim is 35, Max is 10. What is the difference between their ages?'*
  (35 – 10 = 25)
- *'When Jim is 40, Max is 15. What is the difference between their ages?'*
  (40 – 15 = 25)
- *'Is the difference between their ages always going to remain the same?'*
  (Yes; difference = 25)
- *'How old will Max be when Jim is 75?'*

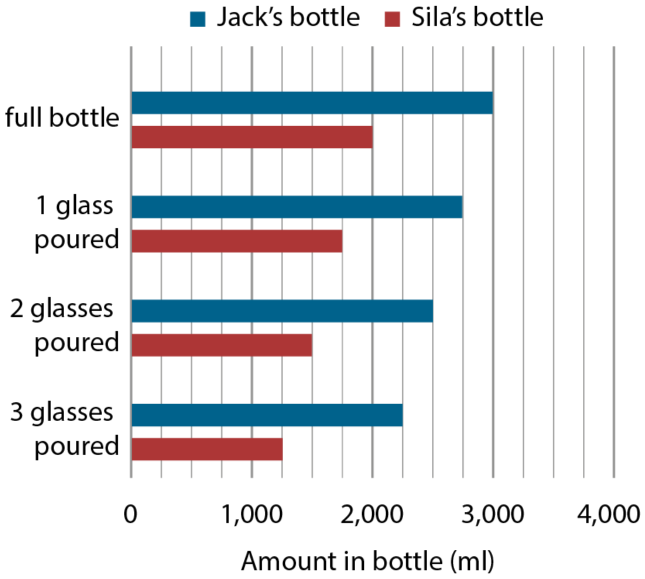
  $(75 - \boxed{50} = 25)$



Example 2:

'Jack has a three-litre bottle of drink and Sila has a two-litre bottle of drink. They both pour out three 250 ml glasses of drink. What is the difference between the amounts in their bottles after each glass is poured?'

Amount in Jack's bottle (ml)	**Amount in Sila's bottle (ml)**	**Difference between Jack's and Sila's bottles (ml)**
3,000	2,000	1,000
2,750	1,750	1,000
2,500	1,500	1,000
2,250	1,250	1,000



3,000 – 2,000 = 1,000

2,750 – 1,750 = 1,000

2,500 – 1,500 = 1,000

2,250 – 1,250 = 1,000

3:3

In order to move towards a generalisation, begin to focus on the language of minuend and subtrahend. Remind children that:

minuend – subtrahend = difference

Present a simple difference subtraction context such as:

- *'There are four apples and one orange. What is the difference between the number of apples and the number of oranges?'*

Ask children to write an equation and identify the minuend, subtrahend and difference. You could use multi-link cubes as shown on the next page, or number lines as shown in step *3:1*, to illustrate same difference.

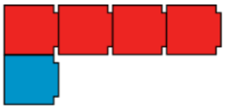
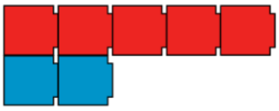
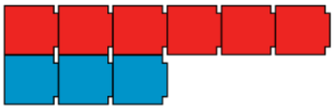
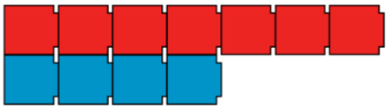
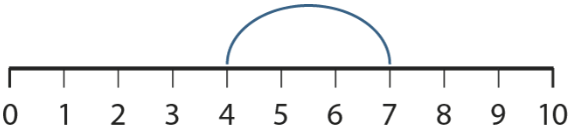
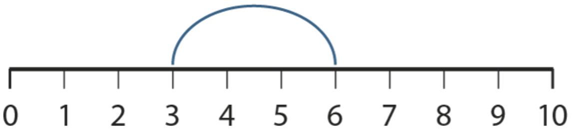
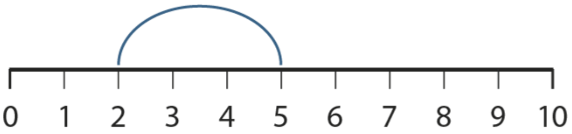
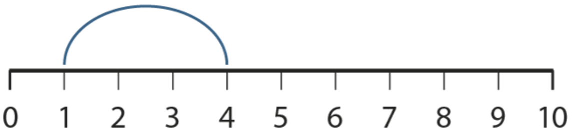
Then repeat, increasing the minuend and subtrahend by one in the story. Ask children to describe how the minuend and subtrahend have changed and

Identifying the minuend, subtrahend and difference:

'There are four apples and one orange.'

- *'How many more apples are there than oranges?'*
- *'How many fewer oranges are there than apples?'*

$$\underset{\substack{\uparrow\\ \text{minuend}}}{4} \quad - \quad \underset{\substack{\uparrow\\ \text{subtrahend}}}{1} \quad = \quad \underset{\substack{\uparrow\\ \text{difference}}}{3}$$

	what has happened to the difference. Continue working through, increasing the minuend and subtrahend by one each time in the story, using the following stem sentence to describe the change each time: ***‘I’ve added ___ to both the minuend and the subtrahend, so the difference stays the same.’***	Increasing the minuend and subtrahend by one:  4 – 1 = 3  5 – 2 = 3  6 – 3 = 3  7 – 4 = 3
3:4	Now start with seven apples and four oranges, write the equation and identify the minuend, subtrahend and difference. Then reduce the minuend and subtrahend by one, repeatedly, using the following stem sentence to describe what happens each time: ‘***I’ve subtracted ___ from both the minuend and the subtrahend, so the difference stays the same.’*** Look at the resulting set of equations and generalise: ***‘If the minuend and subtrahend are changed by the same amount, the difference stays the same.’***	Decreasing the minuend and subtrahend by one:  7 – 4 = 3  6 – 3 = 3  5 – 2 = 3  4 – 1 = 3

3:5

Before moving on to using same difference as a calculation strategy, provide children with some practice, including missing-number problems and contextual problems, for example:

- *'Sam and Ellie are playing a game. Sam had 83 points and Ellie had 94 points; that's 11 more than Sam. Sam and Ellie both scored another 27 points. How many more points does Ellie have compared to Sam now?'*

Use a dòng nǎo jīn question such as the following, which illustrates that we don't need to know the value of the minuend and subtrahend, just the *change* in their value, to calculate the change in difference:

- *'Felicity had 23 more red marbles than blue marbles. Then she gave away 12 red marbles and 12 blue marbles. How many more red marbles than blue marbles does she have now?'*

Missing-number problems:

'Fill in the missing numbers.'

10 – 7 = 3	100 – 94 = 6
12 – 9 = ☐	98 – 92 = ☐
14 – 11 = ☐	96 – 90 = ☐
16 – ☐ = 3	94 – ☐ = 6
18 – ☐ = 3	92 – ☐ = 6
☐ – 17 = 3	☐ – 84 = 6
☐ – 19 = 3	☐ – 82 = 6

3:6

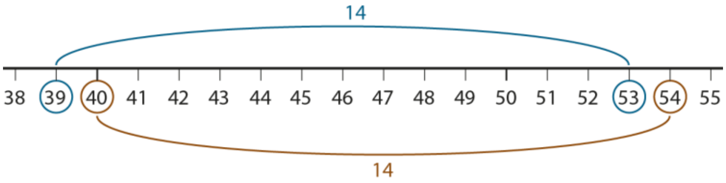
Once children have secure understanding of the generalisation in step *3:4,* begin to explore how same difference (sometimes referred to as 'constant difference') can be used to transform calculations to make them easier to solve.

Look at a sequence of calculations, such as:

50 – 25
49 – 24
48 – 23
47 – 22

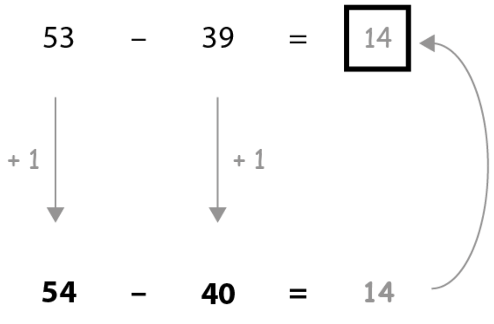
Use a number line to confirm that they all have the same difference, then ask children which calculation in the set they think is the easiest to solve and why (*50 – 25* should be identified). Look for explanations that involve known facts (for example, half of 50 is 25) or

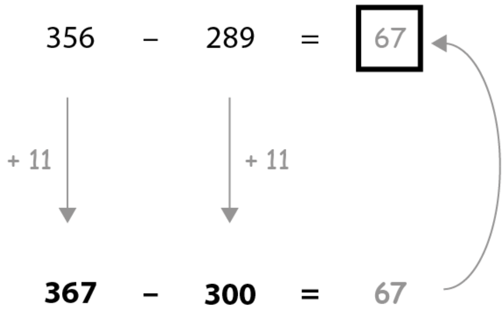
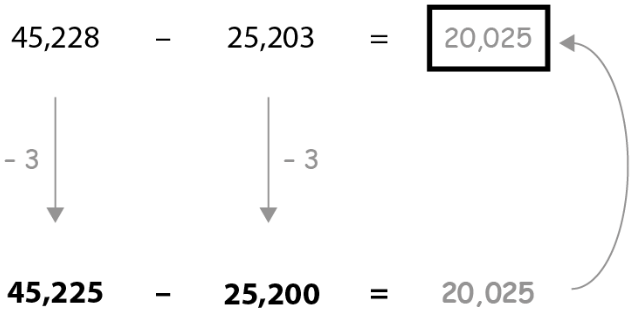
Transforming a calculation using same difference – number line:



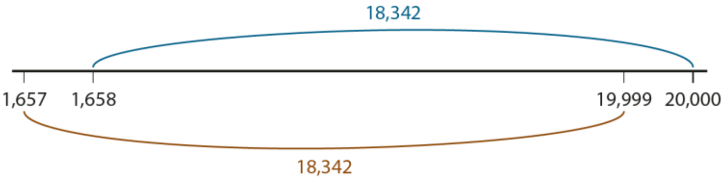
53 – 39 = 54 – 40 = 14

Transforming a calculation using same difference – jotting:

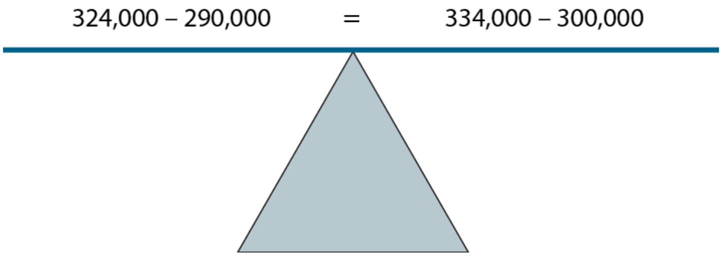
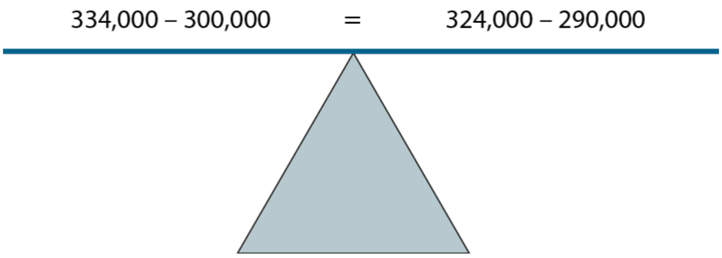
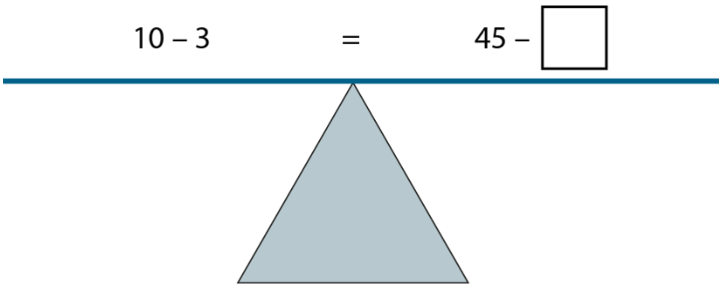
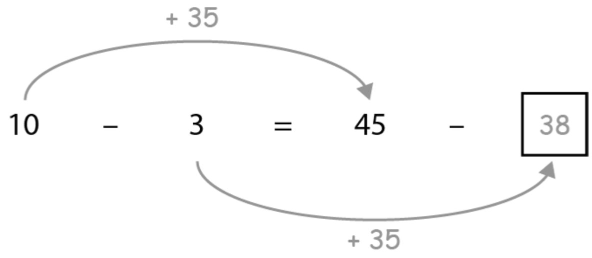


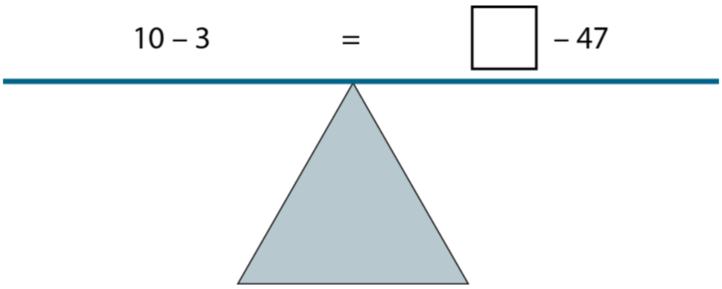
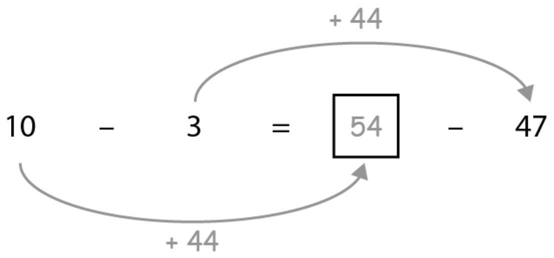
	that refer to the minuend (in this case) being a multiple of ten. Then present a calculation where the subtrahend is close to a multiple of ten (e.g. *53 – 39*). Refer to the 'same difference' generalisation and ask children how they could transform this calculation, keeping the difference the same, but making it easier to solve mentally. Use a number line and jottings to draw attention to how the calculation has been transformed, keeping the sum the same, and discuss why the transformed calculation is easier to solve (subtracting a multiple of ten).	
3:7	After working through several two-digit calculations where the calculation can be transformed such that either the subtrahend or minuend is a multiple of ten, extend to larger numbers and decimal fractions. Encourage children to identify whether the minuend or subtrahend can be transformed into a power of ten (one, ten, hundred etc.).	Transforming calculations using same difference – three-digit numbers:  Transforming calculations using same difference – five-digit numbers: 

		Transforming calculations using same difference – decimal fractions: $$\begin{tikzcd} 19.72 \arrow[d, "+0.06"'] & - & 4.94 \arrow[d, "+0.06"] & = & \boxed{14.78} \\ \mathbf{19.78} & - & \mathbf{5} & = & 14.78 \arrow[u, bend right] \end{tikzcd}$$
3:8	Provide children with practice, including sequences of related calculations and isolated calculations.	Missing-number problems: *'Fill in the missing numbers.'* 121,372 – 10,998 = $\boxed{}$ 121,373 – 10,999 = $\boxed{}$ 121,374 – 11,000 = $\boxed{}$ 121,375 – 11,001 = $\boxed{}$ 48.7 – 6.9 = $\boxed{}$ 986 – 261 = $\boxed{}$
3:9	Next, introduce the idea that calculations that cannot be easily solved mentally and require a written method can also be transformed to make them easier to solve. This is particularly the case when the minuend is a power of ten; in these cases, the calculation can be transformed to avoid exchange through zeros. Present a calculation such as *20,000 – 1,658*. Write it out as a column calculation (shown opposite) and work through it as a class, exchanging through the zeros, thus demonstrating the inefficiency. Then look at the problem again and ask children to suggest how the calculation	Transforming a column subtraction calculation using same difference: - Original calculation – exchange required $$\begin{array}{rccccc} & \overset{1}{\cancel{2}} & {}^{1}\overset{9}{\cancel{0}}, & {}^{1}\overset{9}{\cancel{0}} & {}^{1}\overset{9}{\cancel{0}} & {}^{1}0 \\ - & & 1, & 6 & 5 & 8 \\ \hline & 1 & 8, & 3 & 4 & 2 \\ \hline \end{array}$$ - Transformed calculation – no exchange required $$\begin{array}{rccccc} & 1 & 9, & 9 & 9 & 9 \\ - & & 1, & 6 & 5 & 7 \\ \hline & 1 & 8, & 3 & 4 & 2 \\ \hline \end{array}$$

	could be transformed so that there are nine ones in the ones column. Transform the calculation by subtracting one from both the minuend and subtrahend, reminding children of the same difference generalisation: (20,000 **– 1**) – (1,658 **– 1**) = 19,999 – 1,657 Work through the transformed calculation, demonstrating that it does not require any exchange. Then discuss how the same-difference transformation made the calculation easier. Representing the transformation on a number line can help to reinforce that both pairs of numbers have the same difference. Work through more calculations as a class, including larger numbers and decimal fractions, before providing children with independent practice.	- Number line  Independent practice: - *'Which equivalent calculation is easier to solve using a written method?'* **400,003 – 24,438** ↙ ↘ 400,000 – 24,435 399,999 – 24,434 - *'Transform the following calculations to make them easier to solve using a written method.'* 8,000 – 4,387 42.000 – 34.832 60,100 – 2,758 176 – 22.318 860,005 – 98,297 12.002 – 6.935
3:10	Children might think that this strategy can only be used in subtraction contexts with the difference structure because of the language used (same *difference*). Draw attention to the fact that 'same difference' as a calculation *strategy* can be applied to all subtraction *structures* (partitioning, reduction and difference): - *'A football stadium has a capacity of 90,000. During a match, when it is full, there are 67, 899 home fans. How many away supporters are there?'* (partitioning) - *'Hari had saved £400. Then he bought a bike for £369.99. How much does he have left?'* (reduction)	

	- *'A female African elephant has a mass of 5,050 kg. The mass of her calf is 298 kg. How much less is the mass of the calf than the mass of its mother?'* (difference)	
3:11	To assess and promote depth of understanding, present questions such as those shown opposite.	*'For which of these calculations would "same difference" be a useful strategy? Explain your answers.'* 83 – 49 16.89 – 8.29 53,604 – 40,000 70.003 – 38.391 Dòng nǎo jīn: - *'Which equivalent calculations could you use to simplify these?'* 2.01 – 1.98 3,487 – 1,598 10 – 5.2 *'How do you know that the calculations are equivalent? Is there more than one choice?'* - *'All these calculations are equivalent; true or false?'* 2,001 – 1,998 2,000 – 1,999 2,000 – 1,997 2,003 – 2,000 *'Explain your answer.'* - *'Dorota decides to use a mental strategy to calculate 342 – 96. She starts from 342. Which of these methods could she use? Tick those that are correct.'* add 4 then subtract 90 ☐ subtract 100 then add 4 ☐ subtract 6 then subtract 90 ☐ subtract 4 then subtract 100 ☐

3:12	Now shift the focus to how an understanding of same difference can be used to balance equations. Begin by reviewing some of the calculations previously explored and draw attention to the fact that two expressions that have the same difference are equivalent. As in *Teaching point 1*, you can use the image of scales to emphasise the equivalence and the meaning of the '=' symbol. Encourage children to use the stem sentences from step *3:4* to describe the relationship between the terms in the two expressions, for example: *'If I add 10,000 to the minuend and add 10,000 to the subtrahend, the difference stays the same.'* Demonstrate that it doesn't matter which expression comes first (the calculation that is 'easier' to solve can be on the left as well as the right of the equals symbol).	Balanced equations:  
3:13	Then progress to balancing equations where there is an unknown, encouraging children to use the following stem sentences: - ***'I've added ___ to the minuend (subtrahend), so I need to add ___ to the subtrahend (minuend) to keep the difference the same.'*** - ***'I've subtracted ___ from the minuend (subtrahend), so I need to subtract ___ from the subtrahend (minuend) to keep the difference the same.'*** Note that in the example opposite the numbers are fairly simple to draw attention to the structure; since the difference is a known fact (*10 – 3 = 7*), it is likely just as efficient to calculate the unknown using inverse operations (*45 – 7 = 38*). However, the aim here is to use the same difference structure, which can then be applied when the	Example 1:  

	relationship between the minuends or subtrahends can be seen easily while the difference is more difficult to calculate, for example: 642 – 387 = 630 – □ Work through a range of examples as a class, including larger numbers and decimal fractions. Vary the position of the unknown, and whether the unknown minuend or subtrahend has been increased or decreased. You could model on a number line how the difference is the same, but the number line should not be relied upon as the method to solve these types of questions. Children should be using 'same difference' and looking at the relationships between the numbers.	Example 2:  
3:14	Provide children with independent practice balancing equations using same difference, including contextual problems, for example: - *'Sara has saved £16.90 and Eli has saved £8.35. Then Sara and Eli are both given the same amount of money to add to their savings. Sara now has £18; how much does Eli have?'* To assess and promote depth of understanding present a dòng nǎo jīn problem such as: - *'Jon was born in 2007 and his dad was born in 1982. Jon says that there will always be an odd difference between their ages. Explain whether he is right or not.'*	Missing-number problems: *'Fill in the missing numbers.'* 720,201 – 390,199 = □ – 390,200 □ – 0.57 = 0.87 – 0.61

Teaching point 4:

If the minuend is increased (or decreased) and the subtrahend is kept the same, the difference increases (or decreases) by the same amount.

Steps in learning

Guidance

4:1

In segment *1.13 Addition and subtraction: two-digit and single-digit numbers*, children learnt to apply known subtraction facts within ten to subtraction of a single-digit number from a two-digit number, e.g.:

10 – 3 = 7

so

30 – 3 = 27

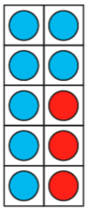
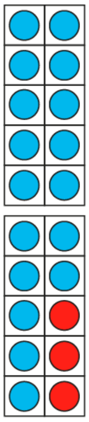
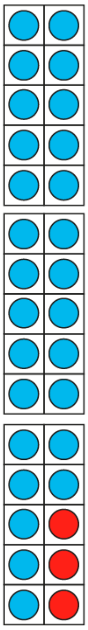
This teaching point uses and builds on this, drawing attention to the structure of the calculations to generalise how the difference changes when only the minuend (and not the subtrahend) is changed. The calculations are intentionally kept simple so that attention can be drawn to the concept being introduced.

Use the number facts and representations from segment *1.13*, along with a context (as shown opposite), to draw attention to the structure of the calculations and how the minuend and difference change by the same amount. Compare the calculations and ask children to spot the pattern, encouraging them to notice that the minuend and difference change by the same amount. You can use a bar chart (as shown on the next page, which extends the given example to the case where Jon has 40 p) or number line (as in step *4:2*) to draw attention to the pattern.

Encourage children to use the following stem sentence to describe the relationship between the

Representations

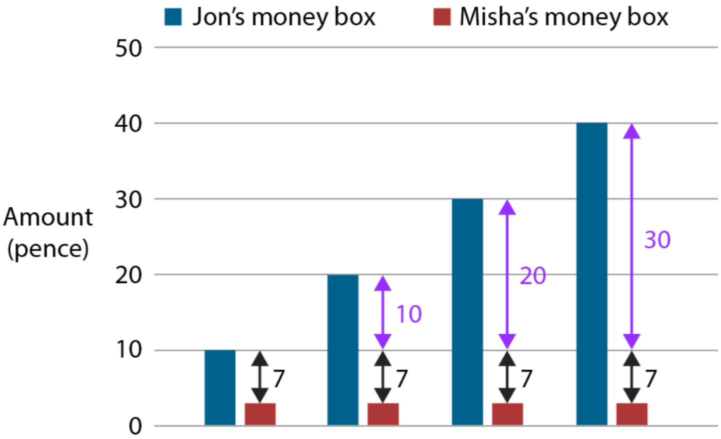
Increasing the minuend – tens frames and counters:

'Jon has 10 p in his money box. Misha has 3 p in hers. Jon has 7 p more than Misha.'	*'Jon adds 10 p to his money box. He now has 17 p more than Misha.'*	*'Jon adds another 10 p to his money box. He now has 27 p more than Misha.'*
 10 – 3 = 7	 **2**0 – 3 = **1**7	 **3**0 – 3 = **2**7

<table>
<tr>
<td></td>
<td>calculations: <i>‘I’ve added ___ to the minuend and kept the subtrahend the same, so I must add ___ to the difference.’</i></td>
<td>Increasing the minuend – jottings:

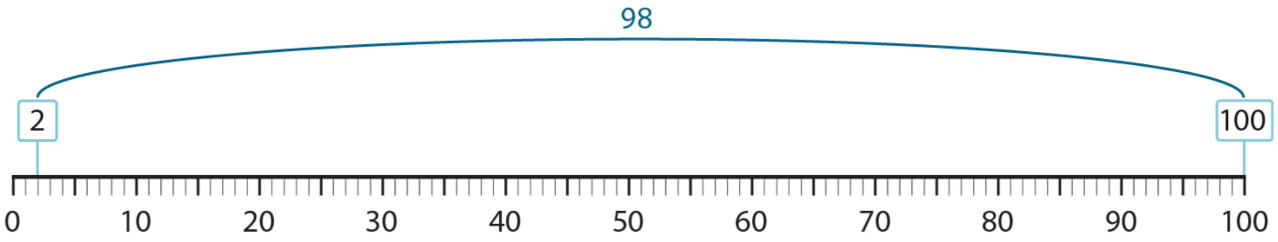
$$\begin{tikzcd} 10 \arrow[d, "+ 10"'] & - & 3 & = & 7 \arrow[d, "+ 10"] \\ 20 \arrow[d, "+ 10"'] & - & 3 & = & 17 \arrow[d, "+ 10"] \\ 30 & - & 3 & = & 27 \end{tikzcd}$$

Increasing the minuend – bar chart:

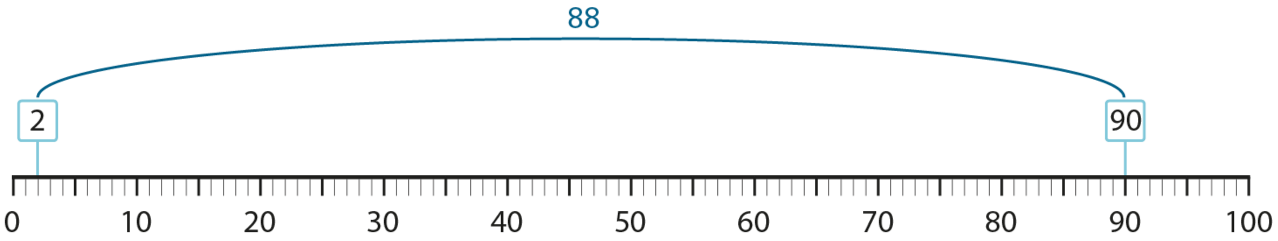

</td>
</tr>
<tr>
<td>4:2</td>
<td colspan="2">Next, explore a context where the minuend decreases, the subtrahend stays the same and the difference decreases. As before, use a representation to draw attention to the structure, compare the calculations and ask children to spot the pattern. Use the following stem sentence to describe the relationship between the calculations: <i>‘I’ve subtracted ___ from the minuend and kept the subtrahend the same, so I must subtract ___ from the difference.’</i>
Draw together the learning from this step and step <i>4:1</i> to generalise: <i>‘If the minuend is changed by an amount and the subtrahend is kept the same, the difference changes by the same amount.’</i></td>
</tr>
</table>

Decreasing the minuend – number lines:

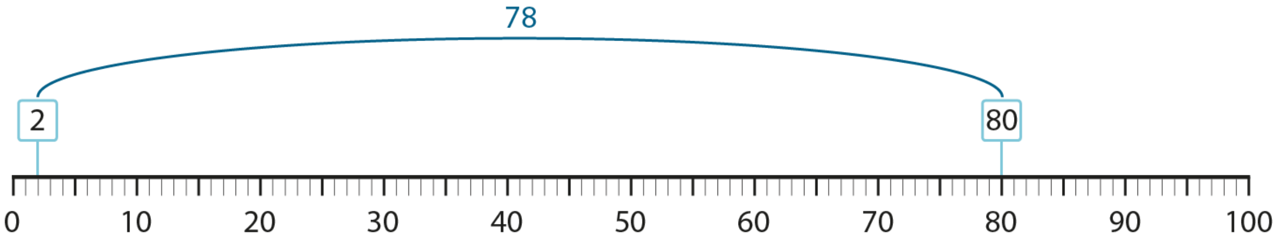
- *'Evie has 100 p in her money box and Hari has 2 p, so Evie has 98 p more than Hari.'*



- *'On Monday, Evie spends 10 p. She now has 88 p more than Hari.'*



- *'On Tuesday, Evie spends another 10 p. She now has 78 p more than Hari.'*



Decreasing the minuend – jottings:

$$\begin{tikzcd}
100 \arrow[d, "-10"'] & - & 2 & = & 98 \arrow[d, "-10"] \\
90 \arrow[d, "-10"'] & - & 2 & = & 88 \arrow[d, "-10"] \\
80 & - & 2 & = & 78
\end{tikzcd}$$

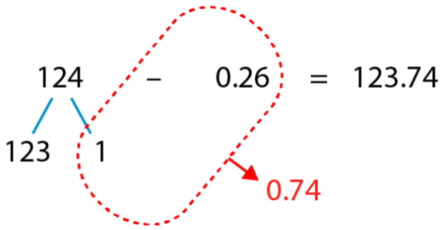
4:3	Now apply the generalisation and stem sentences to solve problems such as the examples opposite. Note that the structure of the problems explicitly links to the generalisation: the minuend is increased or decreased and the difference must be found. Encourage children to identify how the minuend and difference change, rather than recalculating.	Increasing the minuend: *'There were 16 boys and 14 girls in a class. The difference between the number of boys and girls was 2. Now there are 18 boys in the class, but the number of girls is the same. How many more boys than girls are there now?'* $$\begin{tikzcd} 16 \arrow[d, "+ 2"'] & - & 14 & = & 2 \arrow[d, "+ 2"] \\ 18 & - & 14 & = & 4 \end{tikzcd}$$ *'There are now 4 more boys than girls in the class.'* Decreasing the minuend: *'A blue ribbon was 1 m 72 cm long and a red ribbon was 1 m 38 cm long; so the blue ribbon was 34 cm longer than the red ribbon. Then the blue ribbon was cut down to 1 m 52 cm. How much longer than the red ribbon is the blue ribbon now?'* $$\begin{tikzcd} 172 \arrow[d, "- 20"'] & - & 138 & = & 34 \arrow[d, "- 20"] \\ 152 & - & 138 & = & 14 \end{tikzcd}$$ *'The blue ribbon is now 14 cm longer than the red ribbon.'*

4:4	Extend to larger numbers and decimal fractions, varying the position of the missing number, while keeping the subtrahend the same in all the linked expressions. Continue to encourage children to identify how the minuend and difference change, rather than recalculating.	Missing-number problems using given facts: *'Fill in the missing numbers.'* 62,865 – 41,294 = 21,571 63,865 – 41,294 = ☐ 60,865 – 41,294 = ☐ ☐ – 41,294 = 31,571 ☐ – 41,294 = 21,570 23.18 – 0.82 = 22.36 46.18 – 0.82 = ☐ 23.28 – 0.82 = ☐ 123.58 – 0.82 = ☐ 3.18 – 0.82 = ☐ 52.13 – 3.76 = 48.37 ☐ – 3.76 = 18.37 ☐ – 3.76 = 8.37 ☐ – 3.76 = 88.37 ☐ – 3.76 = 88.57

4:5

Now explore how the generalisation can be used as a mental calculation strategy by relating a problem to known facts, rather than linking to a given fact, as in the examples opposite.

Note that this strategy is essentially the same as partitioning the minuend, for example:



Example 1 – step-by-step breakdown:

- Step 1 – identify a related known fact

$$124 - 0.26 = \boxed{}$$

$$1 - 0.26 = 0.74$$

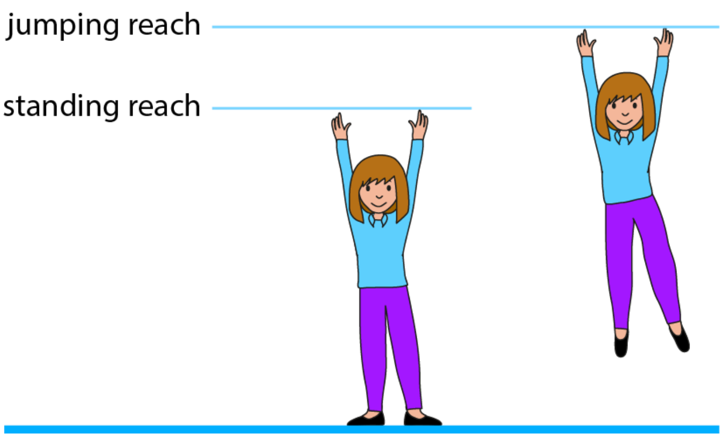
- Step 2 – identify the relationship between the known fact and the problem

$$\begin{tikzcd} 124 \arrow[d, "-123"'] & - & 0.26 & = & \boxed{} \arrow[d, "-123"'] \\ 1 & - & 0.26 & = & 0.74 \end{tikzcd}$$

- Step 3 – use knowledge of inverse operations to calculate the final answer:

$$\begin{tikzcd} 124 \arrow[d, "-123"'] & - & 0.26 & = & \boxed{123.74} \arrow[d, "-123"'] \\ 1 & - & 0.26 & = & 0.74 \arrow[u, bend right=60, "+123"'] \end{tikzcd}$$

		Example 2: $$\begin{tikzcd} 374{,}000 \arrow[d, "-300{,}000"'] & - & 24{,}000 & = & \boxed{350{,}000} \arrow[d, "-300{,}000"'] \\ 74{,}000 & - & 24{,}000 & = & 50{,}000 \arrow[u, bend right, "+300{,}000"'] \end{tikzcd}$$
4:6	Then explore how the generalisation can be used to simplify calculations, not necessarily by relating them to known facts, as in the examples opposite. Draw attention to how the minuend has been changed such that the difference is a multiple of ten and explore how the difference must be adjusted in order to reach the final answer. Note that, for any given calculation, there may not be one 'best' strategy. In the first example opposite, children might prefer to use 'same difference' since the subtrahend is close to a multiple of ten (*183 – 49 = 184 – 50*), while in the second example children might prefer to adjust the minuend so that the difference becomes a multiple of ten, as shown. The aim is for children to develop into confident mathematicians who can think flexibly, rather than remembering a 'fixed' approach for each 'type' of calculation.	Using the compensation property to calculate mentally – example 1: $$\begin{tikzcd} 183 \arrow[d, "+6"'] & - & 49 & = & \boxed{134} \arrow[d, "+6"'] \\ 189 & - & 49 & = & 140 \arrow[u, bend right, "-6"'] \end{tikzcd}$$ Using the compensation property to calculate mentally – example 2: $$\begin{tikzcd} 185 \arrow[d, "+1"'] & - & 46 & = & \boxed{139} \arrow[d, "+1"'] \\ 186 & - & 46 & = & 140 \arrow[u, bend right, "-1"'] \end{tikzcd}$$

4:7	Provide children with practice, gradually decreasing the scaffolding.	Missing-number problems using known facts – scaffolded: *'Fill in the missing numbers.'* $\begin{tikzcd} \mathbf{18} & - & \mathbf{0.75} & = & \boxed{} \\ 1 \arrow[u, "+17"] & - & 0.75 & = & \boxed{} \arrow[u, "+\boxed{}"'] \end{tikzcd}$ 100 – 3 = 97 500 – 3 = ☐ 13 – 9 = 4 2,013 – 9 = ☐ Missing-number problems using known facts – unscaffolded: *'For each calculation, use a known subtraction fact to calculate the difference.'* 13 – 0.23 = ☐ 540,000 – 17,000 = ☐ 523 – 9 = ☐ 600 – 92 = ☐
4:8	To complete this teaching point, you could present the children with an unfamiliar context, such as that shown opposite, and encourage them to use the generalisation they have learnt to work out the missing information.	Dòng nǎo jīn: *'Stephanie wants to learn to jump higher. She records her standing reach height, then the height she reaches when she jumps, to calculate how high she can jump.'* 

'Stephanie tests herself each week for five weeks. Stephanie doesn't get any taller during the five weeks, so her standing reach doesn't change. Her results are shown in the table, but some of the values are missing. Fill in the missing information.'

Week	Standing reach (cm)	Jumping reach (cm)	Score (difference in cm)
1			9
2			8
3			8
4			10
5	143	154	

Teaching point 5:

If the minuend is kept the same and the subtrahend is increased (or decreased), the difference decreases (or increases) by the same amount.

Steps in learning

	Guidance	Representations

5:1

Guidance

Although this teaching point builds on all the others covered so far in this segment, this compensation property is probably the one that children will find most challenging. Children often make errors, assuming that if the subtrahend increases, the difference will increase, and if the subtrahend decreases, the difference will decrease, for example:

85 – 47 = 38

so

85 – 48 = 39 ✗

To avoid such misconceptions, it is important to use clear pictorial representations so that children learn to visualise the relationships.

Begin by using a cardinal representation of a very simple situation, such as that shown opposite, to illustrate that the more items we 'take away' the fewer we are left with (and vice-versa). Ask the children to write inequalities to compare the different situations, working towards the following generalisations:

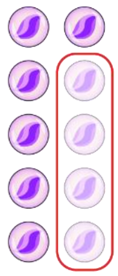
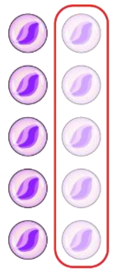
- ***'The more we subtract, the less we are left with.'***
- ***'The less we subtract, the more we are left with.'***

Give the children some practice completing inequalities before moving on. Encourage children to compare the sizes of the subtrahends, asking themselves *'Am I subtracting more or less? So will I be left with more or less?'* rather than calculating the value of the

Representations

Increasing the subtrahend:

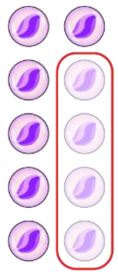
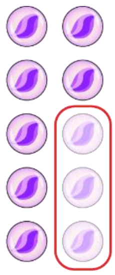
'Sebastian has 10 marbles. He gives away 4 of them. Megan also has 10 marbles. If she gives away 5 marbles, will she be left with more or fewer marbles than Sebastian?'

Sebastian	Megan
	
10 – 4 > 10 – 5	

'Megan gives away more marbles, so she is left with fewer marbles.'

Decreasing the subtrahend:

'Peter also has 10 marbles. If he gives away 3 marbles, will he be left with more or fewer marbles than Sebastian?'

Sebastian	Peter
	
10 – 4 < 10 – 3	

'Peter gives away fewer marbles, so he is left with more marbles.'

expressions on each side of the inequality symbol.

Completing inequalities:

'Fill in the missing symbols.' (>, = or <)

10 – 1 ○ 10 – 2 87 – 33 ○ 87 – 23

10 – 2 ○ 10 – 2 87 – 33 ○ 87 – 33

10 – 3 ○ 10 – 2 87 – 33 ○ 87 – 43

5:2

At this stage, introduce a context in which the minuend is kept the same, but the subtrahend is increased, such as the example shown below. Represent the two calculations on a number line and with bars. Ask children:

- *'What's the same?'*
- *'What's different?'*

Then draw attention to the relationship by asking:

- *'How has the subtrahend changed?'*
- *'How has the difference changed?'*

Use the pictorial representations to draw attention to the fact that the difference decreases because the numbers (the minuend and subtrahend) are closer together; in the example given, the shopping bill is the same, but Harvey has a big discount.

Use the following stem sentence to emphasise the inverse relationship between the change in the subtrahend and the change in the difference: ***'I've kept the minuend the same and added ___ to the subtrahend, so I must subtract ___ from the difference.'***

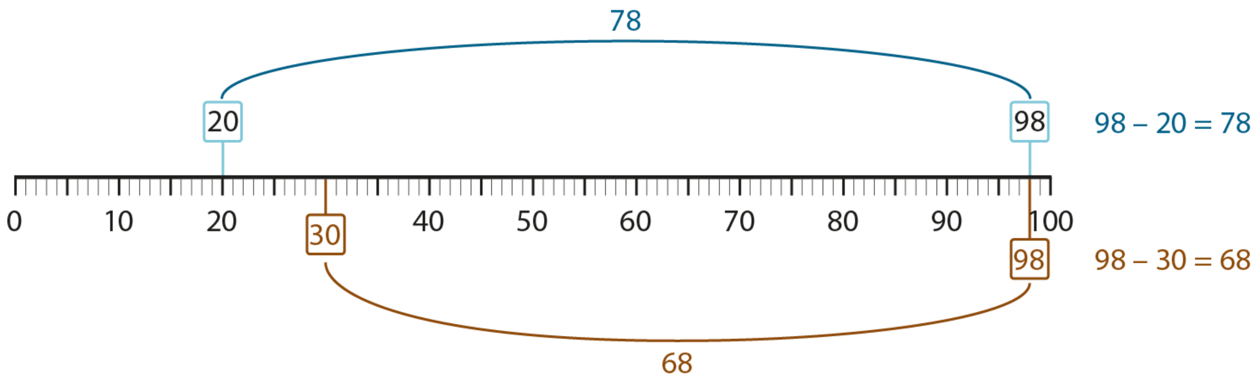
Work through several examples as a class, using contexts with different subtraction structures (partitioning, reduction and difference).

'Harvey is buying groceries. The bill is £98, but he thinks he has a voucher for £20 off so will only have to pay £78.'

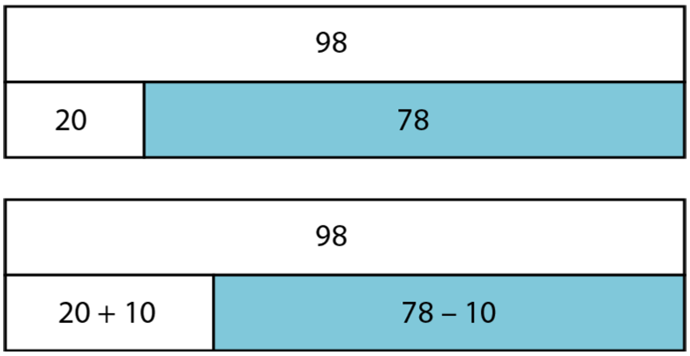
£98 – £20 = £78

'Harvey's voucher is actually worth £30. How much will he have to pay for his groceries?'

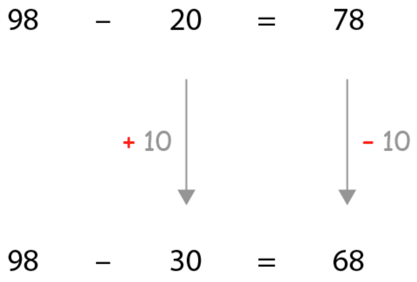
Increasing the subtrahend – number line:



Increasing the subtrahend – bar models:



Increasing the subtrahend – jottings:



5:3 Now examine a context in which the minuend is kept the same, but the subtrahend is *decreased*. Use the same representations as in step *5:2*, and again ask children to describe the similarities and differences between the calculations. Use the stem sentence: ***'I've kept the minuend the same and* subtracted ___ *from the subtrahend, so I must* add ___ *to the difference.'***

Use the pictorial representations to draw attention to the fact that the difference increases because the numbers (the minuend and subtrahend) are further apart. In the example given, the price of the game console is the same, but Raf now has less money than before.

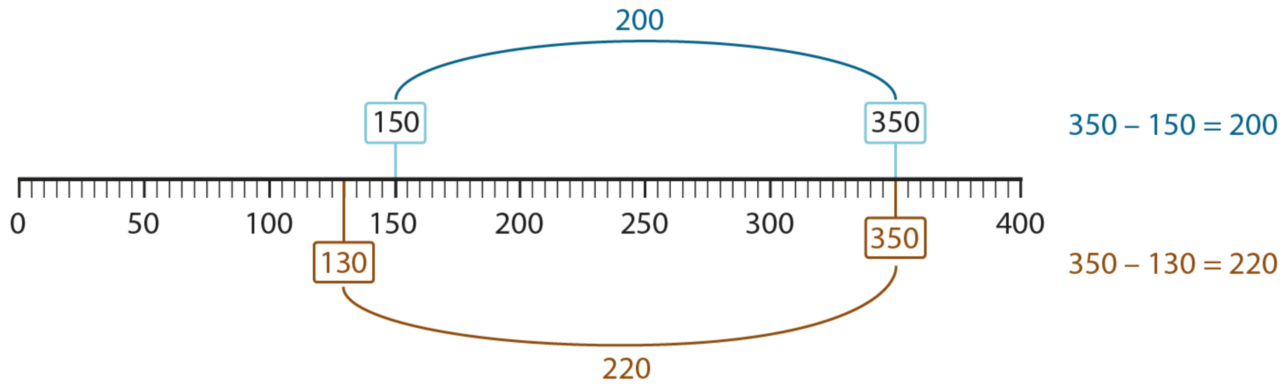
Again, work through several examples as a class, using contexts with different subtraction structures.

'Raf has saved £150. He wants to buy a game console that costs £350. How much more does he need to save?'

£350 – £150 = £200

'Raf goes out with his friends and spends £20 of his savings. He now has £130. How much does he now need to save before he can buy the game console?'

Decreasing the subtrahend – number line:



Decreasing the subtrahend – bar models:

Decreasing the subtrahend – jottings:

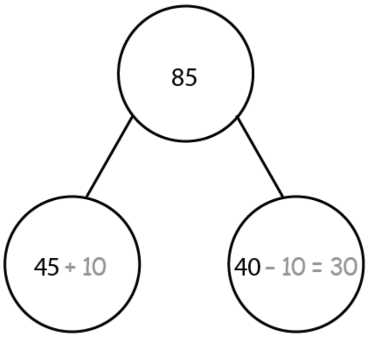
$$\begin{tikzcd}350 & - & 150 \arrow[d, "-20"'] & = & 200 \arrow[d, "+20"] \\ 350 & - & 130 & = & 220\end{tikzcd}$$

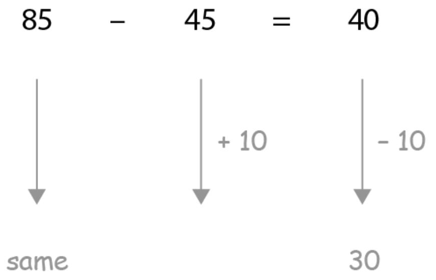
5:4

Now, for both contexts, with an increase in the subtrahend and a decrease in the subtrahend, work through some examples in which only the *change* in the subtrahend is given (and not the new subtrahend). Encourage children to use the change in the subtrahend to work out the change in the difference, rather than calculating the new subtrahend and then the new difference.

Increasing the subtrahend – new subtrahend not given:

'Ciara has 85 marbles; 45 of them are red and the rest are blue, so she has 40 blue marbles. Arjun also has 85 marbles; some of them are red and some of them are blue. He has ten more red marbles than Ciara. How many blue marbles does Arjun have?'
(partitioning structure)

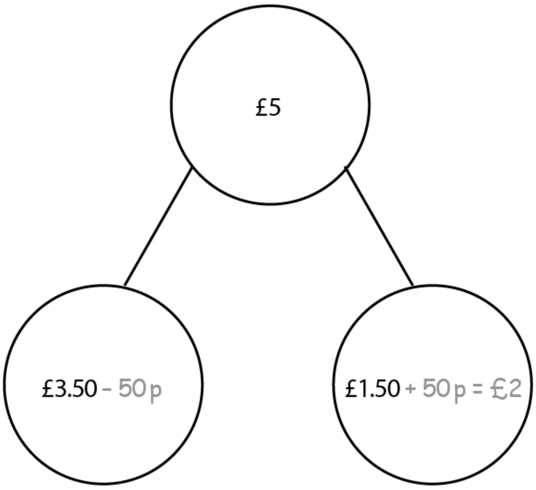


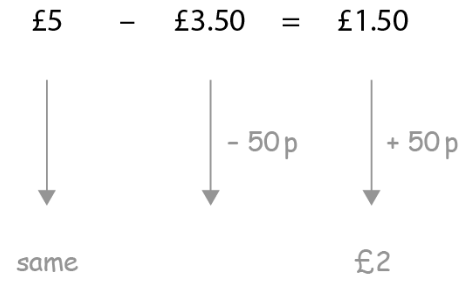
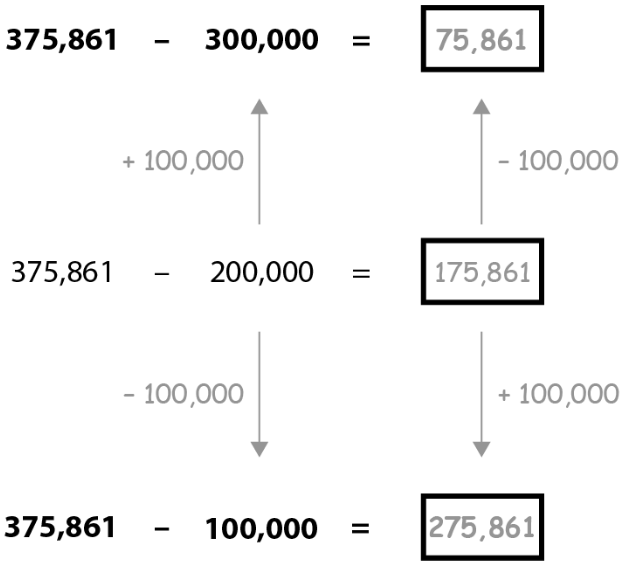


'Arjun has 30 blue marbles.'

Decreasing the subtrahend – new subtrahend not given:

'Leah gets £5 to spend on lunch each day. On Monday she spends £3.50, so she is left with £1.50 to put in her savings. On Tuesday Leah spends 50 p less than she did on Monday. How much does she have left on Tuesday to put in her savings?'
(reduction structure)



		 £5 – £3.50 = £1.50 *'Leah has £2 to put in her savings on Tuesday.'*
5:5	To embed the learning from steps 5:1-5:4, examine sequences of expressions, for example: 156 – 79 = 77 498 – 56 = 442 156 – 80 = 76 498 – 55 = 443 156 – 81 = 75 498 – 54 = 444 156 – 82 = 74 498 – 53 = 445 Ask children to describe the patterns and relationships between the numbers: - *'What stays the same?'* - *'What changes?'* - *'Can you describe the changes?'* Use the stem sentences from steps *5:2* and *5:3*.	
5:6	Now extend to larger numbers and decimal fractions. Use the relationship between the subtrahend and difference to deepen children's understanding of place value by subtracting multiples of 0.001, 0.01, 0.1, 10, 100, 1,000 etc. As well as exploring problems in which the new difference must be found, include examples where the new difference is given, but the minuend or subtrahend is unknown. Working as a class, continue to encourage children to identify what stays the same and what changes, and use the stem sentences to describe the relationships. Emphasise the use of a	Increasing/decreasing the subtrahend – six-digit numbers:  **375,861 – 300,000 =** 75,861 375,861 – 200,000 = 175,861 **375,861 – 100,000 =** 275,861

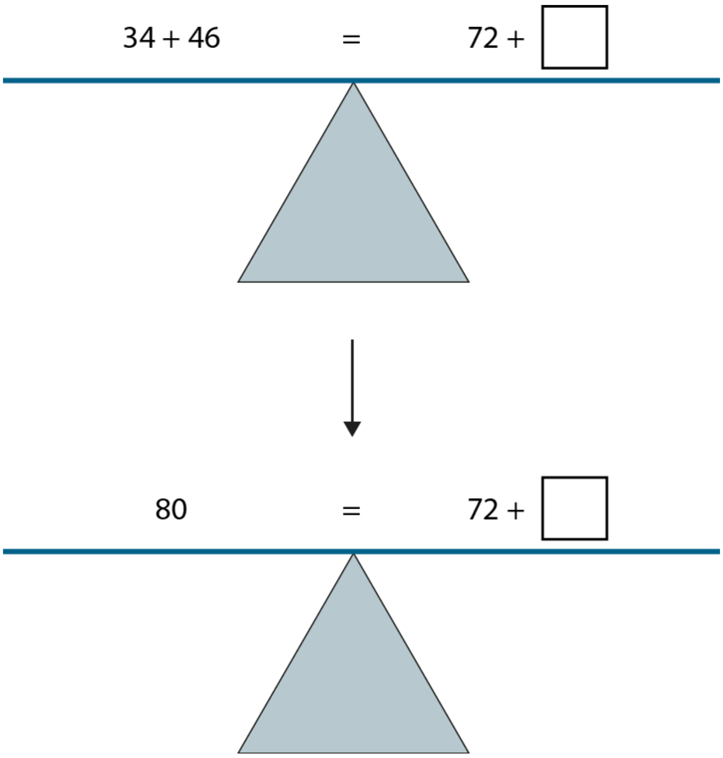
	known/given fact to solve related calculations. You can begin with a more scaffolded approach, before working with just the equations, as shown opposite.	Increasing/decreasing the subtrahend – decimal fractions: 23.964 – 0.9 = $\boxed{23.064}$ 23.964 – 0.8 = 23.164 23.964 – 0.7 = $\boxed{23.264}$ 4,975 – $\boxed{60}$ = 4,915 4,975 – 70 = 4,905 $\boxed{4,975}$ – 80 = 4,895
5:7	Provide varied practice including missing-number problems (varying the position of the missing number).	Missing number problems: *'Fill in the missing numbers.'* 125 – 36 = 89 125 – $\boxed{}$ = 90 2,568 – 367 = 2,201 2,568 – $\boxed{}$ = 2,200 534,974 – 463, 519 = 71,455 $\boxed{}$ – 463, 520 = 71,454 7.209 – 3.87 = 3.339 7.209 – $\boxed{}$ = 3.34

Teaching point 6:

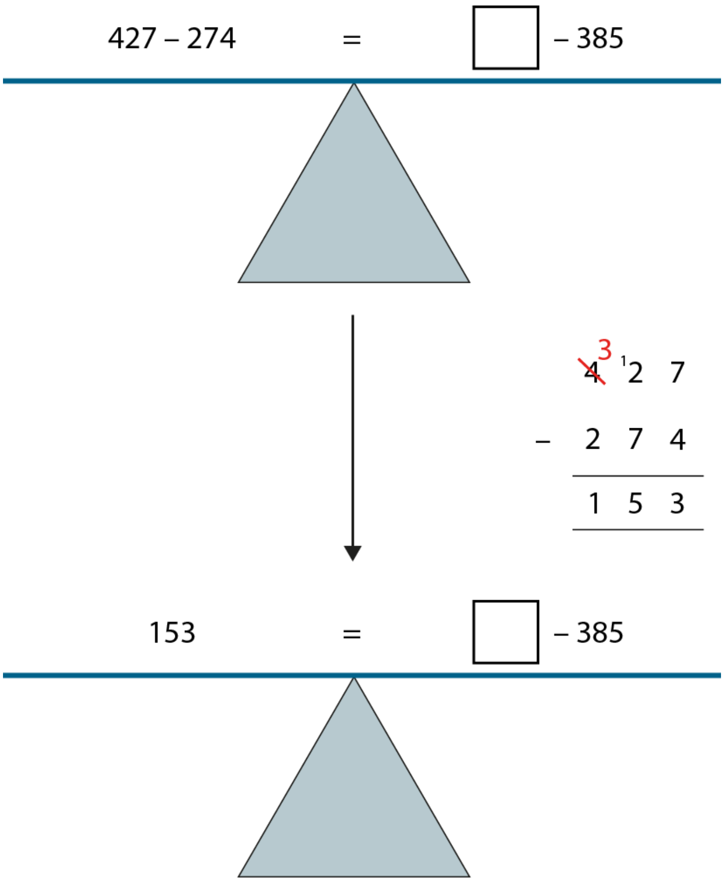
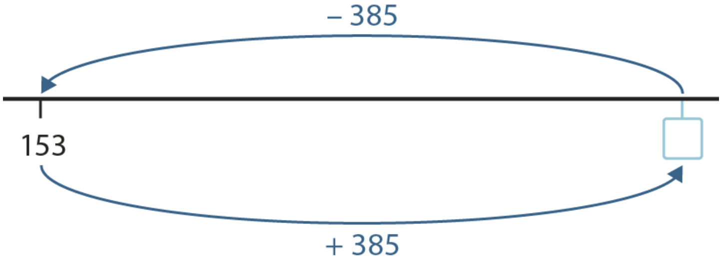
The value of the expressions on each side of an equals symbol must be the same; addition and subtraction are inverse operations. We can use this knowledge to balance equations and solve problems.

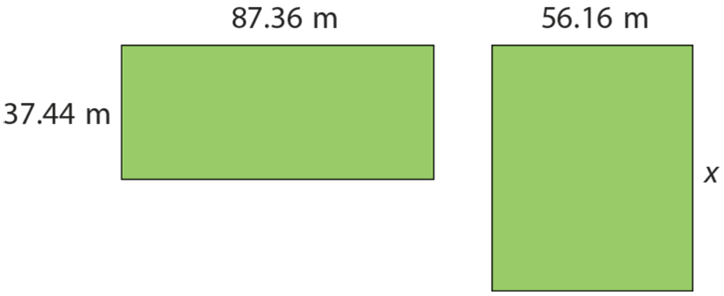
Steps in learning

	Guidance	**Representations**
6:1	In *Teaching points 1* and *3* (same sum and same difference) children practised writing and balancing equations; by now they should have a secure understanding about the meaning of the equals symbol. Remind them of this fact, with a few same sum/same difference examples, using the generalisation: ***'The value of the expressions on each side of an equals symbol must be the same.'*** This teaching point focuses on how to balance equations when there is not an obvious relationship between the terms on either side of the equals symbol, i.e. where the compensation properties of same sum/difference cannot efficiently be applied to find the missing value. Begin by completing inequality problems for addition expressions, such as the example opposite. Encourage children to use estimation and their knowledge of the effect of increasing/ decreasing an addend, to determine the relationship between the expressions. Learning to identify unequal expressions will deepen children's understanding of equal expressions when they come to balance equations in the next step.	$23.63 + 25.34 \bigcirc 35.28 + 29.16$ $23 + 25 < 50$ and $35 + 29 > 50$ so $23.63 + 25.34 < 35.28 + 29.16$ or $23 < 35$ and $25 < 29$ so $23.63 + 25.34 < 35.28 + 29.16$
6:2	Progress to balancing equations with addition expressions. Demonstrate how we now need to calculate the value of one expression to be able to find the missing number in the other. Throughout this teaching point,	

	encourage children to choose efficient strategies for calculation. Work through a range of examples including larger numbers and decimal fractions.	Step 1 – calculating the value of the complete expression:  $34 + 46 = 72 + \boxed{}$ $34 + 46 = 80$ Step 2 – calculating the value of the missing number: $80 = 72 + \boxed{}$ $= 72 + 8$ so $34 + 46 = 72 + \boxed{8}$
6:3	Now look at situations with subtraction expressions on both sides. As for step *6:1*, begin by looking at inequalities, using estimation and knowledge of the effect of increasing/decreasing the minuend or subtrahend, to determine the relationship between the expressions. Note that the symbol '≈' (approximately equal to) has been used opposite for estimated answers; some	Example 1: 89 – 21 ○ 112 – 72 $89 - 25 \approx 90 - 20$ ≈ 70

	children may find it easier to write out 'is roughly equal to' or 'is about', for example *'89 – 25 is about 90 – 20'*.	and $112 - 72 \approx 110 - 70$ ≈ 40 so $89 - 21 > 112 - 72$ Example 2: $42.2 - 7.825$ ○ $125.7 - 6.903$ $42 - 8 < 40$ and $125 - 7 > 100$ so $42.2 - 7.825 < 125.7 - 6.903$ or $42.2 < 125.7$ and $7.825 \approx 6.903$ so $42.2 - 7.825 < 125.7 - 6.903$
6:4	Then explore balancing equations with a subtraction expression on both sides. Demonstrate, again, that we can calculate the value of one expression to be able to find the missing number in the other. It is worth making children aware that there isn't always one 'best' strategy. In the example on the next page, some children may notice that the subtrahend has increased by 111, and be able to apply their knowledge of same difference to identify the missing number as *427 + 111*.	

	Again, work through a range of examples, including larger numbers and decimal fractions.	Step 1 – calculating the value of the complete expression:  Step 2 – calculating the value of the missing number:  so $427 - 274 = \boxed{538} - 385$
6:5	Before moving on to the next step, ensure that children can confidently balance equations with the same type of operation on both sides of the equals sign. For the dòng nǎo jīn problem shown on the next page, children should	

	recognise that they don't have to use the whole perimeter; instead they can balance: $37.44 + 87.36 = 56.16 + x$ (half the perimeter) They could solve this either by: - working out: $124.8 = 56.16 + x$ so $x = 124.8 - 56.16$ (which involves a more complicated column subtraction than the following) - same sum: $87.36 - 56.16 = 31.2$ so $x = 37.44 + 31.2$ This is not as obvious as in earlier examples. Look for reasoning that shows a deep understanding of the relationship between the terms in the equation. There are different strategies for performing the constituent calculations so look for sensible, efficient choices, which may, in this case, include column methods.	Dòng nǎo jīn: *'Both lawns have the same perimeter.'*  - *'How long is the side marked x?'* - *'How many different ways can you find to calculate the value of x?'* - *'Which method do you prefer?'*
6:6	Now look at the final type of equation, i.e. one with an addition expression on one side of the equals symbol and a subtraction expression on the other. Begin by exploring whether a given expression is balanced or not, as in the example opposite. Then look at balancing such equations.	Identifying an unbalanced equation: *'I know that* $49{,}000 + 5{,}000 = 100{,}000 - 46{,}000$ *so* $48{,}000 + 5{,}000 = 100{,}000 - 45{,}000$ *True or false?'* Correcting an equation: *'How could the following equation be changed so that it is balanced?'* $48{,}000 + 5{,}000 = 100{,}000 - 45{,}000$ ✗

<table>
<tr>
<td></td>
<td></td>
<td>Finding an unknown:
'Balance the following equation.'
47,000 + □ = 100,000 – 45,000</td>
</tr>
<tr>
<td>6:7</td>
<td>By now, children have a range of strategies for balancing equations. Provide them with practice, as shown opposite, encouraging them always to first look to see if they can efficiently use a compensation property of addition/subtraction to solve the problem and to only calculate the value of the expressions if that is not the case.
Use a range of contexts such as average monthly rainfall (as shown by the final example opposite) to compare differences and sums, writing calculations to explain statements.</td>
<td>Missing symbol and missing number problems:

'Fill in the missing symbols.' (<, > or =)
7.12 + 0.78 ○ 2.54 + 3.6
7.12 – 0.78 ○ 2.54 + 3.6
392 – 74 ○ 482 – 154
43,216 – 5,412 ○ 32, 216 + 5,588
'Fill in the missing numbers.'
4,924 + □ = 2,447 + 5,472
□ – £4.90 = £23.25 + £6.72
'The table shows rainfall measured in a garden for four months.'
<table>
<tr><th>Month</th><th>Rainfall (mm)</th></tr>
<tr><td>June</td><td>87</td></tr>
<tr><td>July</td><td>67</td></tr>
<tr><td>August</td><td>28</td></tr>
<tr><td>September</td><td>48</td></tr>
</table>

'True or false?
The difference between the amount of rain that fell in July compared to June is the same as the difference between the amount of rain that fell in September compared to August. Write calculations to explain your answer.'
'Explain which difference shows an increase in rainfall and which shows a decrease.'

</td>
</tr>
</table>

6:8	To complete this segment, it is important to provide children with practice covering learning from *all* teaching points, so that they learn to confidently select relevant efficient strategies when presented with problems outside the context of teaching on a particular compensation property.	