Secondary Magazine 107

National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 107 of the Secondary Magazine

In this issue of the Secondary Magazine our series of *Key Ideas* articles continues with a focus on statistical reasoning which is also featured in our linked set of statistical reasoning problems. Elsewhere there are some timely things to do as mathematics teachers. As the green shoots of daffodils and crocus are coming through outside, this issue contains some ideas to stimulate your professional learning.

Contents

Image credit

Page header by David Saddler (adapted), some rights reserved

From the editor: Catch them doing something good

It is that time of year when the mock examinations have been sat, papers marked and results analysed. You as teachers are carefully designing intervention and revision activities designed to help every pupil reach their potential. Pupils have a range of reactions which include:

relief - at having got the mock examination out of the way
pleasure - having achieved success at this stage in their education
determination - to tackle the issues that have been identified through the examination
despair - having received confirmation that they are no good at maths - as they'd always thought
fear - of the size of the problem they have to tackle
reluctance - to summon up the amount of grit and determination to overcome the problem.

I'm sure you can list more reactions from your pupils. The author of our Tale in Issue 106 stated that:

> *getting back a mock paper is probably one of the biggest 'home-truths' a student will have to accept during the run-up to exams and likely to have a considerable impact on their self-esteem.*

It is a tricky balance for the mathematics teacher between giving pupils a realistic picture of where they are now and enabling them to move forward to achieve their potential when they sit their real GCSE

Most people, pupils and teachers, like compliments - they especially like compliments when they are true! As a teacher, finding things to compliment pupils on is vital in managing their self-esteem and the relationship between teacher and pupil. Sometimes this is easy – the pupil plays in a school team or has a part in the school play. At other times we may need to dig a bit deeper to find the subject of our compliment.

Giving compliments to pupils can also be a way of focusing their attention on the things that we value as mathematics teachers. So alongside the 'well played last night Josh' sort of comment, there can also be comments like 'some good work on equivalent fractions yesterday Ben - great stuff' or 'I really think you know how to solve problems using Pythagoras now Tara - well done'. We need to catch them doing something good mathematically and celebrate that achievement.

Some of the compliments will be verbal; the opportunity to have eye contact with a pupil is valuable. Other compliments will be written in exercise books; whether your school has a WWW/EBI (what went well/even better if) system or a PAR (praise action response) system, there is room for a written compliment that boosts pupil self-esteem alongside a point to take forward their learning. Some teachers celebrate these interim, small gains on an achievement wall; other schools give pupils places in their books where they can collect their compliments. This may not work for you – but please share your ideas.

Catch them doing something good today!

Secondary Magazine 107

National Centre for Excellence in the Teaching of Mathematics

Key Ideas in Teaching Mathematics – Statistical reasoning in Key Stage 3

In this and other issues, the Secondary Magazine will feature a set of six articles, written by Anne Watson, Keith Jones and Dave Pratt, the authors of the recent publication Key Ideas in Teaching Mathematics. While not replicating the text of this publication, the articles will follow the themes of the chapters and are intended to stimulate thought and discussion, as mathematics teachers begin to consider the implications of the changes to the National Curriculum. This article is the third in the series and focusses on Statistical reasoning in Key Stage 3. Future articles will feature Place Value, Algebra and Probabilistic Reasoning. Previous articles focussed on similarity, ratio and trigonometry in Key Stage 3 and Geometric and spatial reasoning in Key Stage 3.

In 2009, Hal Varian, Chief Economist at Google and emeritus professor at the University of California, predicted, "I keep saying the sexy job in the next ten years will be statisticians…The ability to take data – to be able to understand it, to process it, to extract value from it, to visualise it, to communicate it – that's going to be a hugely important skill in the next decades, not only at the professional level, but even at the educational level for elementary kids, for high school kids, for college kids. Because now we really do have essentially free and ubiquitous data. So the complementary scarce factor is the ability to understand that data and extract value from it."

The above quote emphasises the importance of statistical reasoning in professional lives. We might add to the argument the critical skills needed in evaluating media claims on for example the impact of eating certain foodstuffs, in making personal decisions about for example whether to have an operation, and in seeking to understand policy statements that affect the society we live in. It is not difficult then to see the teaching and learning of statistics as one of the more important and relevant subjects in school.

Hal Varian's quote above also succinctly captures statistics as a process, sometimes referred to as the data handling cycle. Some researchers have called this the investigative cycle and include identifying the system dynamics of the problem, designing how to measure the identified variables, collecting the data, analysing the data including generating hypotheses and finally interpreting in order to communicate conclusions.

Research consistently argues that statistical thinking lies in that process and, whilst knowing facts such as ways to calculate various types of average or spread, how to draw a pie chart or how to conduct a t-test are important, they are subsidiary to appreciating the cycle as a process within a problem context. Engagement with data, seeking to extract value from it, is key. Some have argued that this type of thinking is quite distinct from mathematical thinking and that, if the nature of proof is one of the hallmarks that distinguishes a discipline, proof in statistics is very different from that in mathematics. Nevertheless, the teaching of statistics will in the foreseeable future lie with the mathematics teacher, though there is a need to make the distinction between statistical and mathematical thinking explicit.

Traditionally statistics as a discipline is divided into what are sometimes referred to as the descriptive and the inferential. Descriptive statistics is important when one merely seeks to capture and convey information about the whole set of data available. This is especially relevant when all or very large sets of data are available as in a census. Inferential statistics is important when one seeks to draw a conclusion about a population based on a relatively small set of data. The essential elements of measurement, collection, analysis and interpretation apply in both 'games' – the descriptive 'game 'and the inferential 'game'. However, the detail of the data handling cycle is different depending on which type of 'game' one is playing. For example, in the inferential 'game' one might apply one or more of a battery of tests to evaluate whether the sampled data is unlikely to have been generated were a certain hypothesis true. Such tests are not relevant in the descriptive 'game' and research has shown that it is important that

teachers and students are aware which of these two 'games' are being played when trying to address any particular problem.

The programmes of study for statistics in the National Curriculum and in the GCSE subject content are structured around these two types of statistics. At Key Stages 1 to 3, the emphasis is very much on descriptive statistics with statements at Key Stage 3 such as: 'describe, interpret and compare observed distributions'; 'construct and interpret appropriate tables, charts and diagrams'; 'describe simple mathematical relationships between 2 variables'. The GCSE content includes statements such as: 'infer properties of populations or distributions from a sample' alongside the use of descriptive statistics.

There is a considerable challenge to understanding in moving from the descriptive to the inferential, which many students will not meet unless their experiences at Key Stage 3 have prepared them. Because of the importance of statistics to future professional lives and in everyday engagement as a citizen and because of the sudden increased expectation at Key Stage 4, we would argue that, notwithstanding the competing claims of other topics and disciplines, mathematics teachers at Key Stage 3 need to introduce students to the inferential 'game' through the data handling cycle whenever possible.

Research shows how this might be done. There has been a substantial body of statistics education research development in the last ten years which points to the pedagogic value of what has become termed 'informal inferential statistics' (IIR). IIR almost always draws on modern software developments. Indeed much of the research has been conducted using TinkerPlots, as an especially well-designed application for younger students, though much of the work could be done through other tools. The key idea in IIR is for even very young students to play the inferential 'game' through the data handling cycle. Students might be challenged with a problem, designed to pique their curiosity, which will require them to collect data and draw inferences. For example, asking how tall younger students might be when they are fully-grown can lead to the collection of growth statistics and their interpretation. Alternatively, students might be given access to a set of data about which the students pose questions and explore. For example, see CensusAtSchool.

One pedagogic approach developed by researchers is referred to as 'growing samples'. Students might be challenged to decide the answer to a question, such as 'How many hours of television per week do children in your year watch?' on the basis of asking fellow students who sit near to them in class. The variation in responses across different groups in the class could be discussed. The sample could then be grown by joining groups and then further by collecting data across the whole class. The process could continue by amalgamating classes until a census of the year was achieved. At each stage questions could be asked about how confident the students were in their answer. Once the results for the year were collected (when confidence should be 100%), the question can be changed to ask about the whole school or even town. Similarly using CensusAtSchool, samples of gradually increasing size could be taken to note the effect of sample size on the confidence with which the inferences can be drawn. This pedagogic approach places emphasis on the inferential 'game' in a way that is accessible to quite young students (it has been trialled in primary as well as secondary schools) because it does not require the knowledge needed to conduct formal tests as in classical inference.

In this short article, we have proposed the need to teach beyond the requirements of Key Stage 3 statistics on the basis that statistics is extremely important and relevant and yet the Key Stage 3 curriculum will not prepare students for the demands of inferential statistics at Key Stage 4. We have pointed to the research on informal inferential statistics as an approach that makes it feasible to address inferential statistics through the data handling cycle at quite a young age.

Keith Jones, Dave Pratt and Anne Watson

In keeping this series of articles brief, there is no space for full references; these can be found in the book Key Ideas in Teaching Mathematics

Image credit
Page header by Jim Rush, some rights reserved

A resource for the classroom – Statistical reasoning problems

This issue of the magazine has an article linked to the recent publication, Key Ideas in Teaching Mathematics. There is a website that accompanies the book which provides links to some relevant resources. Our article this month is related to reasoning about data, so the resources for the classroom are a suite of problems which have been selected to develop reasoning about data. Some of these problems may be familiar whilst others may be new to you; all have been chosen to develop and deepen understanding.

The website identifies seven themes within the context of reasoning about data which it lists as inference, variability, sampling, context, signal and noise, graphs and re-sampling, and states:

> *Statistical reasoning draws on contexts that relate not only to mathematics but to most disciplines in the school curriculum. It is an invaluable tool for rooting out possible causes and associations.*
>
> *There are clear connections with mathematics. For example, variability is a key characteristic of phenomena that students are trying to describe, or model, or simply to comprehend. Variability occurs when considering mathematical functions and other entities that might be described algebraically or geometrically. However, variability that might be modelled statistically has added layers of complexity because the variability may (or may not) be attributable to a number of causes.*
>
> *It is important to develop particular ways of reasoning statistically as a variation on mathematical variability, as well as a tool for making sense of data in other disciplines, or to improve statistical literacy generally.*

The individual problems are:

- How Faithful is Old Faithful?
- Who Has the Heaviest Backpacks?
- Fish-Length Distributions
- Crime Scene Evidence
- Using Random Samples of Real Data
- Data Visualisation
- Data With No Name
- Active Graphing
- Visual Inference Tools (VIT).

What will you do now?

You could:

- select a problem and try it out with a particular class
- select a problem and work with a colleague to consider how you can use the problem to develop understanding for a group of pupils
- include some of these problems in your scheme of work
- consider how these problems develop the powerful aspects of the curriculum and the links between them.

Do tell us what you find out...

Image credit
Page header by Simon Cunningham (adapted), some rights reserved

5 things to do

The NRICH website features the problem, Estimating Time, which has a useful estimation tool. This might link well to some of the other data reasoning problems featured in this issue.

You may like to think about making links with the Science department in your school and running a trip to the Cheltenham Science Festival 2014. It runs from 3 - 8 June, with special sessions for schools 3 - 6 June.

Registering for the EDF Energy website The Pod gives you access to educational resources. You might consider holding your own Waste Week 2014.

The Key Ideas in Teaching Mathematics article mentions the use of TinkerPlots. The free version allows you to run 20-minute sessions.

If you like these interesting patterns Physical Movement Translated into Symmetrical Charcoal Drawings, you may also be interested to read Case Study 8: Mathematics Across the curriculum – performing arts in the publication Engaging mathematics for all learners.

Image credit
Page header by Alan Cleaver (adapted), some rights reserved

Tales from the classroom: Even better if…

I was fortunate to be observing an interview lesson when I overheard a student talking (shouting) about the marking in his book. It was nothing whatsoever to do with the lesson, just that it was the first time he had opened his book since it had been marked.

“Yessss, I actually got a maths ebi!”

The interviewee was mid-flow, so it seemed an inappropriate time to follow up – however, my interest had been captured.

I know I’m pretty poor at keeping on-task. Half my students like that because they enjoy hearing about educational stuff that probably isn’t exactly on the usual curriculum, or part of a particular learning objective but nonetheless gives them an insight into wider maths and life. The other half of my students hate it, because it isn’t exactly on the usual curriculum, or part of a particular learning objective and they are not interested in an insight into wider maths and life. Whatever the standpoint, I knew that at some point during the interview lesson I would “slip” off-task, and find out the reason for that comment. It was inevitable. And surely, as it pertained to a comment in a maths book, it was my duty to know more…

Well sure enough, slip I did. And I’m glad I did. It not so much opened a can of worms, as re-kindled an ember I thought extinguished. It forced me somewhere I had been before, and will no doubt have to venture again.

So Fraser’s comment…

"Yessss [pleasure, also indicated by clenched fist being pulled down]

I [it’s never normally me]

actually [I never thought it would happen]

got a maths [it wasn’t expected to happen in maths of all places]

ebi!" [even better if]

The ensuing conversation…

Me: tell me about the marking in your book Frazer

Frazer: Oh yeah. Sorry ‘bout that.

Barney: Yeah, he always gets a bit excited Sir.

Frazer: Yeah, but not normally in maths!

Laughs all around - me with irony, them more genuine!

Me: Well…

Frazer: Well, you see, since the start of term all our teachers have started those "ebi" things again and 'cos my writing is a bit…well hard to read…all, well nearly all, by "ebi's" have been about my writing and presentation, not actually about the work I have done. I already know it's hard to read, and like I'm not trying? Actually getting an ebi that is about doing maths and that I might actually be able to do something about…

Barney: He won't though Sir 'cos he just forgets everything anyway…

Laughs all around again - this time: me genuine, Frazer, I fear, more irony!

And for me, that was it. Again. Why do I (we) mark? What am I getting out of it? What are my students getting out of it?

But then, who is deciding how I mark? What are they getting out of it?

A big part of this for me is that it exposes a certain lack of modesty that I don't normally like to reveal - they're my kids, I'm their teacher, and for the past x years in my school there's only one (or perhaps two) teachers that are close to me in terms of progress of their students – and certainly no department in the last couple of years can touch us. As a more mature teacher I now know that ignominious rush of emotion comes firstly because I just don't like being told what to do and that is just the way I am, but secondly, because if I am being told what to do, I want to be very, very sure that the person that is telling me what to do has thought about it at least as hard as I have. It is the progress and future of my students they are meddling with! However, as a more experienced practitioner, I also know not to cast aside a diamond amongst the coal. With the rush of emotion ebbing, but modestly still far-flung, I'm beginning to doubt myself… "is there something in this that can push me yet further?"

So back to square one - a square that uncannily appears adjacent to all other squares…

I mark because I am charged with so doing…well of course, but it is more than that.

I mark because it is important that students know what is correct…still more than that.

I mark because I can exert an opinion over students' performance…its deeper than that.

I mark because it is the crucial planning of the next learning…that's definitely a massive and vital part, but of a different vein (I could just look and not write for that...).

I mark with feedback because that takes students further than they would think to go alone…getting nearer.

I mark with feedback because it initiates a dialogue that allows students to reflect on how they have learnt…that is more like it, but it's not enough.

I mark with feedback because it continues a learning dialogue that allows students to reflect on their learning and initiates them to take action to progress…that would be great, but I don't think I am there yet…

Of all the instructions over the years perhaps the most useful came last week. From a teacher with all of one year's experience: "You say your feedback opens a learning dialogue – it might for you, but how do you know it does for them? Why don't you ask them to feedback on your marking – now that would be a dialogue!" Touché! And happily wounded I was. Where fresh eyes see new flesh!

So having danced on square one for a while where was I now? Well, had I enthusiastically adopted all the instructions and advice over the years, I would be a circling skipping rope. However, having that inner dialogue and frequently revisiting it has allowed me to skip enthusiastically. Watching the rope and jumping as it comes, whilst remaining a full part of the game. Sometimes I shout the accompanying rhyme. Other times, I just skip quietly. All the time asking myself is this marking helping my students learn…and now, I might actually ask them too.

So, I will continue to think, that for me, alongside marking being crucial to my planning, a genuine learning dialogue is a vitally important part of marking. Thanks to Frazer, perhaps a little bit from sidekick Barney and of course that immature yet experienced teacher, I will now take more care to attempt to make sure that the dialogue is useful to my students too – by asking them.

I will also thank square one for always being there and always being adjacent to all other squares. So as I now step from it, I do so to the square that is two greater than the square I originally left.

The author is a mathematics subject leader and assistant principal working in the South West