Primary
& Early Years
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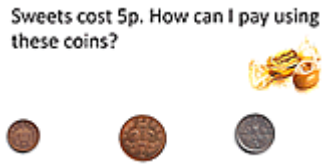
National Centre
for Excellence in the
Teaching of Mathematics

Welcome to the second half of the autumn term. In this issue, we have a thoughtful article by a primary teacher on being observed the 'Shanghai way', and a report from a primary teacher using inspiration from the local beach to create a lesson on estimation.

Don't forget all previous issues are available in the Archive.

This issue's featured articles



The potential of the Teacher Research Group to improve teaching and learning
Primary teacher Jonathan Leeming reports on the powerful CPD he experienced when he had his lesson observed, then dissected, by colleagues in a Shanghai Teacher Research Group (TRG). Jonathan, a teacher and primary Mastery Specialist from Lancaster, working with the North North West Maths Hub, has recently returned from visiting schools in Shanghai. Shanghai teachers are on the return leg of the exchange **now**. Contact your local Maths Hub to find out how you can observe a Chinese teacher at showcase events near you, between 13 and 16 November.

Using local context as a 'hook' to ignite interest and excitement
We also visit a beach in Devon to find out why Y6 pupils have been trying to calculate how long it would take to pick up the grains of sand, one grain at a time, and ask how you might put together a 'Fermi problem' such as this in your local context

And here are some other things for your attention:

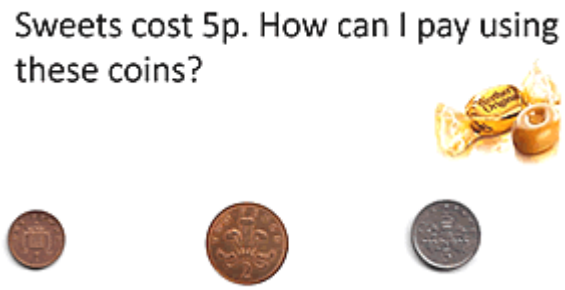
- The NCETM has started to publish professional development materials to support primary teachers introducing a teaching for mastery approach in maths. Written by a team of Mastery Specialists, developed under the Maths Hubs Programme, and other primary maths experts, the materials are not, in any way, lesson plans, but they can be used in conjunction with a high-quality mastery textbook to support planning.
- Are you interested in gaining the NCETM's recognised designation as a leader of maths-specific professional development for primary teachers? The NCETM, working with Maths Hubs, is looking for teachers and others with relevant experience to take part in a programme this academic year, leading to the award of status as an NCETM Accredited PD Lead. More details and application form are here; the closing date for applications is **14 November**.
- Bursaries have been announced to help teachers attend next year's British Congress of Mathematics Education (BCME), taking place in Warwick from 3 to 6 April 2018. It's a four-yearly event where all the maths subject associations combine their annual conferences, with delegates from across the phases and high-profile plenary speakers

The potential of the Teacher Research Group to improve teaching and learning

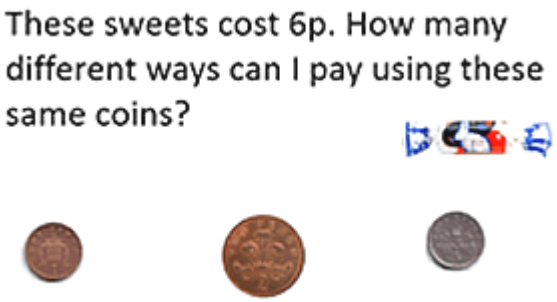
Professional development in Shanghai is largely centred around the Teacher Research Group (TRG). Groups of teachers design a lesson together and then observe one member of the group teaching. The meeting which follows, allows the teacher to reflect on the lesson and for colleagues to provide feedback. All then contribute ideas to improve the design and delivery of the lesson. Key principles can be incorporated into teachers' practice beyond the lesson itself. This is the structure of professional development that the NCETM's Primary Mastery Specialists are now using to work with groups of schools in England.

Jonathan Leeming, a primary Mastery Specialist from Lancaster, working with the North North West Maths Hub, was part of the latest group of teachers to take part in the China England Teacher Exchange in September 2017. As part of the visit, he had the chance to design and deliver a lesson which formed the focus of a TRG meeting. In this article, he reflects on his experience and on what the model can offer for teacher professional development.

"The lesson didn't quite work, but I am not entirely sure why," was the feedback from an English colleague, after I had taught a problem solving lesson to a grade 3 (year 4) class in Shanghai. Bam!!! The next teacher to give feedback, a Shanghai teacher, seemed to nail what had gone wrong with the lesson. The feedback focused on the mathematics and was supportive, frank and thorough.



The lesson focus had been for children to work systematically or "in an ordered way" through a problem. The problem involved English money. Children were introduced to 1p, 2p and 5p coins and asked how they would pay for a sweet that cost 5p. Children's responses were shared using a visualiser and then they were asked if they could find all the different ways that they could pay. Again, children's responses were shown to the class and then I explained how the problem could be solved in a systematic way. I gave one example of what non-systematic looked like and one example of what systematic looked like.



The children then excitedly set about finding all the ways that a Chinese sweet costing 6p could be paid for. From monitoring the room, it was clear that about 50% were using various systematic approaches but some were not. The result raised two interesting consequences of not following a systematic approach. Firstly, that we may not find all the possibilities and, perhaps more importantly, that it makes it hard to know if we have found them all. Examples of children's work demonstrating systematic approaches were shown and then the children were asked to discuss their predictions for how many ways we could pay if a sweet cost 7p. They then proved their predictions, found the rule and learning was summarised. The lesson ended with me introducing the 10p and asking the children to find all the ways that they could pay for a sweet that cost 13p. They completed this problem as a homework task.

The excitement throughout the lesson had been palpable, no doubt in part due to the novelty of being taught by an Englishman. Many of the children had thrived; their work demonstrated a clear systematic approach and the ability to quickly apply a general rule to a mathematical pattern…BUT… approximately 50% seemed not to have worked systematically and a few had struggled to work through the problem. So… How had I failed these children? I was not entirely sure but was hopeful to find answers in the post lesson analysis (TRG).

The TRG included nine teachers and started, as seems to be common practice in Shanghai, with the teacher of the lesson giving feedback. I explained my rationale for the approach that I had taken and that I had been happy with student engagement but was aware that many children had not fully grasped the idea of working systematically. I also explained that, in England, I would have used plastic money so that the children had the practical apparatus to help. I invited the other teachers to give advice on how I could have met the needs of the children that failed to grasp what "systematic" meant in this context. As already mentioned my English colleague gave feedback next. Then it was the turn of the Shanghai teachers…

"You needed to explore more fully what you mean by systematic and non-systematic, giving more than two examples" advised the next teacher. The children needed to "feel" what "systematic" and "non-systematic" are like. Then it would have been beneficial for the children to review their previous work and correct it using a systematic approach. Other advice included using paper copies of the coins so that the children had concrete resources to work with.

Finally, the head of the maths department shared her wisdom. She explained that when some children have not understood, we need to identify when in the lesson this lack of understanding occurred. It was clear that when I was explaining and eliciting how many ways we could pay for the 5p sweets, I did not use enough examples and I did not spend enough time comparing the examples and allowing children to experience how "systematic" compared to "non-systematic". Thus, many of the children floundered with the next problem. The advice was frank but supportive and gave very clear examples of how the lesson could be improved.

The following week I was to deliver the same lesson again but in a different school, so I redesigned it applying what I had learned from the TRG.

This time children had paper coins available to use and I gave several examples of "systematic" and "non-systematic". I also explicitly showed, with one example, what a systematic approach looked like. The outcomes, in terms of student attainment, were much improved. It seemed that all the children had used a systematic approach. It was also interesting to observe some children working with the paper coins as an aid. The lesson had improved dramatically because all children had grasped the learning intention.

So what?

This was an incredibly powerful CPD experience on many levels. It highlighted some apparent differences between the English and Shanghai approach to lesson observation and feedback. In 20 years of class teaching in five different schools, I can only remember being observed three times by peers, yet I have been observed countless times by senior leaders. While the feedback from these leaders has been kind and constructive, it has, at times, felt judgmental and has often been linked to appraisal. The feedback in Shanghai, however, felt more supportive and less threatening (even though it involved eight professionals). It also seemed to be more frank at times. For instance, the Shanghai maths lead said that she wouldn't discuss the positive aspects of the lesson because they had already been mentioned and went straight in with how it could be improved. No "positive sandwich" here!

Crucially, because Shanghai teachers are used to analysing lessons in detail, they seem better equipped to identify areas of weakness and to know how to resolve them. My English colleague and I, with collectively

over 40 years of teaching experience, struggled to identify how to resolve the weakness of the lesson yet, to our amazement, the Shanghai teachers "nailed" it. Having the opportunity to teach the lesson again and seeing the improved outcomes demonstrated the validity of their feedback.

Returning to England, I not only feel better equipped to lead TRGs, but I am also acutely aware of their potential for improving Teaching and Learning.

Using local context as a 'hook' to ignite interest and excitement

Y6 at Woolacombe Primary School have been asking:
"How many grains of sand on Woolacombe beach? … And how long would it take to pick them all up?"

Why it's important to link maths to the environment around us

When a child learns to read, practice carries on outside the classroom as the child can't help but read signs, notices and adverts around them. Perhaps if we can develop children's ability to interpret the world around them mathematically, then we will see this effect with mathematics too.

When we ask mathematical questions about the world around us, maths becomes a tool for finding things out, rather than something that only happens in school.

Dabbling with maths outside the curriculum boundaries, particularly when stimulated by their desire to answer a question or find something out, allows children to taste that there is a whole exciting body of maths beyond the curriculum, waiting to be explored.

By provoking, developing and nurturing children's natural excitement about learning we stand the best chance of helping the child to engage with education and to become a lifelong learner.

Consideration should be given to the context to make sure that the numbers are manageable by the children and that they have the skills, strategies and equipment to handle the calculations.

Woolacombe School Y6 teacher, Dan Polak, tells their story

I love it when children become obsessed about an idea from one of our lessons. A child obsessed with finding answers and applying their understanding of mathematics to enhance their understanding of the world, is a child who knows why we learn maths in the first place.

We look at this beach every day so it gives us a great opportunity to build obsession by asking questions about something familiar. It is possible to ignite interest which builds to obsession when we present mathematics which tells us something more about the world.

We started speaking in class about the idea that there are more stars in the sky than grains of sand in the world. It started the discussion about how many grains of sand might be on Woolacombe beach.

To create a series of lessons to explore this, I needed to carefully consider what my year sixes knew which could help them, and what structures needed to be in place to aid their thinking.

The question I posed was:

"If I picked up one grain of sand per second, what year would it be when I finished?"

Of course, the caveat was that I never took breaks and lived forever! (And for the purposes of this exercise, leap years were excluded)

I decided to give them the first piece of information that the 'sandy' part of the beach was three metres deep so each square metre was the end of a cuboid (1m × 1m × 3m) comprised of around 163 billion grains of sand. I also told them that the area of Woolacombe Beach is 2.4 million square metres.

The maths Dan's class did…

1. **Calculate the number of grains of sand on the beach using Area of beach × number of grains of sand in each cuboid = 163 billion × 2.4 million.** This involved some pretty hefty understanding of the relative size of numbers, some complicated long multiplications, even with the use of equivalent calculations.

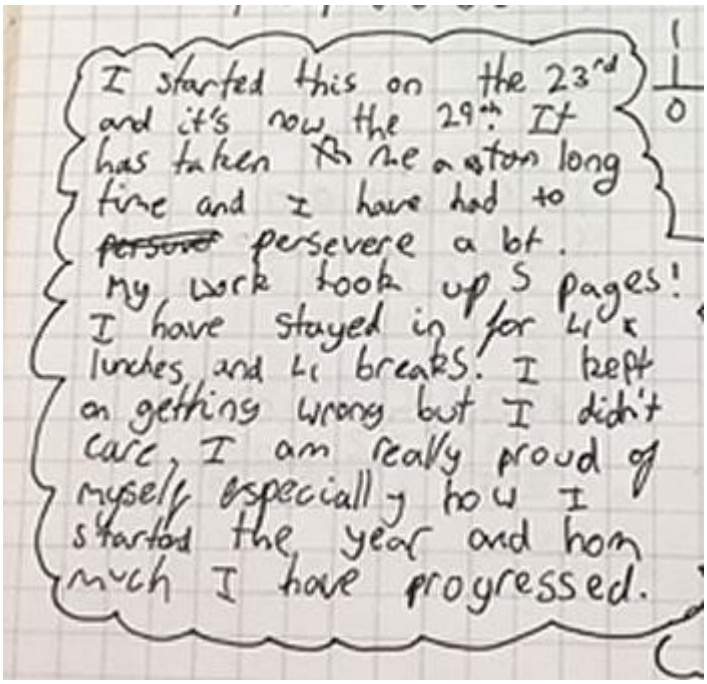
2. **Convert the number of grains (i.e. the number of seconds) into years.** Plenty of extra maths can be generated here. How many seconds have you been alive? What about family members? Can you live to be a billion seconds old? Will anyone ever be three billion? Some ask me whether they can work on this for a bit, and since the general principle of the maths is the same, I encourage them to follow their questions.

3. **Add 2017 to the number of years by dividing by 31 536 000 (the number of seconds in a year)**, giving the year 12 404 872 641 as the answer.

Using Dan's starting point, his class got involved in all sorts of complicated calculations as well as grappling with the concepts of very large numbers, and of time and the difficulties of converting seconds to years. Their knowledge of long multiplication and division, as well as their ability to use equivalent calculations (e.g….) was drawn upon and practised in a context that made sense to them. Whilst the size of the numbers used goes way beyond the curriculum at Y6, and calculating with them is something that would normally suggest the use of a calculator, Dan was able to allow these children to be led by their enthusiasm to handle maths way beyond expectations. As he says:

> *"The brilliant thing about this task was that there was a huge buzz in the room. There is a real sense of curiosity- so the difficulty of the task goes out the window, and the children are busy working out how they can solve this question. This is obsession - it might take a very long time but we will get there* ***because we want to****."*

In the words of one child:



"We spoke about one the most amazing concepts, which is how big these numbers really are. Most children know millions and billions at year six, but the fact that a billion is a thousand times bigger is illustrated dramatically when we think about time. ***A million seconds is just over 11 days, a billion seconds is 31 years.****"*

Dan used time on the carpet to support children in their methods. "I started them all off together and left them for ten minutes to identify for themselves whether they had a strategy to solve this. I then started the first multiplication on the carpet. I normally operate an 'open-invite' approach to guided maths. I give the class an opportunity to chat about how to solve it and then start solving it myself shortly into the lesson. I might say something like "I'm going to start this on the carpet, if you want to see my strategy, come on down." I might have already identified some children who need support and ask them to join us if they don't volunteer!"

So many children asked what the world would be like in the year 124 050 723. This gave us a huge opportunity for more maths, as humans have only been around for a tiny fraction of this time. This gave us more far questions about human history and time than I'd thought about when I first started thinking about this lesson sequence.

Curriculum areas that the children drew on, developed or learned about:

- multiplication
- multiplying and dividing by multiples of ten
- volume
- estimating
- relative size of large numbers
- conversion between different units of time.

Alison Hopper, one of the NCETM's Assistant Directors (Primary) comments on curriculum areas covered:

Once children are fluent in using formal calculation strategies, they need somewhere to apply their learning. In this case Dan looked beyond the classroom to find a context in which to apply learning of multiplication and division strategies to solve an exciting and challenging problem. The fact that the problem was rooted in the school's environment provided an extra level of engagement for the children.

Using representation to break the problem down in to manageable stages made sure that it was accessible to all children in the class - in this case, breaking the beach into sections which the children could visualise easily.

Children were required to draw on other aspects of their mathematical understanding: to consider time, using their knowledge of the relationship between units of time. Fluency in calculations, both mental and written, was required to manage the conversions.

In this particular problem, the numbers went beyond the expectations of the primary classroom but children are often fascinated by really big numbers and the names (both mathematical and not) that we use to label seemingly unimaginable amounts. In this case, those amounts had a physical representation on the beach that the children can see from their school, helping the children to move their learning into the real world. Having said that, the scale of the problem needs to be considered to check that the task is a reasonable one for the children to tackle and that the numbers are appropriate for the strategies that you want them to apply.

Estimation, rounding, application of mental and written calculation strategies and finding the structure of calculations in the world around them are all skills that we want children to have and rooting problems in the children's environment is a way of providing the opportunity to develop them.

Fermi Problems

The problem that Dan set is an example of a Fermi problem. Fermi problems, named after the physicist Enrico Fermi, are problems that involve estimation and reasoned guessing about values that are not known. A famous example is to estimate the number of piano tuners in Chicago. Ideas for using Fermi problems with primary school children can be found on the Fractus Learning and National Council of Teachers of Mathematics websites.

So, what about that beautiful tree by the school gate? How many ants might be living under the bark? Could you pick all the daisies on the school field in a day? Do a thousand buses pass the school gate in a week? Are there a million bricks in the building? How many times round the field/playground do we have to go to get to [nearest city]/Paris/New York/to climb Mount Everest? How many playtimes would it take to score as many goals as are scored in the Premiership in a year?