Secondary
Magazine 120

National Centre
for Excellence in the
Teaching of Mathematics

Like the recent Grand National competitors, we're now entering the last leg of the school year, but before the July finish line there are all the hurdles and waterjumps of public exams that we want our pupils to clear successfully…and of course there's a certain something happening on 7 May that will have impact on all of us. Let us know what you think about the election, either locally in your constituency or on the national scale, by email to info@ncetm.org.uk or on Twitter, @NCETMsecondary.

Contents

Image credit
Page header by Miss Acuarelas (adapted), some rights reserved

Heads Up

Whilst not wanting to define this article as mathematical gossip, it does come close! We've brought together news and current mathematical affairs, all in one place. We do hope it will interest you.

How was Pi Day for you? Did you celebrate at 09:26 (with a slice of Twin-Peaks-homage cherry pie)? Did your pupils or school put on special activities? Do share any pictures or anecdotes that you have.

Did you attend any of the many education conferences over the Easter holiday – perhaps a union conference, perhaps a specifically mathematical one such as the MA or the ATM conference? If you did we'd love to hear your comments: are such conferences worth attending? How do they support you as a classroom teacher, or is their value more that they take you away from the classroom quotidian? Let us know by email to info@ncetm.org.uk or on Twitter, @NCETMsecondary.

The election is already throwing up lots of data double-dutch and statistical sleights-of-hand, so encourage your students to look for independent commentary, analysis and correctives, for example on the BBC news site or the Full Fact site. There is a wealth of fascinating data about each constituency on the Constituency Explorer site and the presentation (including box and whisker plots!) makes it a wonderful resource for discussing data handling and interpretation with pupils across KS3-5: very well worth a visit.

Ofqual has shared some feedback from its on-going review of the GCSE specimen assessment materials.

11 March was the launch of the Cambridge Mathematics Framework. Led by Lynne McClure, this is a new vision for teaching Maths right through from Early Years to university entrance. You can keep up to date @CambridgeMaths.

The Cambridge Mathematics Education Project is a new initiative which aims to support and enhance A-level maths teaching, with lots of rich and challenging resources to deepen students' conceptual understanding. You can register as an affiliate school, or follow @CMEPmaths.

Oxford's celebration of Women In Maths ran 14-17 April as part of the London Mathematical Society's 150th birthday. If you were there please do share your stories and photographs with us; if you couldn't attend, do return to the website to find out more about the events, speakers and activities that were brought together.

The DfE has named the members of the commission on Assessment without Levels. The commission's task is to "*identify and share best practice in assessment with schools across the country and ensure they have information to make informed choices about effective assessment systems. The commission will highlight the great work that is already being done in many schools and will help to foster innovation and success in assessment practice more widely*".

If you teach GCSE re-sit students in Years 12 and 13, and need tips on how to persuade them that maths really does have an important place in everyday workplace contexts, you might be interested in a new online toolkit that aims to help you contextualise GCSE maths. It's been produced by Mathematics in Education and Industry (MEI) and is aimed at both specialist maths teachers and vocational lecturers and trainers who want to embed maths in their teaching.

The DfE and the NCTL are working together to try to attract qualified maths teachers, who left the job for any reason in recent years, back into the profession. Teachers interested in returning to the classroom can register for information and support here, and schools interested in either sharing experiences of successfully recruiting and supporting returning teachers, or in passing on current vacancies that might be filled by returning teachers should email returntoteaching.NCTL@education.gsi.gov.uk.

This interesting blog post from Sir David Spiegelhalter (Winton Professor of the Public Understanding of Risk, and a regular contributor to the marvellous Radio 4 programme More or Less @BBCMoreOrLess) may help your students (and perhaps your non-mathematical friends, relatives and colleagues!) understand uncertainty more clearly.

This beautiful graphic depicts the relationship between Earth years and Venus years...it's very reminiscent of that 70's classic, Spirograph™!

Image credit
Page header by NS Newsflash (adapted),

Building Bridges

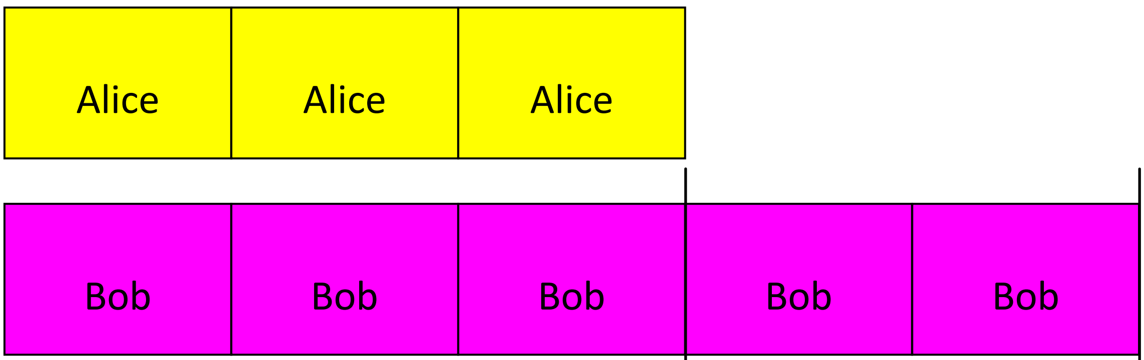
In Issue 118 and Issue 119 I've written about the flexibility and longevity of the area model for multiplication, and I've tried to explain that I think it's a "good" mathematical model because it can be used to help pupils think about multiplication in a range of different contexts over a number of Key Stages. More generally, I believe that when we choose and use models and representations in our teaching to help our pupils understand the mathematical concepts that they are exploring, we need to ensure that the models/representations

- can at first be explored "hands on" by all pupils irrespective of prior attainment
- arise naturally in the given scenario, so that they are salient and hence "sticky"
- can be implemented efficiently, and increase all pupils' procedural fluency
- expose, and focus all pupils' attention on, the underlying mathematics
- are extensible, flexible, adaptable and long-lived, from simple to more complex problems
- encourage, enable and support all pupils' thinking and reasoning about the concrete to develop into thinking and reasoning with increasing abstraction.

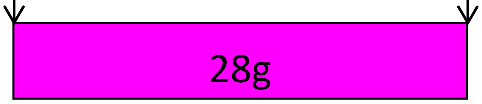
I would argue that the area model for multiplication meets these six criteria, as does the model for division as fitting sticks into gaps (discussed in Issue 116): please do let me know (email or Twitter) if you disagree. Currently there is a lot of discussion about – and advocacy of – bar modelling (sometimes called Singapore bar modelling because it is regularly used in both primary and secondary schools there): does it fit the criteria? Is it the wonder-panacea its most zealous proponents claim?

If you're unfamiliar with bar modelling, there are many online demonstrations and explanations you can easily look up. It's often taught in the context of ratio and fraction problems, for example *"I share some sultanas between Alice and Bob in the ratio 3:5. Alice gets 28g fewer sultanas than Bob. How many grams of sultanas does Bob get?"*

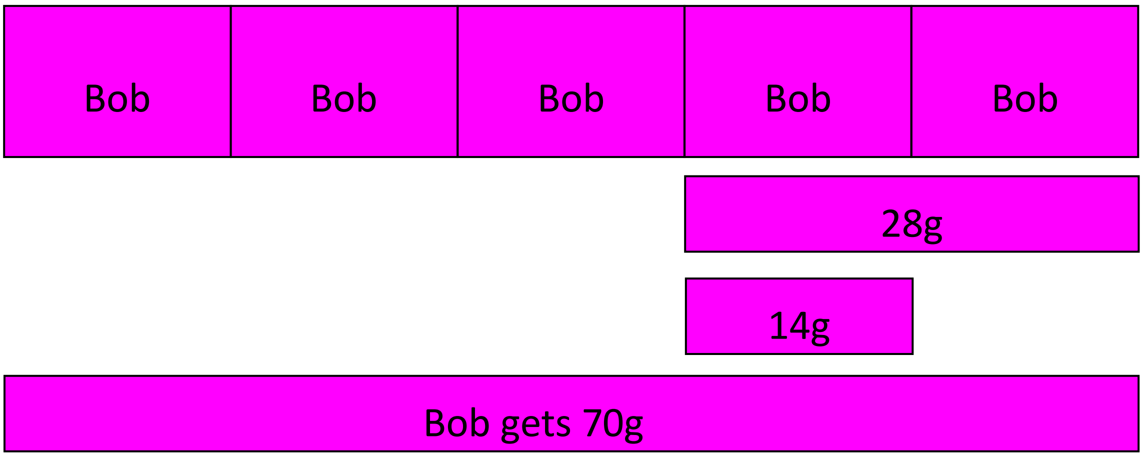
To solve this problem, a pupil might first draw



to represent the 3 portion / handfuls / scoops / boxes that Alice gets, and the 5 that Bob gets. The pupil would then add to the picture that the discrepancy is 28g:



and then see that 2 of Bob's portions weigh 28g and so 1 must weigh 14g. Therefore his 5 portions must weigh 70g. The diagram would end up looking like:



There are two immediately clear and powerful benefits to using this representation to model and answer this question: the pupil has got the right answer, and the diagrams capture precisely the pupil's reasoning. This argument – pictorial though it is – is as incontrovertible as a formal symbolic proof, and it is accessible to pupils with a wide span of prior attainment. I certainly think that the bar model meets the first five of the six criteria I proposed earlier.

To test whether it meets the sixth, let's imagine giving the pupil these subsequent questions:

- I share some sultanas between Alice and Bob in the ratio 3:5. Alice gets 28g fewer sultanas than Bob. How many grams of sultanas does Bob get?
- I share some sultanas between Alice and Bob in the ratio 6:10. Alice gets 28g fewer sultanas than Bob. How many grams of sultanas does Bob get?
- I share some sultanas between Alice and Bob, so that Bob gets $\frac{5}{8}$ of all the sultanas. Alice gets 28g fewer sultanas than Bob. How many grams of sultanas does Bob get?
- I share some sultanas between Alice and Bob, so that Alice gets 60% of what Bob gets. Alice gets 28g fewer sultanas than Bob. How many grams of sultanas does Bob get?

A pupil who doesn't see that, or see why, these are all the same question hasn't yet got a deep conceptual understanding of the question; the pupil who answers the question a further four times is still reasoning about the concrete. My caution about the bar model is that it doesn't naturally or automatically develop the abstract reasoning that enables the pupil to see that these are all flip sides of the same (five sided!) coin, because drawing boxes/bars focuses pupils' attention on the **additive** structure of the Alice and Bob problem (Alice has two boxes/bars **fewer** than Bob, Bob has two boxes/bars **more**) and not on the **multiplicative** structure of the problem, that

- Alice's share is $\frac{3}{5}$ of Bob's share
- Bob's share is $\frac{5}{8}$ of the total
- Alice's share is Bob's share reduced by 40%
- the scale factor from Alice's share to Bob's is $1\frac{2}{3}$.

This is not a fatal flaw in bar modelling, as long as we recognise it and in our teaching address it and overcome it: when we draw bar representations we must ask pupils for multiplicative as well as additive comparisons between the two (or more) bars drawn, so that they get into the habit of looking for multiplicative relationships between numbers and variables.

Whatever the mathematical concept and the associated model, if pupils' thinking and reasoning about the concrete is going to develop into thinking and reasoning with increasing abstraction, they need us their teachers to help make this happen. Crucial to this happening successfully is our choice of representation / model, but equally important are

- the reasoning we cultivate and sharpen through the discussions we foster and steer;
- the misconceptions we predict and confront as part of the sequence of questions we plan and ask;
- the conceptual understanding we embed and deepen through the intelligent practice we design and prepare for the pupils to engage in and with.

Bars, fields and sticks'n'gaps are all contexts in which this can happen easily, naturally and powerfully, hence my advocacy (though not uncritical) of them. As for "BODMAS" … well, the less said about that the better!

Agree or challenge? Let me know robert.wilne@ncetm.org.uk or @NCETMsecondary.

You can find previous *Building Bridges* features here.

Image credit

Secondary Magazine 120

National Centre for Excellence in the Teaching of Mathematics

Sixth Sense
Making More Sense of Probability

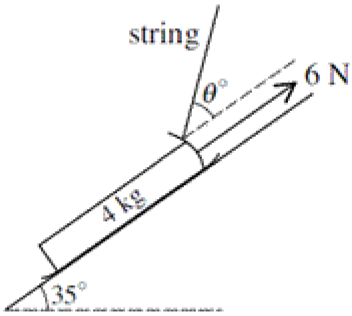
John Partridge, Assistant Head at King's College London Mathematics School, writes:

Towards the end of last term, an NQT-colleague asked me “How do I start the revision process with my mechanics class?” He was keen to discuss more exciting options than those based on his own memory of preparing for exams: could he do something other than dig out some past papers, and the mark schemes, and let the students get stuck in pretty much by themselves? After a little discussion we agreed that giving out a paper and letting the students attempt it in class would probably not be a fruitful use of valuable revision time: they’ve seen so many new ideas in Mechanics this year, and previous experience suggests that every question on the paper would soon become a whole group discussion … lament … along the lines of “what, I have to remember the suvat formulae?!”; “to find the force up the slope do I use cos or sin?”; “but surely the weight is equal to the tension…”

Instead, we decided that concentrating on one topic at a time made sense, and chose to begin with a long (and hence difficult in the eyes of many students!) question on forces. Given that students often find the pure mathematics involved in solving mechanics problems challenging, we thought we would separate the two processes. Consequently, we took a highly structured question and removed the structure, to create an “aide-memoire/starter” question. Here’s an example:

In the diagram, the sledge is held in equilibrium by the string. If I told you the tension in the string, how would you go about finding the normal reaction?

[inspiration: MEI Mechanics 1 January 2011, q6]



Presented with this, what might your students say, and what might you say to them? By removing the value of the tension, you prevent calculation: students have to discuss and develop their plan in words, but they cannot get distracted by – or bogged down by – the numerical manipulation.

Hopefully, the following “recipe” for finding the normal reaction will soon (re-)materialise:

1. draw a force diagram showing all the forces acting on the sledge
2. split forces into well-chosen (meaning?) components where necessary
3. use equilibrium parallel to the slope to find θ
4. use equilibrium perpendicular to the slope to find the normal reaction

If you now tell the students that the tension in the string is 25N, each of these small steps is approachable and “do-able”, and can be discussed with individuals or in small groups if further consolidation is needed. The “pure” skills can be honed now that the mechanics is essentially “done”.

Having got this far, you could ask some extension questions of the students – “How do I make this question harder” [it could be accelerating] “or easier” [the string could be parallel to the slope] – and hopefully this starts to remind them of all the work done [pun fully intended!] on forces and Newton's Laws earlier in the year. Asking the class to "find the speed after 5 seconds if the string is removed and the sledge is released from rest" gives an excuse to get (collectively) the constant acceleration formulae up on the board, and learned before they're needed next lesson!

Now tell them that they've essentially worked their way through a 17 mark question without panicking – indeed, they could see the full version at this point, and asking them to do it should boost their confidence and morale.

Later, my colleague reported back that the 20 minutes of "recipe building" had been very productive, and that after the students got past the scary diagram, they were pleased to have come up with a plan that would get them such a big chunk of their M1 marks. The next lesson was then valuably spent working through whole questions selected from other papers on the same topic (a couple of hours literally "cutting and pasting" a few papers into topics is time very well spent – and you only have to do it once before re-using year after year!) and my colleague is now thinking how to plan the next few revision lessons to be structured similarly but on different M1 topics. For example:

> *Two ships P and Q are moving with constant velocities. Ship P moves with velocity (2i – 3j)kmh-1 and ship Q moves with velocity (3i + 4j)kmh-1. At 2pm, ship P is at the point with position vector (i + j)km and ship Q is at the point with position vector (-2j)km.* (Edited from Edexcel, June 2011, q7)
>
> *You have 3 minutes to write down what the question might go on to ask you to do.*

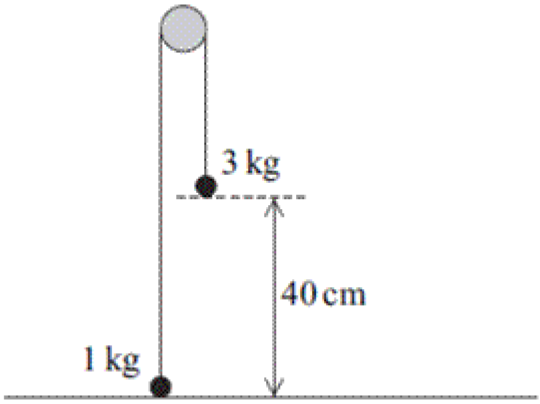
This could lead to a think-pair-share discussion and collectively students might agree on some specific questions such as:

- Find the bearing on which each ship is moving
- Find the position of each ship at 3pm
- Find the distance between the ships at 5pm
- Find the bearing of P from Q (or Q from P) at 4pm

You could encourage them to answer these, before requesting more general questions (ideally with answers!):

- Where are the ships *t* hours later?
- When is Q due north of P?

Or, for example, a picture like this (from AQA, June 2013, q5):



should prompt the students to ask themselves (a) what happens when I let this go? (b) how long until the 3kg mass hits the ground? (c) how high does the 1kg mass go after the 3kg mass hits the ground?

Although this does delay the lesson when you do hand out a whole past paper to do, I'd suggest that it makes that lesson much more valuable for the students (and much less traumatic for them too!) because all the preparation has been done recently. The students have been thinking what questions they might get asked, so when these questions are indeed the ones that are asked, the students tackle them with confidence, enthusiasm, and success.

How do you support your students to revise? Let us know @NCETMsecondary or email to info@ncetm.org.uk.

You can find previous *Sixth Sense* features here.

Image credit
Page header by Richard Gray (adapted), some rights reserved

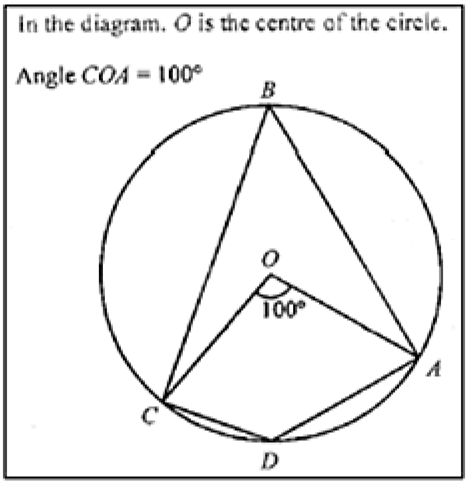
Secondary Magazine 120

National Centre for Excellence in the Teaching of Mathematics

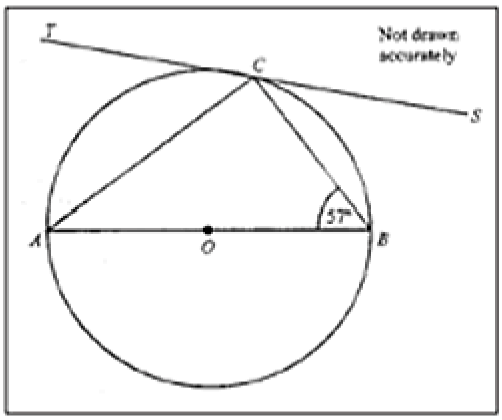
From the Library
Shh! No Talking!

Our regular feature highlighting an article or research paper that will, we hope, have a helpful bearing on your teaching of mathematics. You can find previous features in this series here.

In this issue we share with you a paper first presented at the BSRLM Proceedings in 2006. Celia Hoyles's (the founding Director of NCETM) and Dietmar Küchemann's paper Secondary School Pupils' Approaches to Proof-related Tasks in Geometry gives an account of two pupils' attempts to solve two GCSE geometry questions involving circle theorems. The paper identifies some of the characteristics of such tasks, some of the pupils' emerging strategies, and some of the difficulties the pupils encountered, especially with using the givens and extracting information contained in diagrams.



The paper starts by recording how the pupils find (a) angle CBA and (b) angle CDA in the diagram above. Subsequently they work on a different problem:



Here AOB is a diameter, TCS is a tangent, angle ABC = 57˚ and pupils are asked to find (a) angle CAB and (b) angle ACS.

The paper presents a transcript of the pupil/researcher conversations as they work through the problem: we think it's a conversation that will sound familiar to you, and that you'll recognise many of the misconceptions and difficulties that the pupils encounter. We hope that this paper will help you steer your pupils' development of more secure conceptual understanding and procedural fluency, so that they start to tackle circle theorem questions, not least on GCSE exam papers, successfully and confidently. Let us know what you think. Let us know, @NCETMsecondary or info@ncetm.org.uk.

You can find previous *From the Library* features here.

Image credit
Page header by Abhi Sharma (adapted), some rights reserved

It Stands to Reason

Solving Equations

In this regular feature, an element of the mathematics curriculum is chosen and we collate for you some teaching ideas and resources that we think will help your pupils develop their reasoning skills. If you'd like to suggest a future topic, please do so to info@ncetm.org.uk or @NCETMsecondary. You can find previous features in this series here.

Why is "pencil and paper" curve sketching still an important activity in mathematics when we have such amazing graph plotting software available in the 21st century such as Autograph and Geogebra?
The answer lies in the process that learners go through in order to provide information for the sketch graph and the mathematical reasoning that underpins this process and facilitates the establishment of the key features of the sketch graph. That's why a question along the lines of "How many roots does the equation $x^2 = 2^x$ have?" has been such a stalwart of university entrance interviews for so many years.

This process of reasoning and discovery, for example answering questions such as

- "Does the curve cross either or both of the coordinate axes, and if so, how many times and where?"
- "Does it have any turning points, and if so, how many, what type and where?"
- "Are there any asymptotes and/or what happens for large values of x?"

can take place before or after the actual features of the graph are suggested via graph plotting software, and the two can even be used hand in hand.

Here is an example of some student thinking, working and reasoning with *teacher comments, and prompts for further reasoning*, in italics. It's a complicated example, but one which demonstrates the full power of curve sketching because it entails so many steps. Some simpler examples are suggested at the end of the article.

Problem. Investigate the graph of y = 6ln(x^2+1) – x.

Student 1

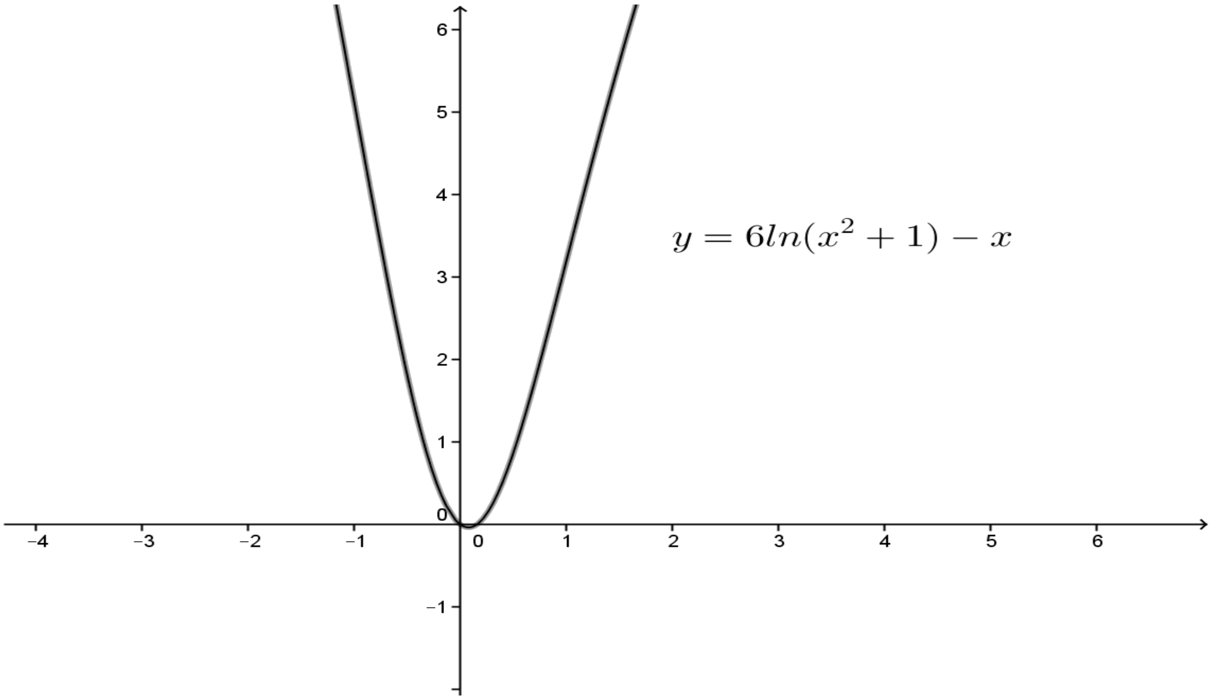
Thinking. Start by just drawing the graph in Geogebra.

Teacher

To get a feel for an unfamiliar graph, and then start to investigate the features more fully by asking questions like those above. In answering these questions, mathematical reasoning will come to the fore.

Having provided pupils with opportunities to explore circle theorems with specific angles, you will want to move on to generalising these specific relationships and showing that these specific relationships also exist in general cases – proof indeed!

Student 2



Teacher

Some colleagues will no doubt balk at this initial use of software and insist on investigation and reasoning first in a curve sketching activity. However, the students can discuss straightaway the benefits but also the limitations of this computer-generated graph: a key one is that we only see the shape of the graph over a very limited domain.

Student 1

Thinking. It looks like the graph crosses the coordinate axes near the origin but where and how many times?

Student 2

Working. When x = 0 (i.e. where the graph crosses the y-axis), y = 6ln1-0 = 0.

Reasoning. This tells me that the graph crosses the y-axis at only one point and this is at (0,0). There are no other intercepts with the y-axis.

Student 2

Working. When y = 0 (i.e. where the graph crosses the x-axis), $6\ln(x^2+1) - x = 0$.

Thinking. This is a much trickier equation to solve or indeed even to be able to say how many roots it has.

Student 1

Reasoning. One of the solutions is x = 0 as we already know that the graph passes through (0,0). So the graph crosses the x-axis at least once.

Thinking. Are there any more intercepts with the x-axis? There might be but 6ln(x^2+1) – x = 0 is an equation that will need an approximate method of solution using an accurately plotted graph (*which we are trying to avoid at the moment*) or numerical methods.

Working. Writing the equation as x = 6ln(x^2+1) and using an iterative method along with a calculator or spreadsheet or graph plotting software leads to roots x = 0 or x = 45.93 (4 s.f.).

Reasoning. There are at least two intercepts with the x-axis at (0,0) and (45.93,0).

Student 2

Thinking. The root x = 45.93 is a surprise! I can see two intercepts with the x-axis at present but neither of them is 45.93. What's the value of the root between 0 and 1? Are there any more?

Student 1

Working. Rewriting the equation as x = $\sqrt{e^{x/6}-1}$ (*Further Mathematics students need to be able to explain why different iterative functions converge to different roots*) and using an iterative method along with a calculator or spreadsheet or graph plotting software leads to the root x = 0.1690 (4 s.f.).

Reasoning. I have found three intercepts with the x-axis at (0,0), (0.1690,0) and (45.93,0).

Student 2

Thinking. How can I be sure that there are no more intercepts? After all, I wasn't expecting 45.93.

Teacher

At this point, colleagues will breathe a sigh of relief as you know that we are going to need more analysis after all.

Student 1

Thinking. Are there any turning points, and if so can I deduce anything from them?

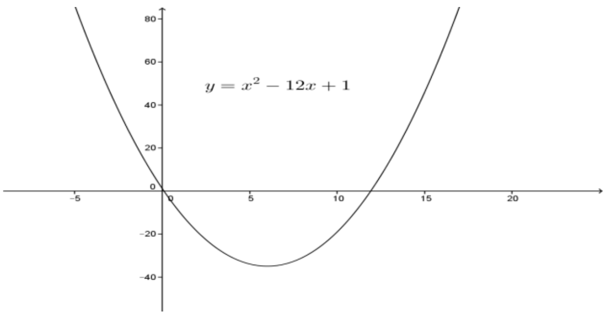
Working. $\frac{dy}{dx} = \frac{12x}{(x^2+1)} - 1 = -\frac{(x^2-12x+1)}{(x^2+1)}$ and so or turning points $x^2 - 12x + 1 = 0$ and so x = 6 ± $\sqrt{35}$.

Reasoning. This tells me that there are exactly two turning points on the graph. One of them is where x = 6 - $\sqrt{35}$ which is small (close to zero) but positive, and y = 6ln($(6-\sqrt{35})^2+1$) – (6 - $\sqrt{35}$) < 0 (*explain why*) and small (*explain why*) so this turning point is just below the x-axis. The second is where x = 6 + $\sqrt{35}$ which is much larger, positive and close to but less than 12, and y ≈ 6ln$145 - 12$ > 0 (*explain why using approximations for powers of e like* $e^3 \approx 20$) so this turning point is well above (*explain why)* the x-axis.

Student 2

Thinking. What type of turning points are they?

Working. Let's look at another, more familiar, sketch graph of y = $x^2 - 12x + 1$ in order to ascertain the nature of $\frac{dy}{dx}$ for the original function.



Reasoning. For x < 6 - $\sqrt{35}$, $x^2 - 12x + 1 > 0$ and so the original $\frac{dy}{dx}$ < 0 i.e. negative.

For $6 - \sqrt{35} < x < 6 + \sqrt{35}$, $x^2 - 12x + 1 < 0$ and so the original $\frac{dy}{dx} > 0$ i.e. positive.

This tells me that the turning point at x = $6 - \sqrt{35}$ is a minimum point as the gradient of the graph changes from negative to positive through zero at this point.

For $x > 6 + \sqrt{35}$, $x^2 - 12x + 1 > 0$ and so the original $\frac{dy}{dx} < 0$ i.e. negative.

This tells me that the turning point at x = $6 + \sqrt{35}$ is a maximum point as the gradient of the graph changes from positive to negative through zero at this point.

I can also deduce from the information about the turning points that there will be an intercept with the x-axis between the two turning points at x = $6 - \sqrt{35}$ (minimum point) and x = $6 + \sqrt{35}$ (maximum point), and another intercept with the x-axis beyond the (maximum) turning point at x = $6 + \sqrt{35}$. Unfortunately I can't deduce where these intercepts are exactly from this information (I need my numerical methods for this), but I now know that there will be a total of three intercepts with the x-axis and no more.

Student 1

Thinking. Can I get a rough idea where the x-intercepts are? I know one is definitely at x = 0. What about the other two?

Reasoning. The second intercept with the x-axis is approximately twice the distance (assuming approximate local symmetry of the graph)(*explain why this is possible*) from the origin to ($6 - \sqrt{35}$, 0) i.e. $2(6 - \sqrt{35}) = 0.1678$ (4 s.f.) compared to the actual value of 0.1690 (4 s.f.).

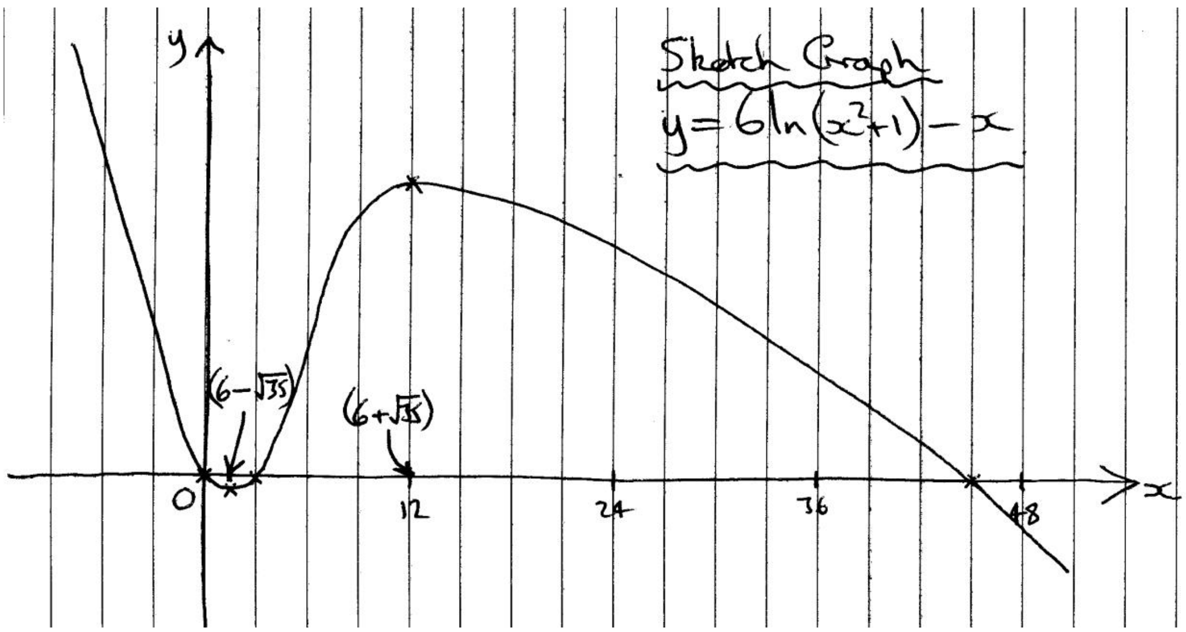
Student 2

Thinking. What about the third intercept which has a larger value of x?

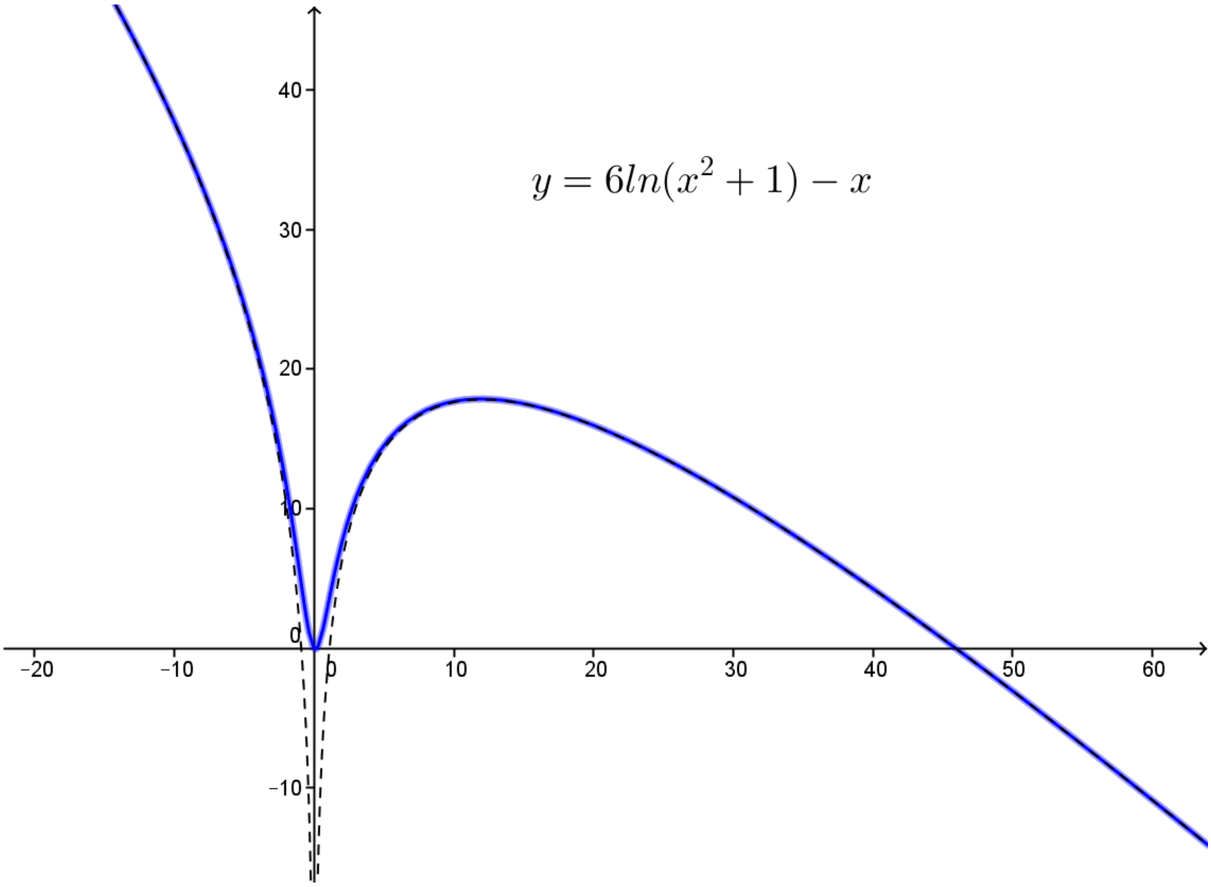
Reasoning. For x with large modulus, $y = 6\ln(x^2+1) - x \to 12\ln|x| - x$ (*explain this including why you have introduced the modulus function here*) , so our graph will approach the graph of $y = 12\ln|x| - x$ for x with large modulus. For positive x, the equation $12\ln x - x = 0$ has a root between 36 and 48 (*explain why*) which is much closer to 48 than 36 (I previously showed the actual root is x = 45.93 (4 s.f.).

Student 1

Thinking. Finally I can sketch my graph using the intercepts with the axes, the turning points and the behaviour for large, positive values of x.



The student reasoning here is very good but you could probably improve it or add to it (*or ask your students to improve it or add to it*). Similarly, the sketch graph is OK but could be improved (*how?*). The actual graph is below.



"Paper and pencil" curve sketching gives students multiple opportunities to develop their reasoning abilities in gradual stages, from simple graph sketching of basic functions in GCSE Mathematics or at the start of AS Level Mathematics to much more challenging problems such as this example in A Level Mathematics or those found in A Level Further Mathematics, and they do so in more than one area of mathematics. They have to link the consequences of algebraic and numerical thinking to their geometric and graphical understanding of the function whose graph is being sketched. It is this interweaving of reasoning that makes curve sketching such an important and worthwhile activity.

So why not start your next lessons asking your students to sketch and reason about related curves such as

- $y = x^2 - 4x + 1$
- $y = x^4 - 4x^2 + 1$
- $y = 4^x - 2^{x+2} + 1$
- $y = (x^2 - 4x + 1)^2$

or

- $y = sin^2 x$
- $y = \sin(x^2)$
- $y = 1/\sin(x)$
- $y = \sin(1/x)$

- y = x sin(x)
- y = x sin (1/x)

Your students don't need to be able to differentiate exponential or trigonometric functions (or know the chain or product rules) to investigate these graphs in considerable detail. You could have a "first pass" at sketching them in Y12, and return to them in Y13 for a "second pass".

Read previous *It Stands to Reason* features here.

Image credit
Page header by Juanedc (adapted),

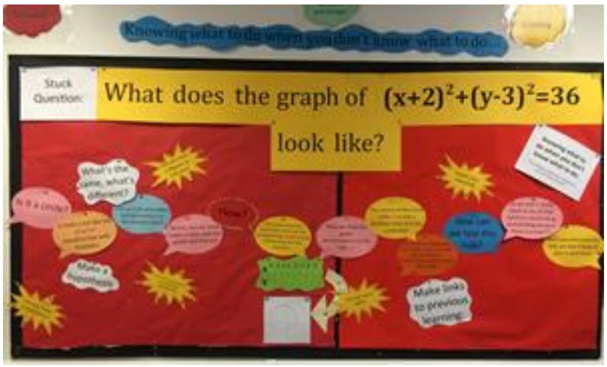
Secondary Magazine 120

National Centre for Excellence in the Teaching of Mathematics

Eyes Down

Our monthly picture that you could use with your pupils, or your department, or just by yourself, to make you think about something in a different way. You can find previous features in this series here.

What do your pupils do when they get stuck? Is there a forest of hands asking for help, or have they developed some alternative – some more resourceful – strategies? Katie Holt at South Dartmoor Community College has done some work with her Year 10 pupils on getting stuck and now has this display in her classroom *[ctrl+click image to enlarge]*. Thank you for sharing this with us Katie.



Having had some initial exploration of the equation of a circle, the pupils had been asked the question, 'What does the graph of $\mathbf{(x+2)^2 + (y-3)^2 = 36}$ look like?' The teacher recorded their responses and stuck them on the wall to model "knowing what to do when you don't know what to do".

Along similar lines, you could:

- Talk to your pupils about what they should do when they get stuck.
- Create a similar "I'm stuck – what now?" display of your pupils' strategies in your classroom. What will you use for the stimulus question?
- Make a video of your pupils talking through a problem where they get stuck, and how they respond to this, and upload this to your YouTube channel.
- Talk to your pupils about the benefits of getting stuck, and explain to them that it's when they are grappling with difficulty that they learn the most. Nobody got fitter/stronger/faster without getting sweatier first!
- Spend some of a department meeting sharing with colleagues how you manage the plaintive "I'm stuck" cry, and agree some common cross-department responses so that there is consistency between classes.
- With your colleagues look into the research of Jo Boaler and Carole Dweck, and then share this with your pupils. They need to have a growth mindset, just as we teachers do – and their parents/carers do too.

Let us know what you try, and what works well – and what doesn't.

If you have a thought-inducing picture, please send a copy (ideally, about 1-2Mb) to us at info@ncetm.org.uk, with a note of where and when it was taken, and any comments on it you may have.

Image credit
Page header by llreadll (adapted), some rights reserved