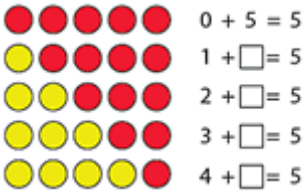
Primary
& Early Years
Magazine 87



National Centre
for Excellence in the
Teaching of Mathematics

Welcome to another issue of our Primary Magazine, which lands electronically on your desks just when Year Six are embarking on their National Curriculum tests. May we wish them all the very best, and pay tribute to all the teachers, over the last six years who've got them to this stage of their school journey. Meanwhile Issue 87 of our magazine offers a varied collection of articles related to maths education and mathematics professional development - all of which are available in the Primary Magazine Archive. This month we have slightly reorganised the magazine, introducing two new sections: Digging Deeper and Aspects of…

Contents

In Digging Deeper each month we will explore an element of mathematics teaching linked to current developments and research: in this issue we look at ideas around planning.

Aspects of… will provide a number of bitesize ideas related to a specific element of mathematics: this month the topic is using understanding of cardinal value.

We continue with our Seen and Heard feature, which provides a specific example of a child's response to mathematics in a classroom to stimulate thinking and provoke questions about how you would react to similar events in your own classroom. In this issue we consider understanding of the place value of decimal numbers.

If you have a photograph, or an account of a classroom conversation, that might stimulate similar thought, please email it to us. If we publish your suggestion, we'll put a £20 voucher in the post.

Next month we will have articles focused on learning table facts and understanding of place value. But first, as always, we have a News section, bringing news from the NCETM and beyond to keep you up to date with the fast-changing world of mathematics education.

News

The NCETM has announced details of the recruitment of a second wave of **teachers** to become specialists in mastery approaches to teaching primary mathematics. 140 primary teachers will be selected to join the Mastery Specialists Programme, run by the NCETM in conjunction with Maths Hubs. Each Maths Hub will select four teachers to take part in the development phase of the programme during 2016/2017, following in the footsteps of the same number recruited a year ago.

Information about this second wave of recruitment and the application form can be found here. The closing date for returning applications to the NCETM is **1 June 2016**.

At the same time, details of a separate process led by Maths Hubs, have been announced, to recruit **primary schools** to work with the first wave of Mastery Specialists, who are now approaching the end of their development year. In 2016/2017, each of the Mastery Specialists will work with six schools in his/her Maths Hub area, to develop teaching for mastery approaches in those schools. This is a significant opportunity for the professional development of teachers in these schools, for which there will be no charge. In addition, a small subsidy will be available towards release time for these teachers. Each group of six schools will form a Work Group as part of the wider Maths Hub programme of teaching for mastery activities.

Information and details of how to apply to take part in this valuable collaborative school development opportunity can be found here. Applications must be returned to your Maths Hub by **8 June 2016**.

The Standards & Testing Agency (part of the DfE) has now produced exemplification material to support teacher assessment at the end of KS1 for 'working towards the expected standard' and 'working at greater depth within the expected standard'. There is no requirement for teachers to use the materials: they have been produced to support teachers as necessary

All teachers of mathematics (including those in the early years and primary phases) are being invited to complete a survey that seeks to gather evidence on their professional learning journey, from the time they entered the profession to the present day.

The survey comes from the independent Advisory Committee on Mathematics Education (ACME) as part of its project to produce new guidance for teachers and those responsible for teachers' professional development. The survey remains open until mid-May. Read more

More or Less (Radio 4's programme about numbers in the news) broadcast an episode on 24 April which included a chance to hear about the UK team on their recent journey to Romania to compete against 38 other teams from Europe in the European Girls Maths Olympiad (EGMO). The episode can be accessed via BBC iPlayer.

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Digging Deeper

Planning in Mathematics

As teachers are now familiar with the content of the National Curriculum, there is an opportunity to take stock and consider some key principles around planning, underpinned by the aims of the National Curriculum and in line with the demands of a mastery curriculum.

The NCETM Teaching for Mastery booklets state:

> *'Mastery of facts, procedures and concepts needs time; time to explore the concept in details and time to allow for sufficient practice to develop fluency' and 'one set of mathematical concepts and big ideas for all. All pupils need access to these concepts and ideas and to the rich connections between them'.* (Pages 5 and 6)

Therefore, planning starts with identifying the focus for a learning journey; this is the big idea or ideas, rather than individual statements from the programmes of study, which will be the focus for a teaching sequence. One of the challenges when planning is to shift away from a focus on coverage of the individual statements within the programmes of study, and instead focus on the key mathematical ideas which are the reason for the learning, giving purpose to the programmes of study. For example, for a Year 3 teaching sequence on fractions teachers might choose to consider the 'big ideas' in the NCETM Teaching for Mastery booklet and decide that the focus for the sequence will be the children demonstrating understanding of fractions as equal parts of a whole, in different contexts where the equal parts might look different but are still the same and understanding of fractions as numbers and where they fit in the number system. This would connect together more than half a dozen statements from the programmes of study, ensuring children make 'rich connections' between mathematical ideas.

It is important to ensure that the aims of the National Curriculum are also reflected in the planning and one way to do this is to be explicit about the expectations of how children will demonstrate understanding. The checklist on page 7 of the Teaching for Mastery booklets provides some support with this:

A pupil really understands a mathematical concept, idea or technique if he or she can:

- describe it in his or her own words;
- represent it in a variety of ways (e.g. using concrete materials, pictures and symbols – the Concrete-Pictorial-Approach (CPA)[1])
- explain it to someone else;
- make up his or her own examples (and non-examples) of it;
- see connections between it and other facts or ideas;
- recognise it in new situations and contexts;
- make use of it in various ways, including in new situations[2].

A clear focus for a teaching sequence can be shared with children so that they know what they are working towards and are able to assess their own understanding at the beginning, during and at the end of a sequence. The use of elicitation tasks to find out what the children already understand provides

valuable information for planning the sequence (for more information see Maths in the Staff Room from Issue 85 on assessing progress using elicitation and application tasks).

When thinking about and planning the learning journey there are decisions to be made. Key decisions are summarised below and exemplified for a sequence focussed on number sense in Year 4

- Focus of the teaching sequence (linked to big ideas for number and place value in Teaching for Mastery in Y4):
    - Children are able to make appropriate decisions about when to use their understanding of counting, place value and rounding for solving problems including adding and subtracting.
- Key conceptual ideas and making connections:
    - What are the steps in understanding needed along the journey?
    - What is the best way to order these steps?
    - How are these steps going to be connected?
    - How is this journey going to be connected to prior learning
      Possible Steps:
        - Place value – naming of numbers, value of digits – adding and subtracting (e.g. 1240 – 40 uses understanding of place value), reading numbers in different ways, importance of zero
        - Counting linked to addition and subtraction - +/- 10, 100, 1000 with four-digit numbers
        - Ordering and comparing numbers and fitting them onto a number line, including identifying mid-points of marked intervals
        - Rounding – build on fitting the numbers onto a number line
        - Negative numbers – extending the number system – fitting negative numbers on to a number line
- Difficult points:
    - What are the common misconceptions within this area?
    - Which parts are difficult to teach and difficult to learn?
      For example:
        - Naming the parts of a number without appreciating the value of the parts
        - Understanding a number only in relation to the columns; for example understanding 1240 only as one thousand and two hundred and forty
        - Focussing on one column when considering 'How many…are there in?'; for example responding to 'How many tens are there in 1240?' with '4' rather than '124'
        - Crossing boundaries particularly when adding ten to or subtracting ten from a number where the hundreds or thousands will change; for example 3996 + 10, 4703 - 10
        - Fitting numbers onto a number line taking account of other numbers and the scale of the line and identifying mid-points; for example identifying that halfway between 3400 and 3500 is 3450
        - Identifying how close numbers are to each other
        - Linking understanding of place value to addition and subtraction
- Representations:
    - Which models and images will best support understanding of the different parts of the journey?
    - Which models and images will expose the difficult points and misconceptions and support understanding in these areas?

    - Which contexts will support the children to make sense of the maths and give the maths meaning and purpose?
    - What language will the children be expected to make sense of and use?
    - How will the children be expected to represent their thinking and understanding at different points on the journey?
        - Models and images - place value arrow cards, place value counters, base ten, place value mat, place value chart, number lines
        - Contexts – data from topic area, crowds at events, reporting crowds in newspapers (rounding), money
        - Language – support using sentence starters (for example 'I have noticed that…', 'I know that…') and sentence stems (for example 'The whole number is one thousand two hundred and forty; one thousand is a part of the number and two hundred and forty is a part of the number).
- Variation:
    - How can variation be used to support the understanding of the structure of the mathematics?
    - What needs to be varied to expose the difficult points and misconceptions?
    - How can variation be used to ensure depth of understanding?
      For example, for place value it might include:
        - Representing a number with an image; for example 1240 using a one thousand block, two hundred flats and four ten sticks.
        - Linking representation with one image to other images; for example explaining what is the same and what is different about 1240 with base ten, with arrow cards and with place value counters
        - Recording an addition to match the way the number has been represented with an image, for example 1000 + 200 + 40 = 1240
        - Identifying numbers which are easy to subtract and explaining why, for example 1240 – 240
        - Explaining how a number can be read in a different way, using an image to support the explanation, for example explaining that 1240 can be read as one thousand two hundred and forty but it can also be read as one hundred and forty tens and using base ten to explain where the one hundred and forty tens can be seen.
- Going deeper
    - What opportunities are there for demonstrating creativity and imagination?
    - What contexts would provide opportunities to explore and generalise about the mathematics at a deeper level?
        - Comparing the structure of the base ten number system with other number systems and considering how negative numbers could have been recorded in these systems
        - Compare the structure of the base ten number system with other bases and consider what is different, for example, how does it change what is easy to subtract from a number?

The resulting plan should be a flexible model which helps to shape the teaching sequence but is responsive to ongoing formative assessment.

Wherever possible, it is beneficial for teachers to plan together; planning in this way with other teachers provides a meaningful professional development experience as teachers discuss and grapple with the elements above. Including teaching assistants either in the planning process or through sharing the

planning with them will support them in their role. In particular, if the teaching assistants understand the purpose of the teaching sequence, the likely difficult points and the models and images which would be best for supporting the understanding at these points, they will be better placed for supporting learning rather than supporting children to complete tasks.

Next time in Digging Deeper we'll explore elements of understanding place value in the context of decimal numbers.

[1] The Concrete-Pictorial-Abstract (CPA) approach, based on Bruner's conception of the enactive, iconic and symbolic modes of representation, is a well-known instructional heuristic advocated by the Singapore Ministry of Education since the early 1980s. See here for an introduction to this approach.

[2] Adapted from a list in 'How Children Fail', John Holt, 1964

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Aspects of…

Cardinal Value: six ways of looking at six (plus one for luck)

(The number six is the focus of this article, but it could have been any single-digit number)

1 Defining cardinalilty

> *An understanding of cardinality is built on an ability to subitise and attach number names to small numbers and includes recognition of the fact that these small numbers can be partitioned in a variety of ways including previous number +1, leading to an understanding of the relationship between successive numbers, as well as being able to say the number name for the whole set.*[1]

Cardinality is a key idea in KS1 and underpins understanding of number and calculation.

2 Six and its close friends - relationship between numbers

Children need to understand that when counting forwards they are adding one and when counting backwards they are subtracting one; if they don't understand this they will not understand the relationship between consecutive counting numbers. A child in Y1 who says 'I know that five add one is six because six comes after five when I count' has made this connection. Laying out Numicon plates or ten frames in order from one to ten and exploring how they change when moving along the line left to right and then right to left can be used to support understanding of this relationship and can be connected to written calculations, for example 6 = 5 + 1 and 6 = 7 – 1, and the position of a number on a number line (six lies between five and seven). Children should be able to explain the relationship in familiar contexts, for example 'there were five children in the classroom and then one more arrived, so there were six children altogether'.

3 Six without counting

Subitising is recognising the number of items in a small group without counting and is something humans, like many animals, are born able to do. Most people can subitise up to three or four things. Iconic representations are arrangements of items in a familiar way which makes the number evident without needing to count; for example, the arrangement of dots on dice and dominoes. This allows groups which are bigger than four to be recognised without counting and often brings an understanding of ways to 'compose the number'; for example the representation of six on dice models the three plus three composition of six.

4 Give me six

Understanding of cardinality is crucial not only when counting to find out 'How many?' but also when asked to create a set of a given number; for example asking a child to get six spoons from a bigger collection rather than giving the child six spoons and asking them 'How many spoons are there?'. When creating a set there is no action to imitate in the way that reciting number names and pointing can be imitated and the child has to decide

when to stop counting. It gives a good indication of whether cardinality has been understood.

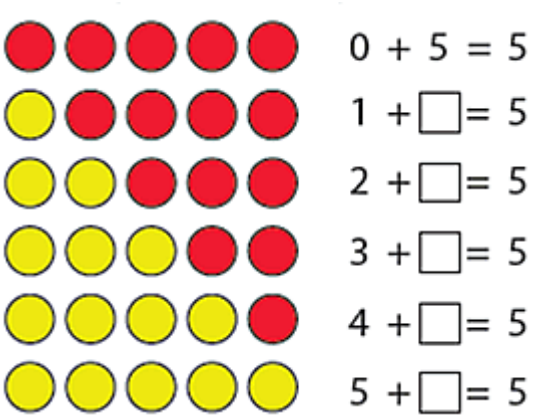
5 All (six) eggs in one basket

Children need to understand how a number can be composed of other numbers, for example

6 = 4 + 2 = 3 + 3 =…

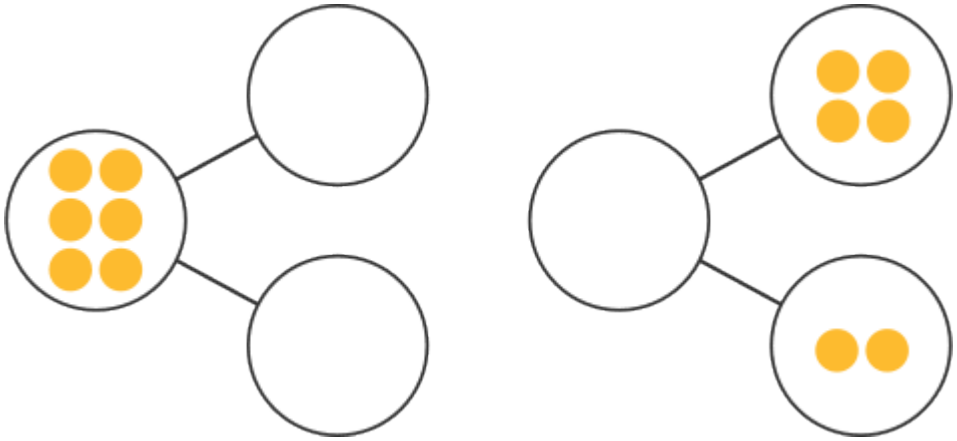
Providing a context for exploring additive composition will help children understand the relationship between the numbers and what is being modelled; examples of contexts are collecting eggs from hens, or on an Easter egg hunt, in two baskets ('six eggs have been collected, how many could there be in each basket?'), children choosing between two fruit options for their snack ('six children are choosing either an apple or an orange for their snack, how many could have chosen each fruit?') and the germination of seeds for two different beans ('we planted runner beans and broad beans and six beans have germinated, how many could there be of each type of bean?').

This could be represented in a number of ways. It is worth noting here that colour can be a major distraction for young children and can be an additional barrier which prevents them from seeing structure. If six assorted coloured cubes or counters were used for exploring the different possibilities for six children choosing an apple or an orange as their snack, then the number of each colour can mask the different ways of splitting six into two parts. To remove this barrier either use cubes/counters of one colour, a different resource where colour is not a factor when looking at six (such as a bead string), or two-coloured counters where the two colours are used to match to the two options (see below)



Teaching for Mastery Y1

The NCETM is currently exploring the use of the 'part-part-whole' model which is used in Shanghai and can be used to explore additive composition:



6 Six means six

Children need to understand that there are six in a set as long as nothing is added or removed and it doesn't matter where you start the count, it will still be six. Explore this by starting the count of a set in different places and asking the children to say what they think will happen and why.

7 Six as a springboard

The power of understanding cardinality, and in particular the additive composition of small numbers, is that it can be applied to many situations. For example, if you know 6 = 4 + 2 then you can use this for:

a. 6 – 4 - this can be modelled using images such as Numicon and the Singapore bar.
b. 24 + 2, 34 + 2, 74 + 2 etc. - the connection can be modelled with base ten, place value counters and Numicon
c. 40 + 20, 400 + 200, 0.4 + 0.2 etc. - the connection can be modelled with base ten and place value counters
d. 58 + 6, 76 + 6 etc. – the connection is exposed when modelling with a bead string
e. 74 – 6, 82 – 6 etc. – the connection is exposed when modelling with a bead string.

For more reading about cardinality….try these:

Gelman, R. and Gallistel, C.R. (1978) *The Child's Understanding of Number*, Harvard University Press
Sarnecka, B., Cerutti, A., and Carey S. (2005) *Unpacking the cardinal principle of counting: A last-word rule + the successor function*; Poster presented at fourth biennial meeting of the Cognitive Development Society (available here)

[1]Trundley R (2008) THE VALUE OF TWO article in MATHEMATICS TEACHING 211 ATM available at http://nrich.maths.org/11527

Seen and Heard

Seen and Heard *will shine a light, via photographs and conversations from classrooms, on a specific example of the mathematics learning experience, the aim being to stimulate thought and questions about how you would react to similar events in your own classroom*

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

- Why do you think these children have placed the counters where they have?
- What do they understand about decimal numbers?
- How has the language used to read decimal numbers influenced their thinking?
- What mathematical resource might you use to support the child's understanding of place value in decimal numbers?
- What might you ask them next?

If you have a thought-inducing picture, please send a copy (ideally, about 1-2Mb) to us at info@ncetm.org.uk with 'Primary Magazine: Seen and Heard feature' in the email subject line. Include a note of where and when it was taken, and any comments on it you may have. If your picture is published, we'll send you a £20 voucher.

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