Secondary Magazine Issue 79

National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 79 of the Secondary Magazine.

Contents

Contributors to this issue include: James Grime, Mary Pardoe, Richard Perring and Peter Ransom.

From the editor

Welcome to Issue 79 of the Secondary Magazine.

In this issue the writer of our Subject Leadership Diary refers to the 2008 Ofsted report, Mathematics: understanding the score, in which we read – *"Subject expertise is highly significant in teaching and learning. Key characteristics of good subject expertise include … giving pupils responsibility for their own understanding by …* ***helping them to think mathematically*** *… ."* (Paragraph 98)

In Thinking Mathematically by John Mason, with Leone Burton and Kaye Stacey, we are reminded that, in order to focus clearly on our ***mathematical thinking***, it is helpful sometimes to address questions and problems which require a minimum of mathematical background, so that our attention is focussed away from mathematical 'content'. We are then more able to direct our attention to the processes that are fundamental to successful mathematical thinking. This applies to all learners of mathematics including our students.

The themes that are addressed in the 'Focus on…' articles in the Secondary Magazine are, quite often, sources of such questions and problems – in order to start thinking about the problems the learner does not need to draw on a great deal of prior mathematical knowledge. Therefore these articles may sometimes be helpful when you are looking for contexts in which to focus students' attentions specifically on thinking processes.

In this issue Focus on…paper folding provides opportunities for students to make and test predictions, to specialise, generalise, conjecture and convince (themselves and others). This context also throws up phenomena that students can try to explain. Their attempts to convince can be in the form of short, fairly simple, formal proofs – although for that they *will* need to draw on some prior mathematical knowledge.

The subject of our Interview is someone who draws the attention of learners all over the world to many fascinating ideas and phenomena that provide rich opportunities for ***thinking mathematically***.

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It's in the News!
Mortality Statistics

The fortnightly *It's in the News!* resources explore a range of mathematical themes in a topical context. The resource is not intended to be a set of instructions but as a framework which you can personalise to fit your classroom and your learners. Each year the Office for National Statistics publishes a set of data showing the causes of death for the previous year. In January 2011 the data for 2009 was released.

This data was represented by The Guardian as an infographic showing the most frequent causes out of the 491 348 registered deaths in 2009.

The data shows some interesting statistics which might challenge our intuitive understanding of, and the way that the media presents, the dangers facing us (for example, 645 people died from 'alcohol' while 149 died from 'swine flu').

This resource uses the infographic as a context for students to explore the way that data is presented and interpreted.

This context addresses issues that may be upsetting for some students (exploring fatalities due to motorcycle, car and air travel) and it is a good idea to check that students are comfortable with this before starting with the resource.

Download this *It's in the News!* resource - in PowerPoint format

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The Interview

Name: James Grime

About you: I run the Enigma Project for the University of Cambridge. I travel the UK and the world talking to schools and adult audiences about the fascinating history and mathematics of codes and code breaking – including a demonstration of a genuine World War II enigma machine!

The project grew from the talk and book of Simon Singh on the history of code breaking, and it is now one of the many things offered by Cambridge's Millennium Mathematics Project. It's wonderful to be able to share such an interesting and important application of mathematics with the general public. Getting to look after and play with an enigma machine every day is just a bonus!

The most recent use of mathematics in your job was…
While working as a research mathematician I studied group theory and combinatorics. Although I was only concerned with the mathematics, this research had applications in particle physics.

These days I work in maths communication, so now I get to do things just for fun. For example, recently I was asked to analyse the game Top Trumps. Specifically, I was asked to find out if the game was designed to be non-transitive. In other words, is there a chain of cards where each card beats the next card more than half the time - like a giant game of rock, paper, scissors. It turns out it isn't quite. This disappointed me so I decided to design my own game of Top Trumps using mathematicians. As you can imagine, I'm a hoot at parties.

Why mathematics?
The real answer is because I am lazy. I was naturally good at mathematics, and by doing mathematics at A-level and university I found that there were no books to read and no essays to write. I also discovered that, if you go to lectures you can do the homework, and if you can do the homework you can do the exam. It's the perfect subject for the lazy student.

But the reason I continued to study mathematics is that I began to understand the beauty of mathematics. Mathematicians talk about beauty and elegance, in the same terms as someone would talk about music or art, and it's true. Mathematics requires intuition, guess-work and creative thought, and in that way it is much closer to art. I would not have stayed interested if it wasn't.

Some mathematics that amazed you is...
I think what is amazing is how not amazing it is. Mathematicians, and people like myself who try to communicate the joy of mathematics, are very drawn to the counter-intuitive examples, like the now famous but still brilliant Monty Hall problem. But I think to spend too much time on these sorts of problems can be misleading. We are drawn to them because these are the exceptions, and in reality the majority of mathematics is not counter-intuitive and in fact fits together exactly how you think it should. This is important because it allows us to make hypothesises and conjectures to begin with and, more often than not, things do work out how we thought they would. Now, I think that is amazing.

Square-roots are pretty good though

A significant mathematics-related incident in your life was…
A significant mathematical object in my life was probably The Little Professor. It was an electronic toy

made by Texas Instruments in the late '70s. It would give you arithmetic problems on an LED display, and was made with the look of a professor, with glasses, a big bushy moustache and wearing a mortar board – a look I style myself on today. I was doing the hard questions from a young age, in fact I blame it for my entire career. I still have it and one day I'm going to put it in a glass case in my office. I saw one recently on display at the Science Museum, that did not make me feel good.

Who inspired you?
Television inspired me. I do not come from a family of mathematicians or scientists and my school was an ordinary comprehensive. I never had the opportunity to attend events like the Royal Institution's maths masterclasses, which I'm sure I would have loved if I had.

Instead, I lapped up programmes by Johnny Ball, the Royal Institution's Christmas Lectures, and Horizon. And this love for these presenters of my childhood has a lot to do with what I do now, as I want to inspire children in the same way that I was enthused and motivated by these programmes.

Continuing in the vein of the programmes that inspired me, in my spare time I make videos on mathematics. It's just another way to reach people and cause them to be creative about mathematics. When I book a table at a restaurant I try to explain I'm big on YouTube, but it doesn't seem to improve the service!

If you weren't doing this job you would…
I think this follows on from the previous question. As a kid, there were two things I wanted to do when I grew up. One actually was to be a doctor of mathematics. I kept that to myself as it seemed very unlikely, but it was a quiet ambition at the back of my mind.

The second thing was to work in television. If you had asked me what I wanted to do when I was five years old I would have said "cameraman". In other words, I wanted to make television, and that's probably what I would have ended up doing. I might have been the next Director-General of the BBC… maybe I'm better off where I am after all.

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Focus on…paper folding

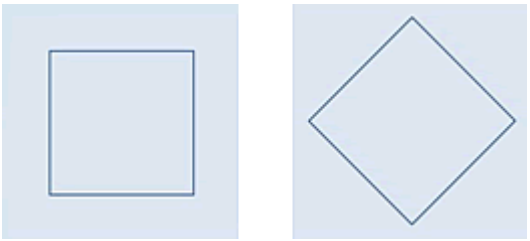
Did you know that it is possible to cut out *any* straight-line drawing on a sheet of paper with a *single straight cut*? You just need to **fold** the paper appropriately several times before you make the cut.

Origami crane by Binnette

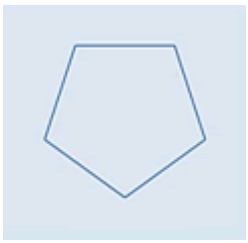
That is one of the many interesting findings of explorations in origami mathematics – the mathematics of the Japanese art of paper folding.

Of course, given a particular straight-line drawing on a particular piece of paper, the problem is to work out how and where to make the folds so that you can cut out the depicted shape with one straight cut. Students will find some such problems much easier to solve than others.

For example, most students will quickly see how they can fold square pieces of paper twice and then cut out, with one straight cut, squares drawn centrally and symmetrically on the pieces of paper.

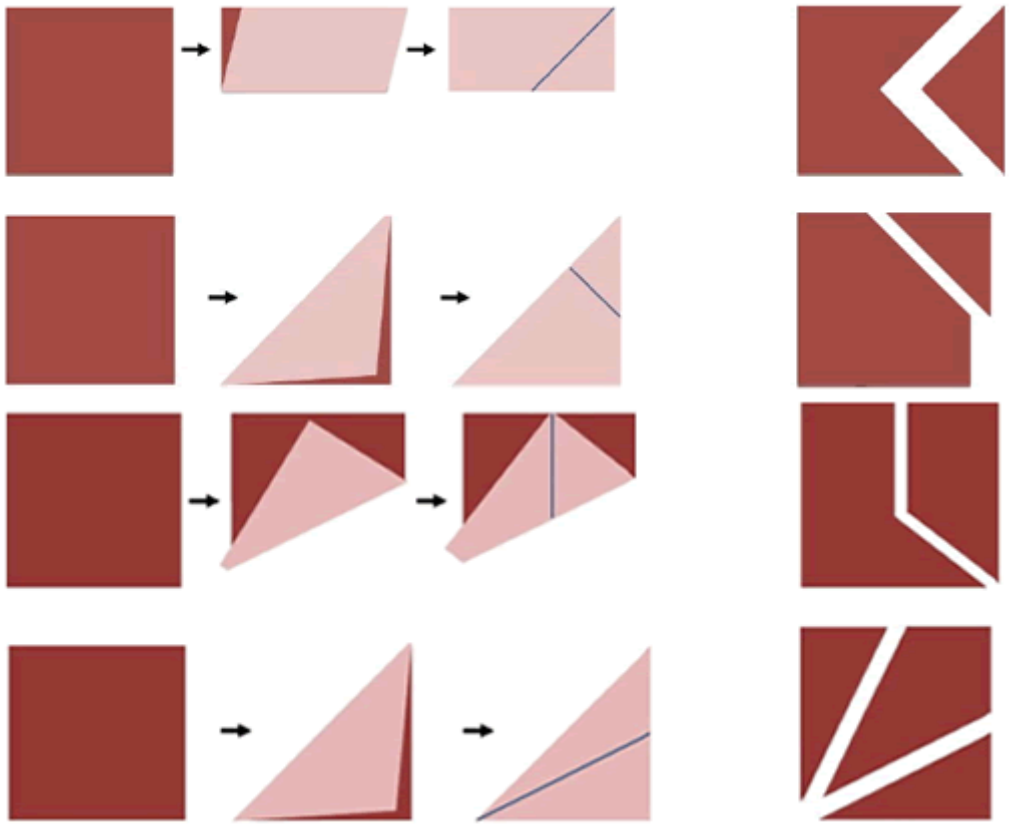


But a regular pentagon centred on a square sheet of paper poses a harder problem.



Alternatively, students can approach this general result about folding and cutting from the opposite direction – what shapes can they make by folding a piece of paper one or more times, making one complete straight cut, and then unfolding?

The possible outcomes of a very simple first challenge may surprise students – fold a square sheet of paper flat once in any way that you like, make a straight cut across it, unfold the pieces and see what you have! A right-angled triangle and a pentagon, perhaps, or maybe a trapezium and a pentagon, or a kite and two right-angled triangles, or … … .?

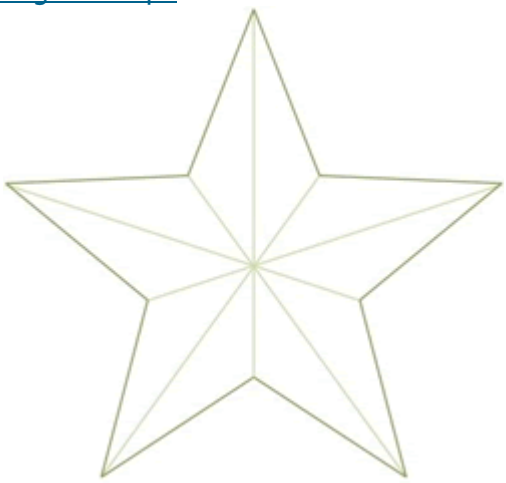


As students become more adventurous they may fold square sheets of paper more than once, again making just one straight cut each time. These are opportunities for predicting, and then unfolding and seeing, what they have created.

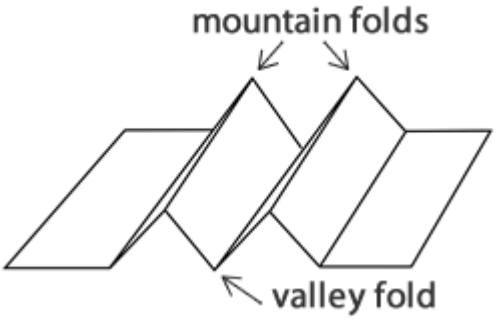
Coloured paper by Sebastian Hartlaub

They might set themselves some challenges.

For example, could your students fold a square sheet of paper so that with one cut they create a five-pointed star – the Betsy Ross pentagram shape?

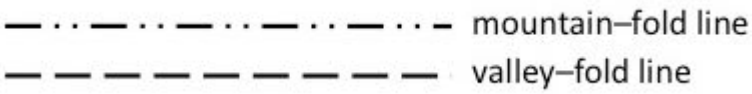


A fold is either a *mountain fold* or a *valley fold*…

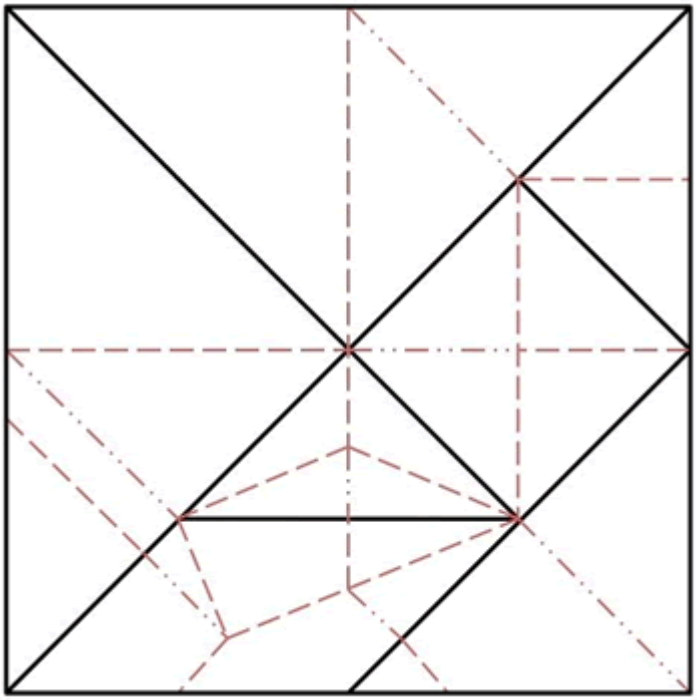


Mountain and valley folds, based on image by Heron2

…and this is the conventional, distinguishing, representation of them in diagrams:



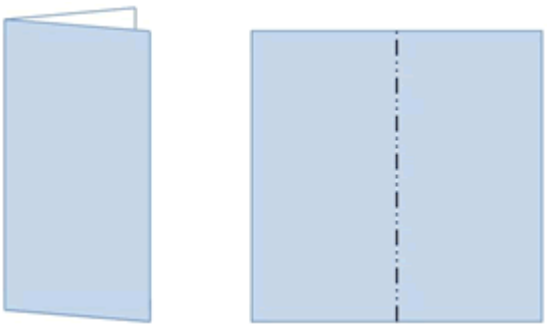
This entire *Tangram Set from a Square* is an example of an arrangement of shapes that are created by one single cut of a square piece of paper that has been folded in a special way. The folds for this ingenious example are indicated conventionally.



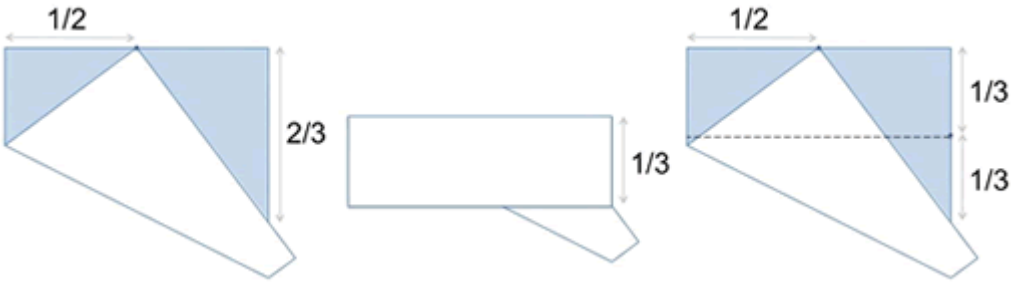
This fold and cut pattern was designed by Erik Demaine or his father, Martin Demaine - Erik is a Professor in the Department of Electrical Engineering at the Massachusetts Institute of Technology, and Martin is a mathematician and artist in residence at the Massachusetts Institute of Technology. You may admire some of their other foldcut examples.

Dividing a line segment into equal parts by paper folding

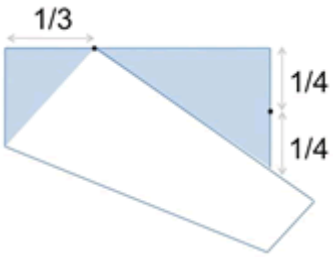
If students have found the mid-point of a side of a square piece of paper by folding one side of the square onto the opposite side…



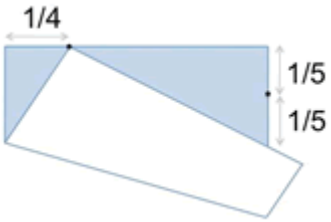
… they may be surprised that, with one new fold, they can then find the point 2/3 of the distance along an adjacent side – and so, by folding again, find the point 1/3 of the distance along that side (the point dividing the side in the ratio 1 : 2) …



… then, by doing two new folds, they can use that point to find the point 1/4 of the distance along a side (the point dividing the side in the ratio 1 : 3) …



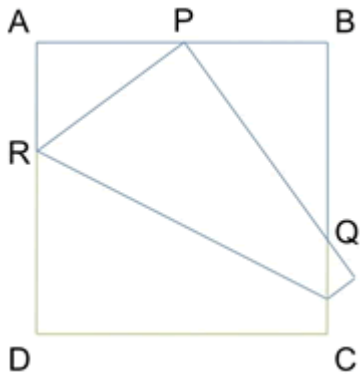
… then, with yet two more folds, they can use that point to find the point 1/5 of the distance along a side (the point dividing the side in the ratio 1 : 4) …



… and so on forever!

In this screencast Square Folding in Thirds, students can watch the folding of a square into thirds, and in this related screencast Geogebra is used to show what is done – although there is no explanation of ***why*** the method works.

Could your students explain why this folding method of finding points at fractional distances along a line segment works? Using a diagram such as this …



… could they use reasoning involving Pythagoras' Theorem, the similarity of two right-angled triangles and simple algebraic manipulation to show that if AB is 1 unit and AP is 1/k units then BQ must be 2/(k +1) units for any (not necessarily integer) value of k?

There is a proof of this on the Cut The Knot website.

As Dr. Alex Bogomolny, who is a former associate professor of mathematics at the University of Iowa and currently the developer of the *Cut The Knot* website, *Interactive Mathematics Miscellany and Puzzles, writes on that website, if 'students … learn mathematics via doing it … it's quite probable that, now and then, (they) will make novel discoveries'.*

In this Bouncy Unit video, in which Tom Hull shows how to make his origami Bouncy Unit, he also demonstrates how to use a method of estimation to divide a side of a square into fifths.

Origami made with PHiZZ units photograph by SirJective

This is made with PHiZZ units, which Tom mentions in his Bouncy Unit video.

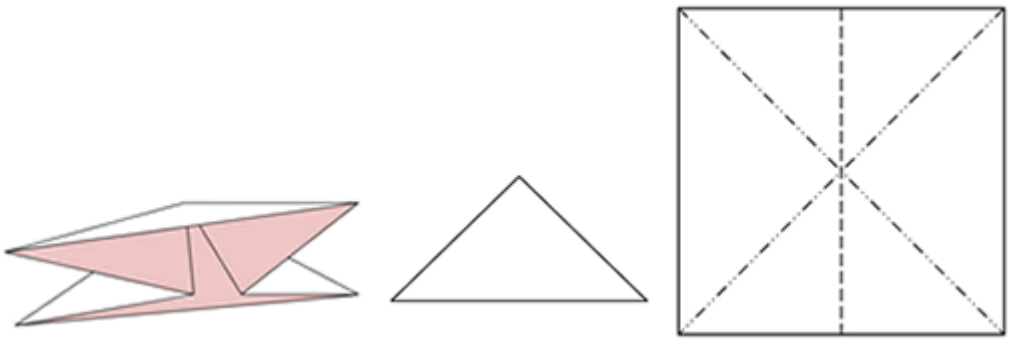
Axioms and theorems of paper folding

Some folds, and combinations of folds, replicate geometric constructions and proofs. The origami axioms describe particular folds on which these replications are based.

Christian Lavoie, while an undergraduate studying at McGill University, created an internet resource which is an illustrated explanation of Axiomatic Origami.

Flat folds are combinations of folds (fold patterns) resulting in origami models that can be pressed flat without adding new creases. *Flat vertex* folds are fold patterns that are *flat folds* and have only one vertex, such as these:

Screenshots from video of origami paper crane folding by Tavin



Thomas Hull, an associate professor of mathematics at Western New England College, is known for his expertise in the mathematics of paper folding. In his Notes on Flat Folding he gives the following 'exercise' that you might pose as a challenge for students.

> *Get some pieces of paper and create your own flat vertex folds. How many creases can you make meet at a vertex and have them fold flat? Can you ever get an odd number of creases? What about the number of mountain creases you see and the number of valley creases? Do you see a pattern?*

Students who take up this challenge might discover the following theorem of paper folding for themselves. This is how Tom Hull states the theorem…

> Maekawa's Theorem: The difference between the number of mountain creases and the number of valley creases in a flat vertex fold is always 2.

… of which he then provides a proof, before moving the learner on to think about a second theorem …

> Kawasaki's Theorem: The sum of every other angle about the vertex in a flat vertex fold is always 180 degrees.

Paper folding demonstrations of mathematical relationships, objects and actions

Paper folding geometry includes the generation of mathematical objects, the doing of mathematical actions and the proving of relationships. Below we give links to some paper folding geometry that you might like to investigate on Alex Bogomolny's *Cut The Knot* website.

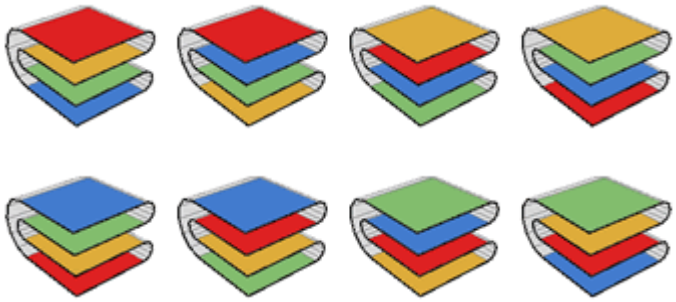
- The angles in a triangle add to 180° – an applet that illustrates a proof by paper folding

- Angle trisection
- Creating a parabola – a parabola emerges as the envelope of paper folds (creases)
- A regular pentagon inscribed in a circle
- The Broken Chord Theorem
- Egyptian Triangle By Paper Folding – paper folds (creases) form a pattern in which there are 32 instances of the 3,4,5 right-angled triangle.

Folding stamps and maps

Students could investigate the number of ways of folding a strip of stamps.

Map folding is a little more complex.



Stamp folding and Map folding images by Robert Dickau

The Miura Map Fold is an unusual origami model – it pulls open with one pull, and closes almost automatically with one push, like this. Students may like to see how to fold a Miura-Ori Pattern.

Flexagons

Students usually enjoy making flexagons. Martin Gardner's book, Hexaflexagons and Other Mathematical Diversions is free to download – and your students might like to watch Dr James Grime showing how to make a cyclic hexa tetraflexagon - A Flexagon for Martin Gardner.

Further exploration and activities

A good place to start exploring paper folding in more depth is Erik Demaine's Folding and Unfolding Page.

At NRICH you will find a variety of interesting folding and origami activities for students.

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Examples of flat vertex folds from video of origami paper crane folding by Tavin some rights reserved
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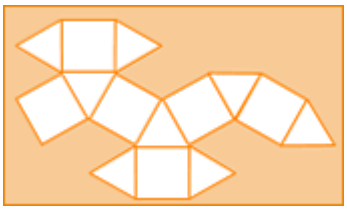

5 things to do this fortnight

- Are you planning to take part in the National STEM Centre two-day event on 17 and 18 March, Promoting STEM Careers in Your Classroom? This conference will provide teachers of STEM subjects – in KS3 through to Post-16 – with a wealth of resources and information about exciting and rewarding careers for students in science, technology, engineering and mathematics. If you are a teacher, tutor or lecturer in a maintained school or college you are eligible for an ENTHUSE Award of £351 + VAT – to cover the conference attendance costs and to help you implement new ideas in your classroom.

- On Saturday 5 March two ATM/MA meetings will be held. One is a meeting of The Marches Network when Stephanie Prestage and Pat Perks will lead a whole morning workshop – at Tenbury Wells School, for teachers from all sectors – on beginning algebra without equations. The other is an East Midlands ATM/MA Branch Meeting in Leicester, at which Sue Pope and Tung Ken Lam will lead sessions on using origami in the mathematics classroom.

- The closing date for applications for British Science Association Media Fellowships (2011) is 1 March 2011. Media Fellows will spend up to eight weeks working with a national press, broadcast or internet journalist, learning how to produce accurate and well-informed pieces about developments in mathematics and science. Then they will attend the British Science Festival.

- You have until 14 April 2011 to respond to the National Curriculum Review - Call for Evidence. This review of the primary and secondary National Curriculum in England is an opportunity for you to voice your opinions and evidence about the National Curriculum, and thereby help to start a debate about what is taught in our schools.

- If you cannot go to the East Midlands ATM/MA Branch Meeting, you can see Tung Ken Lam folding a cuboctahedron in this video.

(In this issue of the magazine the Focus On... is on paper folding).

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Red and yellow cuboctahedron image by McLoaf in the public domain

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Subject Leadership Diary

Nearly there – 0.5 term starts at the end of this week! Have you ever thought about how often we use fractions rather than decimals in our everyday language? Challenge students to think of lots of ways in which we use fractions in our everyday descriptions of things – such as half term, half mast, quarterdeck, quartermaster, quarterpounder, wholemeal, and so on – and use their meanings to help explain simple fractions.

Last week I did all the assemblies. Generally it is the assistant heads, deputy heads and head-teacher that do these, but the call went out to see if volunteers from middle management had anything to offer. I've heard the advice about never volunteering, but when I considered all the support I get from the senior leadership team and heads of year, I felt it was only fair to give them a break – and when I volunteered, the week I opted to do was still some distance in the future! But it came around all too quickly, and I wondered what to do. Now, both my daughters went to the school where I teach (and I taught one of them in key stage 4), so I thought it might be good to talk about what they were doing now. My elder daughter teaches in a primary school in our catchment area, so I knew that at least three year-groups would contain some students whom she had taught. My younger daughter finished her BA in animation a few years ago and her final degree work was a documentary animation about the miners' strike of 1984/85. I thought I would talk about their careers after their time at our school, and also about how people helped each other during the 1984/85 time of strife. Some of the students while at their primary school had provided artwork of footwear which feetured (!) in my daughter's animation. The interest shown was considerable – well done family!

Back to my elder daughter, teaching in the primary school – what impact has her learning and teaching had on my mathematics teaching? The impact has been twofold. First, when I was teaching her I got invaluable regular feedback about learning in my lessons from someone who I knew very well. No amount of peer observation can be better than this. Students don't always describe to their teacher their true feelings about a lesson – but I soon learnt from my daughter which lessons had not been fully understood. This invaluable, and unusual, source of knowledge enabled me to plan to spend time on topics that needed more assimilation. Secondly, her teaching has helped me greatly in easing pupils' transitions from key stage 2 to 3 – some of her pupils in key stage 2 are taught by me in later key stages. I therefore get to know some strengths and weaknesses of the students, as well as becoming familiar with what is happening in key stage 2. We have also worked together on mathematics masterclasses for pupils in Year 5 and in Year 9 – so we are acquainted with what each of us is doing with gifted pupils and students.

This leads me on to professional development. Over the past year or so I noticed that fewer schools seem to be allowing teachers out to develop their skills. I often hear teachers mention problems about supply cover costs, or that all their development is generated 'in house'. But many schools now employ cover supervisors to allow teachers to get out to professional development events and to other schools, so surely the cost has reduced over the past few years. 'In house' might be fine for whole school issues, but mathematical pedagogy is rarely adequately addressed solely 'in house' (read the 2008 Ofsted report Mathematics: understanding the score) - it is essential to get fresh ideas into your faculty from other sources. Faculties should be fighting for freedom to go, for example, to events run by the NCETM, the Mathematical Association and the Association of Teachers of Mathematics (see what's available). Our students should not be denied the richness of activities gleaned and developed from such courses, and our teachers will thrive on the enthusiasm of educators like themselves. Make it a priority this year to attend a course in your area!