Mastery Professional Development

Fractions

3.7 Finding equivalent fractions and simplifying fractions

Teacher guide | Year 5

Teaching point 1:

When two fractions have different numerators and denominators to one another but share the same numerical value, they are called 'equivalent fractions'.

Teaching point 2:

Equivalent fractions share the same proportional (multiplicative) relationship between the numerator and denominator. Equivalent fractions can be generated by maintaining that relationship through the process of multiplication and division.

Teaching point 3:

Fractions can be simplified by dividing both the numerator and denominator by a common factor.

2019 pilot

Overview of learning

In this segment children will:

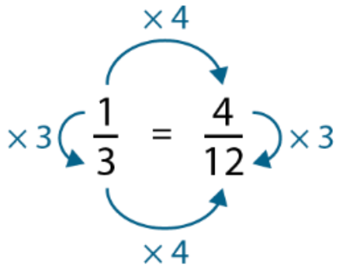
- understand that fractions may have the same value but a different appearance
- understand that having the same value means that they sit at the same place on a number line
- explore the multiplicative relationships between the denominators and numerators of equivalent fractions
- learn how to reduce a fraction to its simplest form
- apply the concept of simplification to calculating with fractions.

So far in this spine, children have met instances where one quantity can be represented by two different fractions. For example, in segment *3.3 Non-unit fractions: identifying, representing and comparing*, children may have noticed that the fraction of a shape that is shaded can be written in two ways (for example, as both $\frac{3}{12}$ and $\frac{1}{4}$). In this segment, the phrase *'equivalent fraction'* is introduced and the relationship between pairs of equivalent fractions such as $\frac{3}{12}$ and $\frac{1}{4}$ is fully explored.

It is important that teachers first understand for themselves the relevance of the proportional relationship between the numerator and the denominator. This relationship will be easier to identify when dealing with unit fractions, and it is something children will have encountered when looking at $\frac{1}{2}$ as having the same value as $\frac{2}{4}$ in previous practical contexts. Some children may have already made the link that if a fraction has the same value as a half, then the numerator is half the denominator, or similarly that the denominator is double the numerator.

The idea that it is the multiplicative relationship *between* the numerator and denominator that determines the value of a fraction, rather than the absolute values of the numerator and denominators per se, is a really challenging idea for children. Fractions are essentially multiplicative comparisons between a part (expressed by the numerator) and a whole (expressed by the denominator). Children will need to understand that it is the sizes of these *relative to each other* that are critical in determining the value of a fraction. For example, $\frac{7}{11} < \frac{5}{6}$ even though both the numerator and denominator in the first fraction are larger than the numerals in the second fraction. $\frac{7}{11}$ is less than $\frac{5}{6}$ because seven *is a smaller part of* eleven than five is of six. Consider another example: $\frac{5}{6} = \frac{10}{12}$. Here, the numerator and denominator in $\frac{10}{12}$ are both bigger numerals than those in $\frac{5}{6}$. However, $\frac{10}{12}$ is *equal* to $\frac{5}{6}$. These fractions are equal because the multiplicative relationship between ten and twelve is the same as the multiplicative relationship between five and six.

The concept of preserving the ratio between the numerator and denominator is key in equivalent fractions, and the impact on both the horizontal and vertical relationships within a pair of equivalent fractions should be explored. In the example on the next page, the numerators and denominators have both been scaled by a factor of four, but the denominator is also three times the numerator in both fractions.



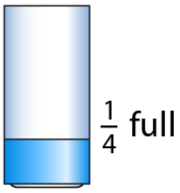
Throughout earlier segments, we have already started to draw children's attention to the relationship between a part (numerator) and whole (denominator), for example using informal language to discuss whether a part was *'quite a large part of the whole'*, or *'quite a small part of the whole'*. It is not necessary to teach the language of ratio in this segment, but vertical and horizontal multiplicative relationships will be explored.

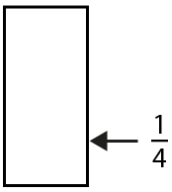
Throughout this spine, emphasis has been placed on seeing fractions as numbers. In this segment it is imperative that children not only consolidate the concept that fractions have a place on a number line, but that equivalent fractions *share the same place* on the number line. Even after children are able to identify pairs of equivalent fractions, including writing an equals symbol between them, they can sometimes still hold onto the idea that, for example, $\frac{2}{6}$ is actually greater than $\frac{1}{3}$. Placing fractions on number lines and seeing that equivalent fractions all sit at exactly the same position will help address any persistent misconceptions that $\frac{2}{6}$ is greater than $\frac{1}{3}$ because the numerator and denominator in $\frac{2}{6}$ are larger numbers. Where problems such as $\frac{12}{24} = \frac{1}{2}$ are presented, it is a prompt for children to reason using the equivalence of $\frac{12}{24} = \frac{1}{2}$, rather than employing methods involving common denominators. This segment will not deal with addition and subtraction using common denominators – that will be explored in segment *3.8 Common denomination: more adding and subtracting*.

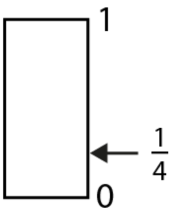
Through the course of this segment, use of a circular area model will be reduced. Children have been taught throughout this spine that fractions can be both operators (e.g. finding $\frac{2}{5}$ *of* a number) and numbers (with a position on a number line). A linear area model conveys the similarities between these two functions more powerfully than the circular area model and will also provide a good grounding for the secondary curriculum. For example, if we shade $\frac{2}{5}$ of a bar (fraction as an operator), then we can also make the bar length '1', and the end position of the shading will represent the position of $\frac{2}{5}$ on a number line (fraction as a number).

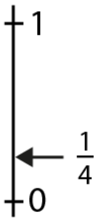
(Note that when models and diagrams are being used to help children make sense of equivalent fractions the '=' symbol should not be used to compare images. The focus of a representation can sometimes be unclear, for instance *'Is it showing that red is the same colour as blue?'* The equals symbol should only be used with numbers to ensure precision. Numbers (or letters in algebra), always refer to numeric values and thus it is clear and precise.

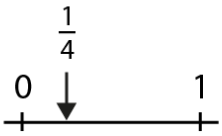
In this segment, we continue to develop children's fraction sense. Practical activities, such as pouring liquids or rice, will support this. When such practical activities include equivalent fractions, children can 'see' how pairs of fractions such as $\frac{1}{4}$ and $\frac{2}{8}$ represent the same part of the whole. A pictorial representation, such as a measuring jug, can act as an intermediate step to estimating on an empty number line, both vertically and horizontally. It will help children to recognise how pairs of fractions that represent the same part of the whole, also have the same value.









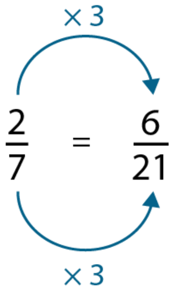


Be aware that because children have already experienced multiplying a whole number by a fraction in segment *3.6 Multiplying whole numbers and fractions*, such as $\frac{2}{7} \times 3 = \frac{6}{7}$, there is potential for confusion.

The idea that when we find an equivalent fraction, we are multiplying the numerator by three *and* we are multiplying the denominator by three, but we are *not* multiplying the fraction by three, can be quite challenging. Through your teaching, you will need to ensure that children understand the fundamental differences between these two concepts:

- $\frac{2}{7} \times 3 = \frac{6}{7}$

  ($\frac{6}{7}$ is 3 times larger than $\frac{2}{7}$.)



($\frac{6}{21}$ is *not* 3 times larger than $\frac{2}{7}$. It has the *same value* as $\frac{2}{7}$.)

Throughout this segment we will discuss multiplying and dividing both the numerator and denominator, as well as discussing scaling the numerator and denominator by the same factor. This language is one tool which can support children to recognise the distinction. The language of *'scaling both the numerator and denominator by the same factor'* and knowing that this preserves the value of the fraction can be helpful.

Once children have learnt about equivalent fractions and are able to identify whether fractions are equivalent, the remainder of the segment introduces expressing fractions in their simplest form. This will then be applied to previous learning from segments *3.4 Adding and subtracting within one whole, 3.5 Working across one whole: improper fractions and mixed numbers*, and *3.6 Multiplying whole numbers and fractions*, which focused on calculating with fractions. This presents an ideal opportunity to revisit prior learning, allowing children to apply their newly-acquired skills of simplification.

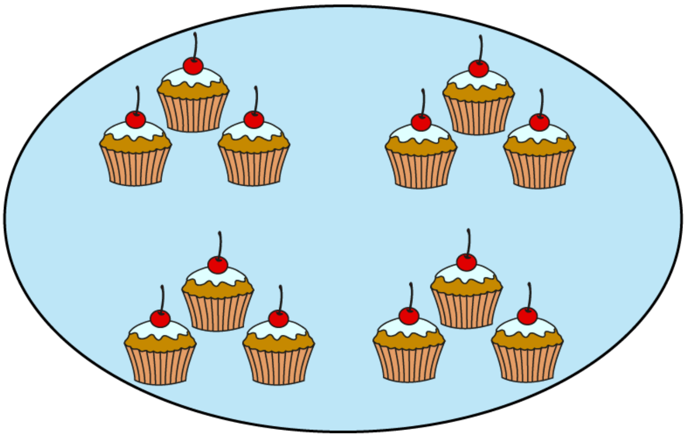
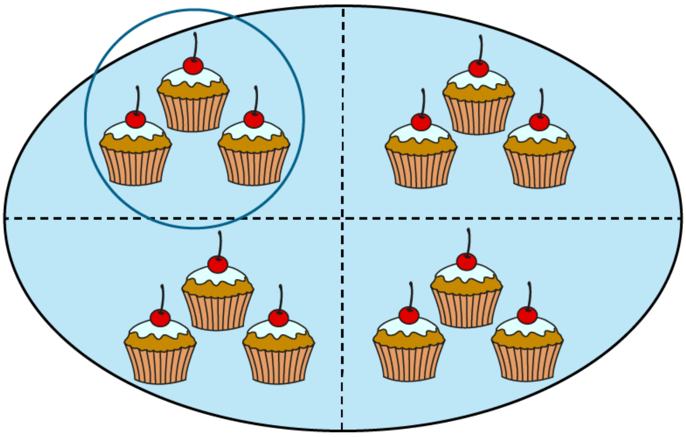
Learning throughout this segment is very much reliant on children having a solid grasp of multiplication and division facts. Ensure this knowledge is secure for all children before starting this segment. Children will also draw on their understanding of factors and multiples from *Spine 2: Multiplication and Division*.

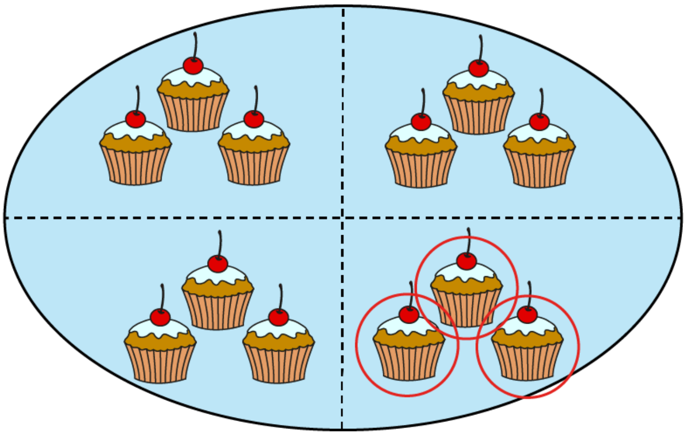
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

When two fractions have different numerators and denominators to one another but share the same numerical value, they are called 'equivalent fractions'.

Steps in learning

	Guidance	**Representations**
1:1	In segment *3.3 Non-unit fractions: identifying, representing and comparing*, the children learnt how to identify a fraction of an amount with the support of the stem sentence: **'The whole is divided into ___ equal parts and we have ___ of them.'** Start this segment by revisiting this concept. Present the children with a scenario or 'story', such as the example given opposite. Display the image of the tray of cakes and ask children to come to the board and circle the cakes that Sabijah eats. They then circle the cakes Jess eats. Children will be very familiar with using the stem sentence. Encourage them to apply it to the examples you provide. It is important to draw attention to the detail in the stem sentences. You might do this by asking children probing questions, such as: - *'Where is the four?'* - *'Where is the one?'* - *'Where is the twelve?'* - *'Where is the three?'*	*'Sabijah and Jess go to the café. They share some cakes.'*  - *'Sabijah eats $\frac{1}{4}$ of the cakes.'*  - *'The whole is divided into four equal parts and one of the parts is circled.' (Sabijah)*

		- *'Jess eats $\frac{3}{12}$ of the cakes.'*  *'The whole is divided into twelve equal parts and three of the parts are circled.' (Jess)*
1:2	Look again at the story: *'Sabijah and Jess go to the café. They share some cakes.'* - *'Sabijah eats $\frac{1}{4}$ of the cakes.'* - *'Jess eats $\frac{3}{12}$ of the cakes.'* To explore children's understanding, add further detail to the story, such as: - *'Sabijah says, "Jess ate more than me!".'* - *'Jess says, "We ate the same amount."'* Challenge the children to decide who is correct: Sabijah or Jess? Show the two fractions opposite and ask children to compare them using one of the symbols <, > or = to make the statement true. This might be a good point to ask the children to discuss the story, images and statements with a partner. Encourage them to justify their answer. - Some children may refer to the picture to prove that Sabijah and Jess ate the same amount. - Some children may refer back to their fractions of quantities work, and say	$\frac{1}{4}$ ○ $\frac{3}{12}$ $\frac{1}{4} = \frac{3}{12}$

that as $\frac{1}{4}$ of 12 is 3, and $\frac{3}{12}$ of 12 is also 3, they ate the same amount.

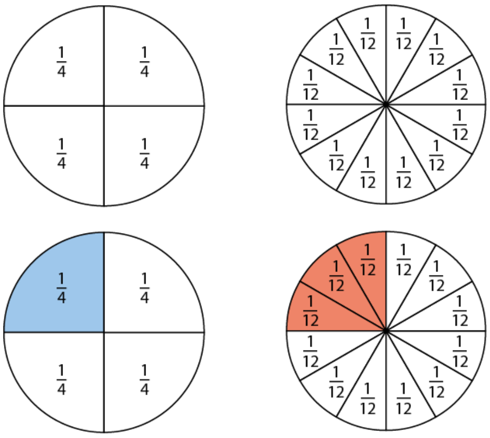
- Some children may look at the numerals in the fraction $\frac{3}{12}$ and (incorrectly) think that $\frac{3}{12}$ must be more than $\frac{1}{4}$ because the numerator and denominator are larger numbers, and so Jess ate more.

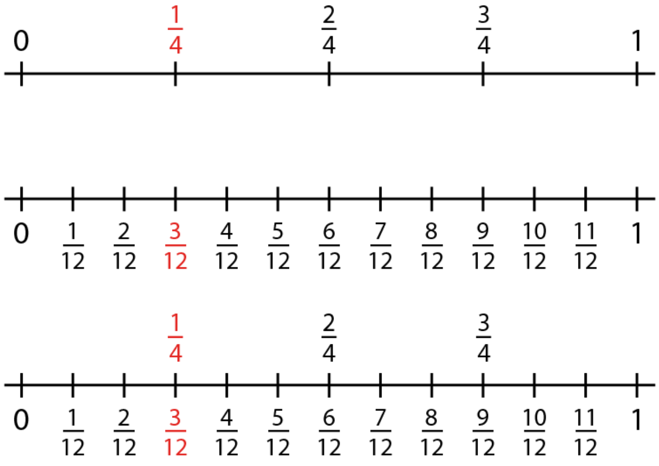
Refer back to the image and establish that Sabijah and Jess have eaten the same number of cakes. $\frac{1}{4}$ of the cakes is the same number as $\frac{3}{12}$ of the cakes. Reveal the '=' symbol to show that the two fractions are equal.

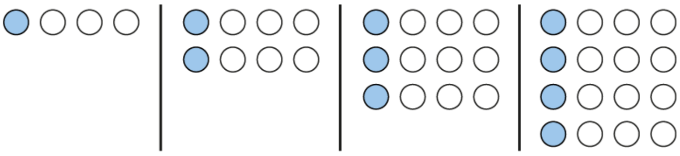
Whether or not the misconception arises in your class, discuss why someone might reason that Jess (who ate $\frac{3}{12}$ of the cakes) ate more than Sabijah (who ate $\frac{1}{4}$ of them). We can't tell just by looking at the size of the numerator and denominator whether the fractions are equal or not. Explain to children that here, $\frac{1}{4}$ and $\frac{3}{12}$ have *the same value, but a different appearance.*

At this point, some children may identify other fractions with the same value, such as $\frac{2}{8}$. Although equivalent, this is not easy to see in the image, so don't rush on to generating other equivalent fractions just yet. The visual stimulus is still vitally important for children to be able to 'see' fractions with the same value.

(Note that the language of equivalence is not yet used. At this stage, children are encouraged to observe that quantities can be expressed by more than one fraction.)

	You may wish to use a practical activity to reinforce this, such as a 'people maths' activity. For example, invite 12 children to sit down at the front of the class. Say to the group *'You are one whole.'* and then ask a child, *'What are you?'*. If needed, prompt the answer *'one-twelfth'*. Ask each individual child what they are, and involve the rest of the class in this questioning. Then say to the group, *'One-quarter of you stand up'*. Allow the group to discuss, without comment from the rest of the class, noting that they need to split the group into four parts with an equal number of children in each part. One of those four parts must then stand up. Notice that this is three children and remind them that individually they are each still $\frac{1}{12}$. Equivalent expressions can then be generated, such as $\frac{1}{4}$ and $\frac{3}{12}$.	
1:3	Using the same fractions of $\frac{1}{4}$ and $\frac{3}{12}$, ask children to shade and compare area models. (A circular model is used here, but through this segment we will start to phase this out in favour of a linear area model. The linear model offers stronger links between fractions as operators and fractions as numbers.) Repeat the stem sentence: ***'The whole is divided into ___ equal parts and we have ___ of those parts.'*** Again, children will see that the fractions $\frac{1}{4}$ and $\frac{3}{12}$ represent the same proportion of the circle. Spend some time really unpicking this and checking that children are comfortable with the idea that $\frac{1}{4}$ and $\frac{3}{12}$ represent the same part of the whole, even though they have different numerators and denominators. One of four equal parts is the same proportion as three of	 $\frac{1}{4} = \frac{3}{12}$ - *'The whole is divided into four equal parts and we have one of those parts.'* - *'The whole is divided into twelve equal parts and we have three of those parts.'*

	twelve equal parts, and because they represent the same proportion, they have the same value. We saw this in our 'cakes' story, and we can see this on the area model. (Note that the '=' symbol is only used to compare the numbers $\frac{1}{4}$ and $\frac{3}{12}$, and *not* to compare the images. The equals symbol should only be used with numbers to ensure precision. With images it may be unclear what is being compared, for instance, *'Is the "=" suggesting that red is the same colour as blue?'*. Numbers always refer to numeric values and so are precise.)	
1:4	So far, children have compared $\frac{1}{4}$ and $\frac{3}{12}$ using a quantity and an area model. Finally, compare these two numbers on a number line. Children will by now be very familiar with seeing fractions on number lines. Look at number lines showing quarters and twelfths separately to begin with, and then combine them. Emphasise that one quarter and $\frac{3}{12}$ sit at the same position on a number line as they have the same value: $\frac{1}{4} = \frac{3}{12}$. Spend some time looking at this combined number line and discuss what it tells us about the numbers on the number line. At this stage, there is no need to look at 'converting between' the two fractions by multiplying numerators and denominators. We can tell they are equal because they sit at the same point on the number line. Notice other pairs of fractions that sit at the same place on the number line. They sit at the same place on the number line because they have the	 $\frac{1}{4} = \frac{3}{12}$

	same value. For example, $\frac{2}{4}=\frac{6}{12}$. (Children may also recognise the equivalence of $\frac{1}{2}$ to these two numbers as well.) Also look at $\frac{3}{4}=\frac{9}{12}$. You may choose to present some or all of the following challenges or discussion points: • Ask half the class to count up in twelfths and the other half to count in quarters, making sure that the equivalent fractions are said at the same time. Then swap over. • Identify the smallest and largest numbers marked on the number line ('0' and '1'). • Identify the smallest and largest non-whole numbers ($\frac{1}{12}$ and $\frac{11}{12}$). • Notice how, for the numbers nearer zero, the numerator is a small part of the whole (denominator), and for the numbers nearer one, the numerator is a large part of the whole (denominator). • For the numbers near the middle of the number line, the numerator is about half of the whole (denominator).	
1:5	Now look at these same fractions using an image, such as the one opposite. Ask children to identify fractions where the same value is represented. Again, do not yet refer to the language of equivalent fractions, but encourage them to see how the quantities can be expressed by more than one fraction. For example, $\frac{1}{4}$, $\frac{2}{8}$, $\frac{3}{12}$ and $\frac{4}{16}$ all have the same value.	

1:6

Children have now explored the concept of fractions expressed in more than one way. They have done this using the three models of quantity, area and number line. Next, repeat steps *1:1–1:4* with a different fraction pair, such as $\frac{1}{2}$ and $\frac{5}{10}$. The detail of the progression is not repeated here, so refer back to the previous steps.

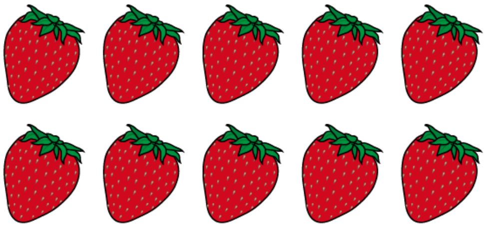
Children could be encouraged to invent their own fraction 'story', using an image, such as the strawberries opposite, and the fractions $\frac{1}{2}$ and $\frac{5}{10}$.

As a starting point, they may wish to follow the structure of the Sabijah and Jess story.

As before, use the stem sentence to reinforce the concept: ***'The whole is divided into ___ equal parts and we have ___ of those parts.'***

Use the number lines to emphasise that $\frac{1}{2}$ and $\frac{5}{10}$ are equivalent numbers and that they share the same position on the number line. Children may notice that in $\frac{5}{10}$ (and in $\frac{1}{2}$), the numerator is half of the denominator.

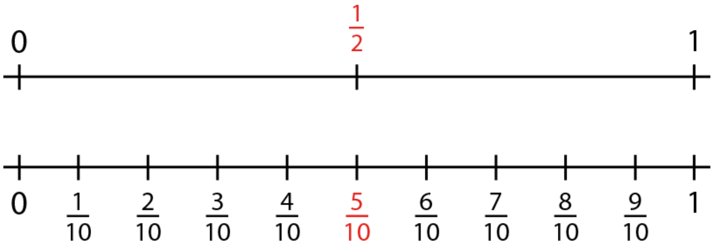
Quantity model:



Area model:

- *'The whole is divided into two equal parts and we have one of those parts.'*
- *'The whole is divided into ten equal parts and we have five of those parts.'*

Number lines:

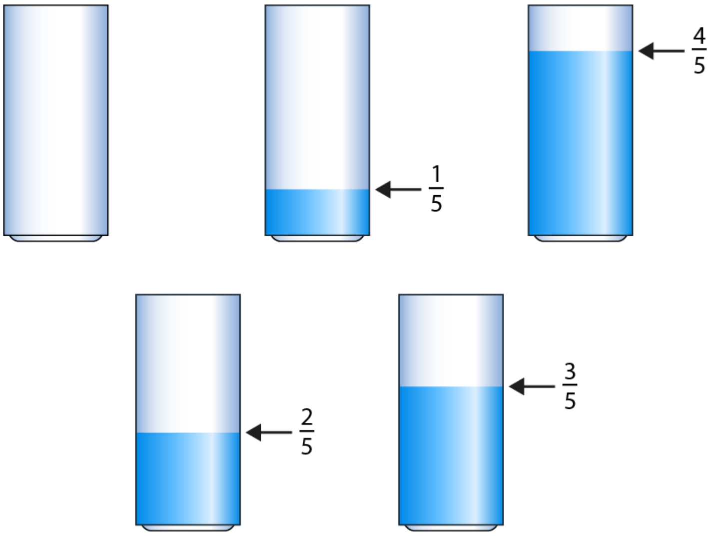


$\frac{1}{2} = \frac{5}{10}$

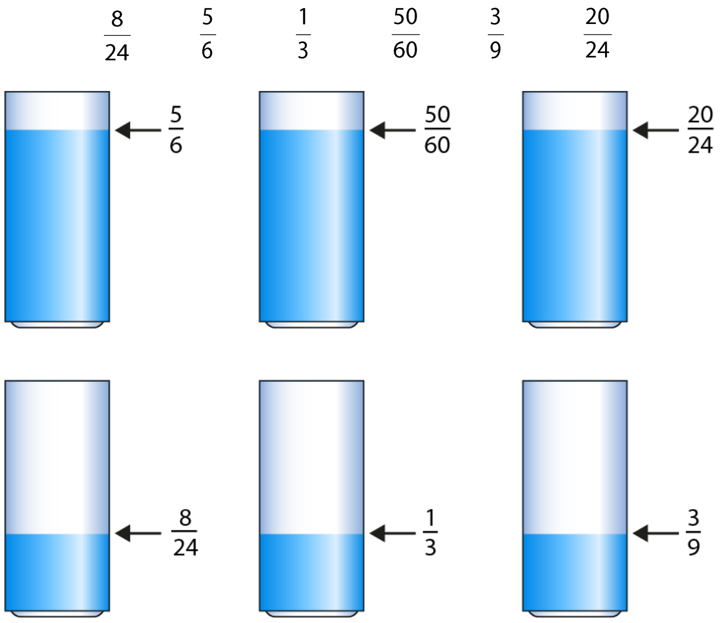
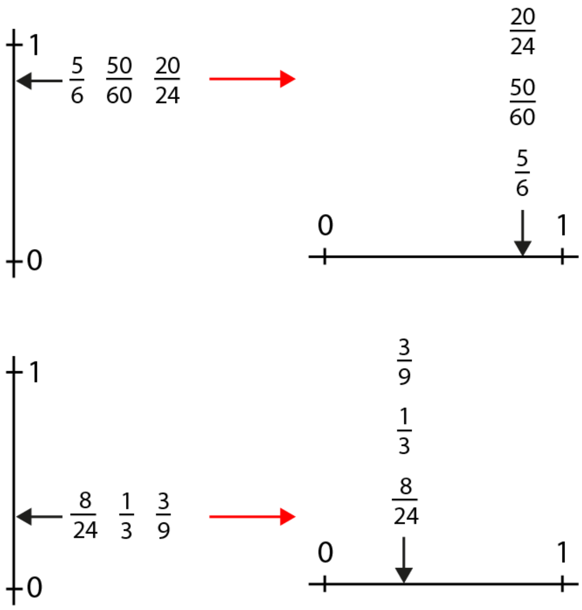
1:7

Another visual way to exemplify equivalent fractions is by returning to the measuring and pouring work from segments *3.1 Preparing for fractions: the part–whole relationship* and *3.3 Non-unit fractions: identifying, representing and comparing*.

Start by revising this work. With a transparent straight-sided jug, vase or measuring cylinder, tell the children that you are going to start pouring rice, water or squash (all work well) and that you would like them to shout *'Stop!'* when the container is $\frac{1}{5}$ full. Have a brief discussion around how they will



	know when it is $\frac{1}{5}$ full. The stem sentence may again prove useful at this point, for example, *'The whole is divided into five equal parts and we want one of them.'* Once you have poured to $\frac{1}{5}$ full, ask one of the children to sketch the rough amount onto an image on the board. As discussed in segments *3.1* and *3.3*, the pouring level indicates where $\frac{1}{5}$ would sit on a 0–1 number line. Empty the container and say you would like them to think about what $\frac{4}{5}$ full would look like. Again, return to the stem sentence. You can roughly indicate the five equal parts with your hand, but don't be tempted to literally draw them onto the container. The key is to develop proportional thinking. The aim is to get the children thinking along the lines of *'So, if the whole thing is five equal parts, whereabouts would four equal parts come in relation to five?'* Repeat the pouring exercise until the children shout *'Stop!'*. Repeat the whole process with $\frac{2}{5}$ and $\frac{3}{5}$.	
1:8	Once the children have re-familiarised themselves with pouring, progress to using pouring to exemplify equivalent fractions. You will need six equal-sized straight-sided containers (six identical glass tumblers would be ideal). The six fraction examples on the next page include three from each of two equivalent fraction families. You can either work as a class to examine each example in turn or allocate one glass and one fraction to each table or group, and ask them to estimate, pour and then bring their container back to the	

	front. Either way, you should end up with six glasses (make sure you note which container represents which fraction). Look at the six glasses. The children should notice that they can be arranged into two groups. The glasses that are $\frac{1}{3}$, $\frac{3}{9}$ and $\frac{8}{24}$ full should contain about the same amount as each other. The glasses that are $\frac{5}{6}$, $\frac{20}{24}$ and $\frac{50}{60}$ full should also all contain a similar amount to one another. As before, shade the amounts on pictures on the board and make the link to number lines. Discuss how all the fractions that represent the same part of a whole, sit at the same place on a number line.	 
1:9	In a list, write the pairs of equivalent fractions that you have encountered so far. Formally introduce the language of *equivalent* fractions. You can summarise this with the following generalisation: ***'Sometimes two fractions have the same value. We call these equivalent fractions.'*** The children may notice relationships within the numerators and denominators in each pair of fractions. This relationship is explored fully in *Teaching point 2.*	$\frac{1}{4}=\frac{3}{12}$ $\frac{2}{4}=\frac{6}{12}$ $\frac{3}{4}=\frac{9}{12}$ $\frac{1}{2}=\frac{5}{10}$ $\frac{8}{24}=\frac{1}{3}=\frac{3}{9}$ $\frac{5}{6}=\frac{20}{24}=\frac{50}{60}$

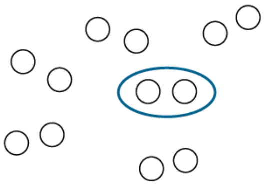
1:10

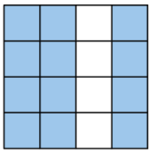
Provide practice opportunities through a variety of questions that use the following three model types:

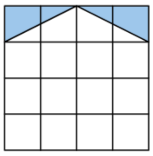
- area
- linear
- quantity

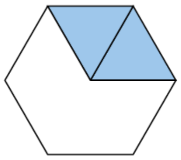
To allow children to demonstrate the depth of their understanding, present dòng nǎo jīn questions along the lines of the examples opposite.

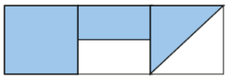
'Find different ways to write the fraction of each shape or group that is shaded or highlighted.'











Dòng nǎo jīn:

$\frac{1}{4} = \frac{3}{12}$

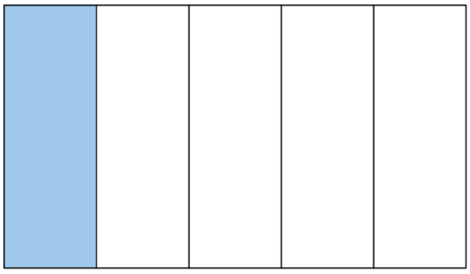
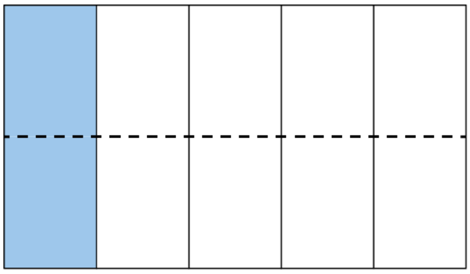
if $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$ then $\frac{1}{4} + \frac{3}{12} = \boxed{}$

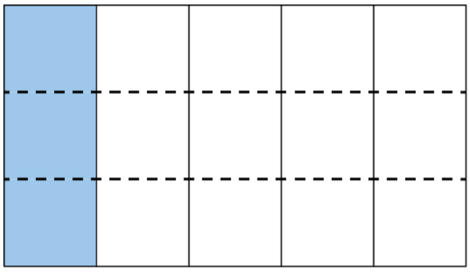
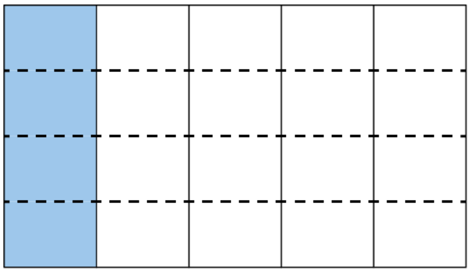
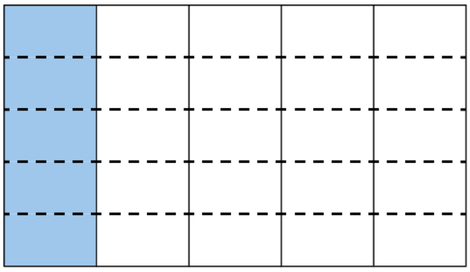
if $\frac{1}{4} - \frac{1}{4} = 0$ then $\frac{1}{4} - \frac{3}{12} = \boxed{}$

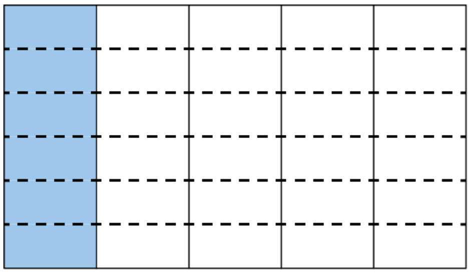
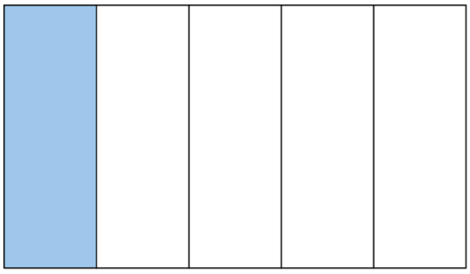
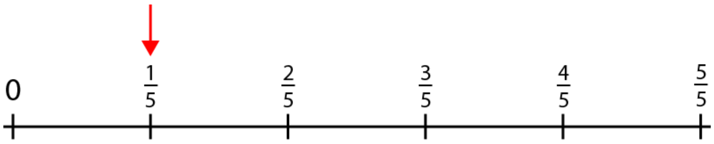
Teaching point 2:

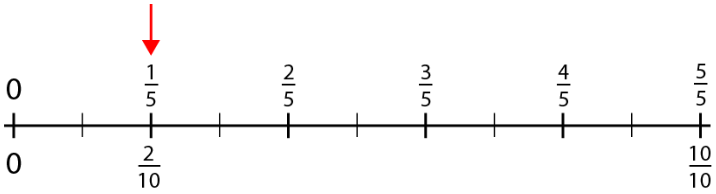
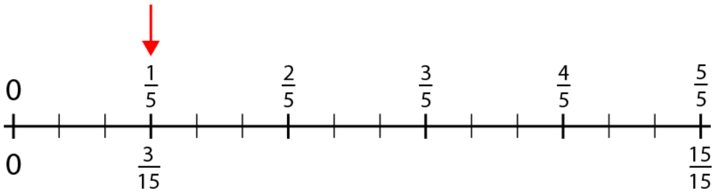
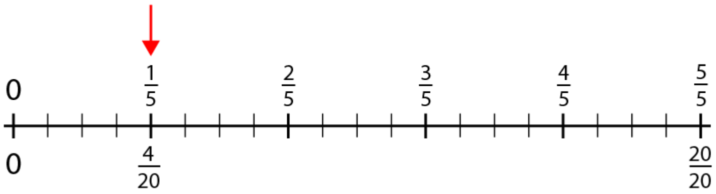
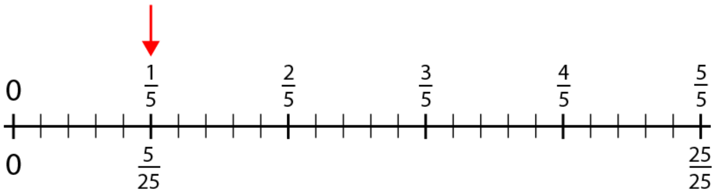
Equivalent fractions share the same proportional (multiplicative) relationship between the numerator and denominator. Equivalent fractions can be generated by maintaining that relationship through the process of multiplication and division.

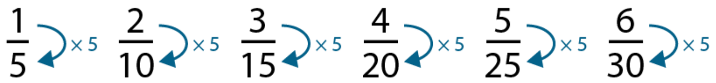
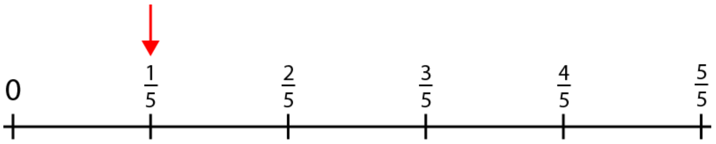
Steps in learning

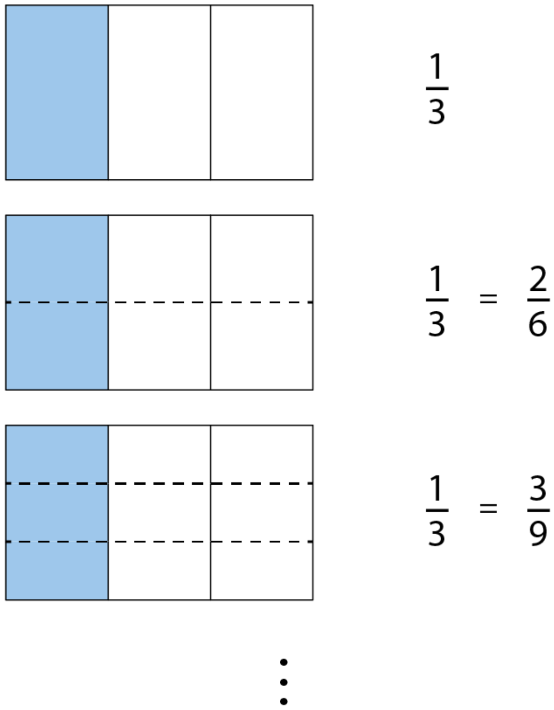
	Guidance	**Representations**
2:1	In this teaching point, we begin to explore the relationship between the numerators and the denominators in equivalent fractions. Notice the repetitive nature of the first couple of steps in securing the connections between the equivalent fractions each time. The following stem sentence, which children have met many times before, will support them throughout this teaching point: ***'The whole is divided into ___ equal parts, and ___ of these parts is/are shaded.'*** Show the children the area model opposite and establish that $\frac{1}{5}$ is shaded. Ask children to use the stem sentence to convince you of this. Further probe their understanding by asking: • *'Where are the five parts?'* • *'Where is the one part?'*	 *'The whole is divided into five equal parts, and one of these parts is shaded.'*
2:2	Progress to the image shown opposite. Notice that the size of the whole remains constant. Ask the children to discuss whether the size of the shaded area has changed. Some may be immediately comfortable in seeing that the same amount is still shaded. Others may think that because the shape is now partitioned into more parts, the amount which is shaded has changed. Using *3.7 Representations*, slide *16*, scroll forwards and backwards between the two images until all of the children are comfortable with the fact that the	 $\frac{1}{5}=\frac{2}{10}$ *'*$\frac{1}{5}$ *of the shape is equivalent to* $\frac{2}{10}$ *of the shape.'*

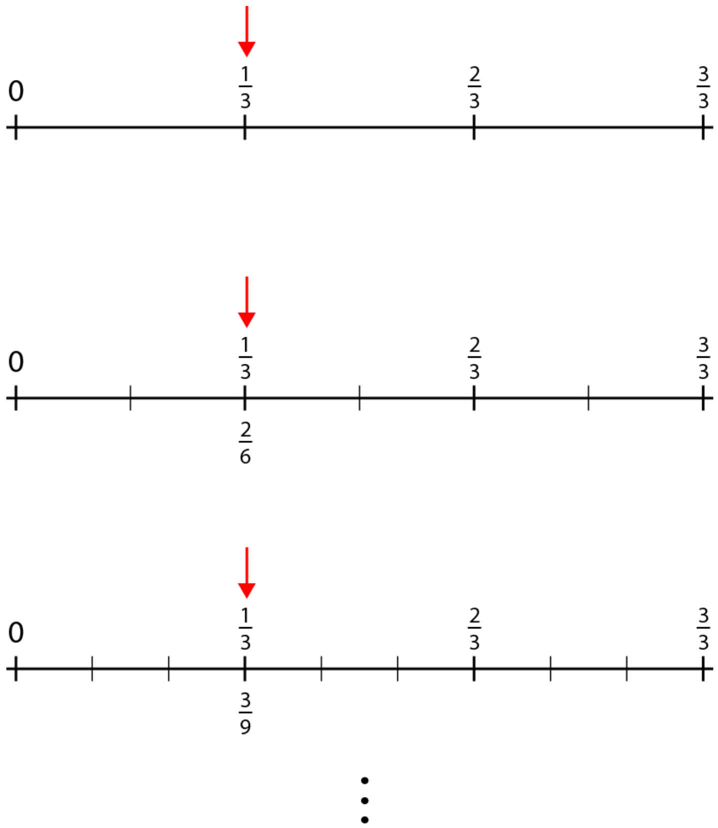
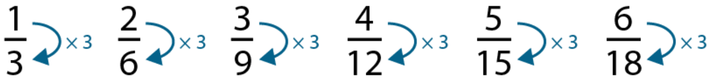
	amount of the shape that is shaded hasn't changed. Ask the children: - *'Can you still see the one-fifth?'* - *'What other fraction can you see?'* ($\frac{2}{10}$) Discuss with them how we can see five equal parts, of which one is shaded, and we can see ten equal parts, of which two are shaded. We can say that $\frac{1}{5}$ of the shape is shaded, and we can also say that $\frac{2}{10}$ of the shape is shaded.	
2:3	Display the next image on the board. As previously, ask: - *'Can you still see the one-fifth?'* - *'What other fraction can you see?'* ($\frac{3}{15}$) Examine with the children how we can see five equal parts of which one is shaded, and we can see 15 equal parts of which three are shaded. We can therefore say that $\frac{1}{5}$ of the shape is shaded, and we can also say that $\frac{3}{15}$ of the shape is shaded. Discuss as a class whether we can also still say that $\frac{2}{10}$ of the shape is shaded. Among the class there will almost certainly be arguments on both sides around this question. In fact, it *is* true to say that $\frac{2}{10}$ of the shape is still shaded. Using *3.7 Representations*, slide *16*, scroll backward and forward between this image and the previous one. If it were true that $\frac{2}{10}$ of the shape was shaded before, and neither the shaded part nor the whole has changed, then it is still true to say that $\frac{2}{10}$ of the shape is	 $\frac{1}{5} = \frac{3}{15}$  $\frac{1}{5} = \frac{4}{20}$  $\frac{1}{5} = \frac{5}{25}$

	shaded, even though it is no longer divided into ten equal parts. Repeat the questions and continue the discussion for the remaining images in the sequence: • *'Can you still see the one-fifth?'* • *'What other fractions can you see?'* ($\frac{3}{15}$, $\frac{4}{20}$, $\frac{5}{25}$ and $\frac{6}{30}$)	 $\frac{1}{5} = \frac{6}{30}$
2:4	Display the original image again, alongside all of the fractions that have been used to express the proportion of the shape that is shaded. These fractions all denote exactly the same proportion of the shape.	 $\frac{1}{5} = \frac{2}{10} = \frac{3}{15} = \frac{4}{20} = \frac{5}{25} = \frac{6}{30}$
2:5	So far, we have focused on fractions as operators ($\frac{1}{5}$ *of* the shape). Progress to looking at fractions as numbers, including where all of these fractions sit on a number line. Display a fifths number line with $\frac{1}{5}$ highlighted. Focus on what happens if the number line is marked in tenths. Using *3.7 Representations*, slide *18*, move forward to the next image, looking at the combined fifths and tenths number line. $\frac{2}{10}$ and $\frac{1}{5}$ sit at the same position on the number line. These two numbers have the same value; they are equivalent to one another. Continue to progress through the number lines on the slides, focusing each time on the fraction that is equivalent to $\frac{1}{5}$. As before, address directly the fact that although the numerator and denominator keep getting bigger, the value of the fraction hasn't changed. Check the children are	

	really comfortable with this, asking each time, *'So which number is bigger, one-fifth or two-tenths?'* ensuring they realise that neither is bigger – they have the same value.	   
2:6	Display an equation such as the example opposite, and ask children to describe how the numerator changes in each term. Children should see that the numerator increases by one each time. Now ask them to focus on the denominator and prompt *'How does the denominator change each time?'* Some children may say that five is being added on each time. At this point, accept this. Ultimately, children should recognise the proportional relationships between the numerators and denominators, and this will be developed in subsequent steps. There are parallels here between this layout of families of equivalent fractions and the ratio tables that children encountered in *Spine 2: Multiplication and Division.*	Highlighting change in numerator: $\frac{1}{5}=\frac{2}{10}=\frac{3}{15}=\frac{4}{20}=\frac{5}{25}=\frac{6}{30}$ Highlighting change in denominator: $\frac{1}{5}=\frac{2}{10}=\frac{3}{15}=\frac{4}{20}=\frac{5}{25}=\frac{6}{30}$

2:7	Refer again to the examples opposite. Ask children to work with a partner and look at the numerator and denominator in each of the equivalent fractions in turn. What relationship can they see? Draw their attention to the vertical relationship: - In each case the denominator is five times the numerator (and the numerator is $\frac{1}{5}$ of the denominator). - Each numerator forms the same part of the whole (the denominator). This is why the fractions have the same numerical value and represent the same part of a whole.	
2:8	Present the children with some other fractions and discuss in turn whether or not each of them would be equivalent to $\frac{1}{5}$. Remember that at this stage we are only looking at the vertical relationship *within* a fraction. Pose questions such as: - *'Is $\frac{9}{45}$ equivalent to $\frac{1}{5}$?'* (Yes, because the denominator is five times the numerator.) - *'Where would $\frac{9}{45}$ sit on the number line?'* (At the same place as $\frac{1}{5}$ because it is equivalent to $\frac{1}{5}$.) - *'Is $\frac{8}{12}$ equivalent to $\frac{1}{5}$?'* (No, because the denominator is not five times the numerator.) - *'Is $\frac{4}{30}$ equivalent to $\frac{1}{5}$?'* (No, because the denominator is not five times the numerator.) The children haven't learnt enough to accurately position $\frac{8}{12}$ and $\frac{4}{30}$ on this number line yet, without trying to	

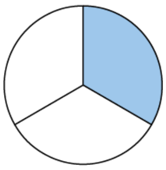
	partition it. But by discussing that in $\frac{8}{12}$ the numerator is quite a large part of the whole, and in $\frac{4}{30}$ the numerator is quite a small part of the whole, they can start to develop the sense that $\frac{8}{12}$ sits nearer to 1 and $\frac{4}{30}$ is nearer to 0.	
2:9	Work through the sequence again (steps *2:1–2:8*), this time with a different unit fraction. You may wish to use the representations for $\frac{1}{3}$, provided opposite. Follow the same steps as for the previous examples, listed here, referring back to the notes above as necessary: - fractions as operators – fractions of a shape (steps *2:1–2:4*) - fractions as numbers – marked on a number line (step *2:5*) - the proportional relationship between the numerators and denominators of these equivalent fractions (steps *2:7–2:8*) - identifying whether other fractions are equivalent to one-third (step *2:8*).	Fractions as operators:  $\frac{1}{3}$ $\frac{1}{3} = \frac{2}{6}$ $\frac{1}{3} = \frac{3}{9}$

		Fractions as numbers:  Proportional relationship:  - *'The denominator is always three times the numerator.'* - *'The numerator is one-third of the denominator.'*
2:10	Provide children with opportunities to practise completing equivalent fractions, still with the emphasis on the vertical relationship. Prominence is placed on the vertical relationship, as it is this relationship that determines the value of a fraction and will therefore help develop fraction sense. If converting between equivalent fractions attending only to the horizontal relationship, children's focus would no longer be on the numerator in relation to the denominator, and therefore no longer on the aspect of a fraction that determines its value.	

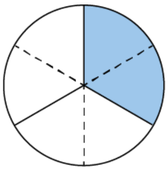
	For fractions equivalent to $\frac{1}{5}$, the denominator was five times the numerator. For fractions equivalent to $\frac{1}{3}$, the denominator was three times the numerator. Ask • *'What can we say about the relationship between the numerator and denominator in fractions equivalent to $\frac{1}{4}$?'* • *'What about in fractions equivalent to $\frac{1}{10}$?'* Writing the vertical arrows on both sides of the equation to show the multiple that the denominator is of the numerator will support this. However, in the second column, the children will need to use division to work from the denominator to find the numerator.	×5 $\frac{1}{5} = \frac{12}{\square}$ ×5 $\frac{1}{5} = \frac{\square}{40}$ $\frac{1}{3} = \frac{8}{\square}$ $\frac{1}{3} = \frac{\square}{30}$ $\frac{1}{4} = \frac{3}{\square}$ $\frac{1}{4} = \frac{\square}{24}$ $\frac{1}{10} = \frac{7}{\square}$ $\frac{1}{10} = \frac{\square}{50}$
2:11	It is also important at this point to look at non-examples, such as asking children to identify whether a pair of fractions are equivalent or not. Provide varied practice, such as in the examples opposite, that will allow children to reason around both the vertical relationship and the horizontal relationships. You may wish to encourage children to use stem sentences such as the following to structure their responses: • ***'The numerator has been scaled up/down by ___.'*** • ***'The denominator has been scaled up/down by ___.'*** • ***'These fractions are/are not equivalent.'***	Missing-symbol problems: *'Are these fractions equal or not? Fill in the missing symbols (= or ≠).'* $\frac{1}{6} \bigcirc \frac{3}{12}$ $\frac{18}{20} \bigcirc \frac{9}{10}$ $\frac{4}{12} \bigcirc \frac{1}{3}$ $\frac{1}{8} \bigcirc \frac{2}{12}$ Matching: *'Match each fraction to its equivalent.'* $\boxed{\frac{1}{5}}$ $\boxed{\frac{3}{12}}$ $\boxed{\frac{1}{4}}$ $\boxed{\frac{4}{20}}$ $\boxed{\frac{1}{3}}$ $\boxed{\frac{3}{9}}$

Writing equivalent fractions:

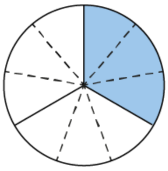
'Write the equivalent fractions to match each image.'



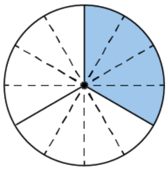
$\frac{1}{3}$



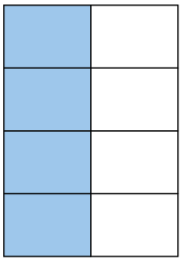
$\frac{1}{3} = \square$



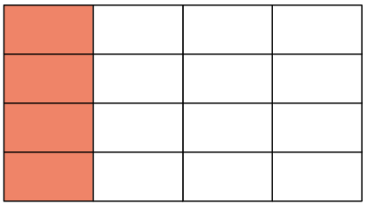
$\frac{1}{3} = \square$



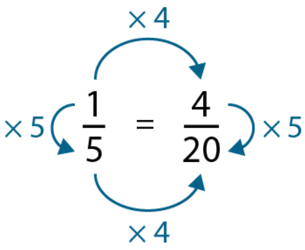
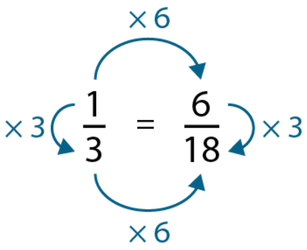
$\frac{1}{3} = \square$

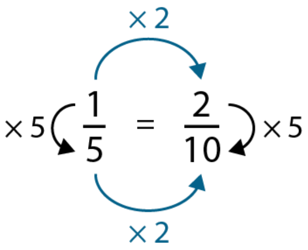
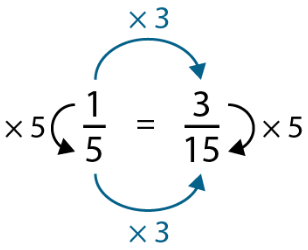
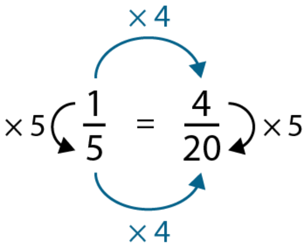
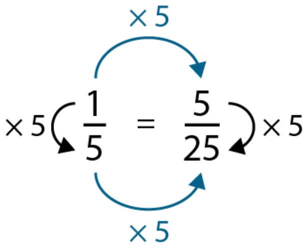
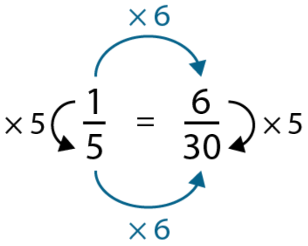
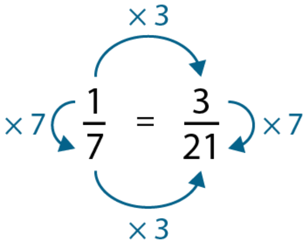
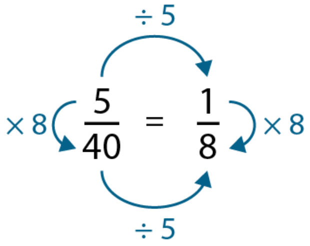


$\frac{\square}{8} = \frac{\square}{4} = \frac{\square}{2}$



$\frac{4}{16} = \frac{\square}{8} = \frac{\square}{4}$

2:12	Once children are secure in their understanding of the vertical relationship of equivalent fractions, focus on the horizontal relationship between each equivalent fraction. Start by looking back at some equivalent fraction pairs the children have already worked with. Children have already learnt that the vertical relationship between the numerator and denominator is the same in equivalent fractions. Now, ask the children to compare the numerator with the numerator and the denominator with the denominator in each pair of equivalent fractions. Some children may look at an additive comparison at first. For example, *'The numerator goes up by three and the denominator goes up by fifteen'*. However, if they look at a multiplicative comparison, they will see that in the first pair, both the numerator and denominator are four times larger in $\frac{4}{20}$ than in $\frac{1}{5}$.	 
2:13	Use the numbers that are equivalent to $\frac{1}{5}$ and $\frac{1}{3}$ again, and draw attention to the horizontal relationship between the numerators and the corresponding horizontal relationship between the denominators. A fraction preserves its value only when both the numerator and denominator are scaled by the same factor. If we multiply the numerator by two, we have to multiply the denominator by two. If we multiply the numerator by three, we have to multiply the denominator by three, and so on.	

		    
2:14	Look at some other equivalent fraction pairs. Identify that there is always a horizontal and vertical relationship in each. Work together as a class to begin with, then give children some independent practice.	  $\frac{4}{24} = \frac{1}{6}$ $\frac{1}{12} = \frac{3}{36}$ $\frac{1}{20} = \frac{7}{140}$ $\frac{1}{33} = \frac{2}{66}$

2:15

By this stage, children should be confident with identifying equivalent fractions for unit fractions. They should be able to scale the numerator and denominator by multiplying and dividing, and reason using both the horizontal and vertical relationships of the numerators and denominators.

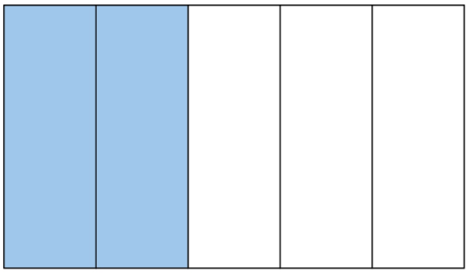
Use a similar representation to that provided in steps *2:1* and *2:2*. This time you may opt to work with $\frac{2}{5}$ as a starting point. As previously, exploit the repetitive nature of these steps in order to secure the connections between the equivalent fractions each time.

Show children an area model for $\frac{2}{5}$, such as the example opposite, and ask:

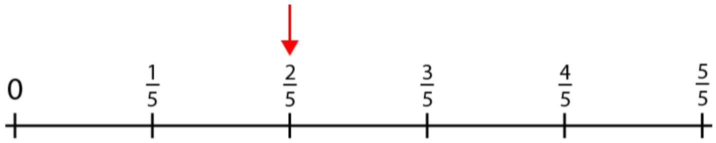
- *'What fraction is shaded?'*
- *'Where are the five parts?'*
- *'Where are the two parts?'*

As the children have already spent some time exploring equivalent fractions on area models and on number lines, this time a number line can be presented at the same time as an area model. Start by reminding children that we can have $\frac{2}{5}$ of a shape or quantity, but that $\frac{2}{5}$ is also a number which can be positioned on a number line.

Fractions as operators – area model:



Fractions as numbers – number line:



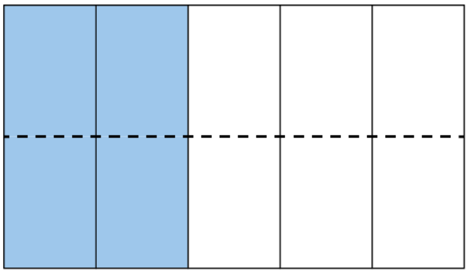
2:16

Reveal $\frac{4}{10}$ on the area model and ask:

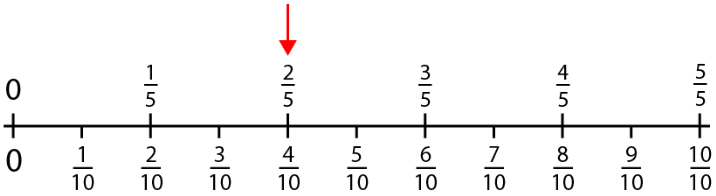
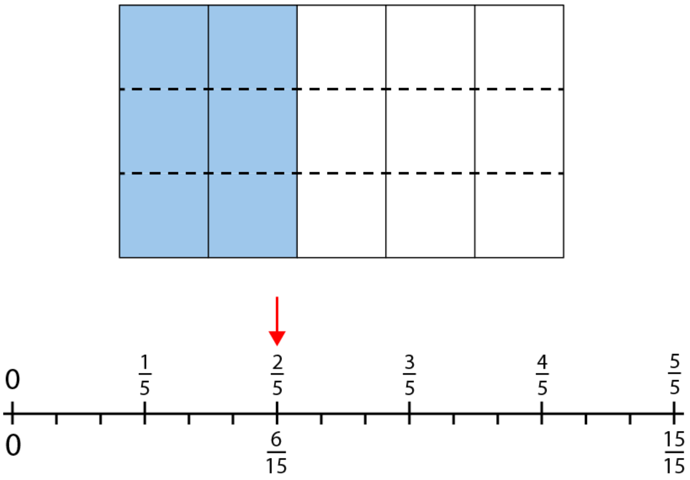
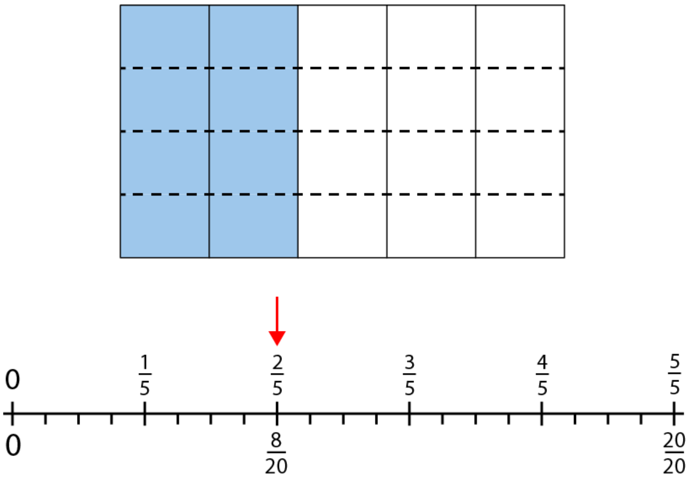
- *'Can you still see the two-fifths?'*
- *'What other fraction can you see?'* ($\frac{4}{10}$)

Display the number lines showing the fifths and the tenths alongside each another. Write $\frac{2}{5}=\frac{4}{10}$.

Discuss that these numbers have the



$\frac{2}{5}=\frac{4}{10}$

	same value but a different appearance. You may choose to use the stem sentence: '$\frac{\square}{\square}$ ***is equivalent to*** $\frac{\square}{\square}$'	 *'$\frac{2}{5}$ is equivalent to $\frac{4}{10}$.'*
2:17	Display each new equivalent fraction with its corresponding notation. Each time, ask the children *'Can you still see the two-fifths?'* *'What other fraction can you see?'* For each example, make the link to the new fraction and discuss how it has same value but a different appearance. Again, use the stem sentence from step *2:16*: '$\frac{\square}{\square}$ ***is equivalent to*** $\frac{\square}{\square}$'	 $\frac{2}{5}=\frac{6}{15}$ *'$\frac{2}{5}$ is equivalent to $\frac{6}{15}$.'*  $\frac{2}{5}=\frac{8}{20}$ *'$\frac{2}{5}$ is equivalent to $\frac{8}{20}$.'*

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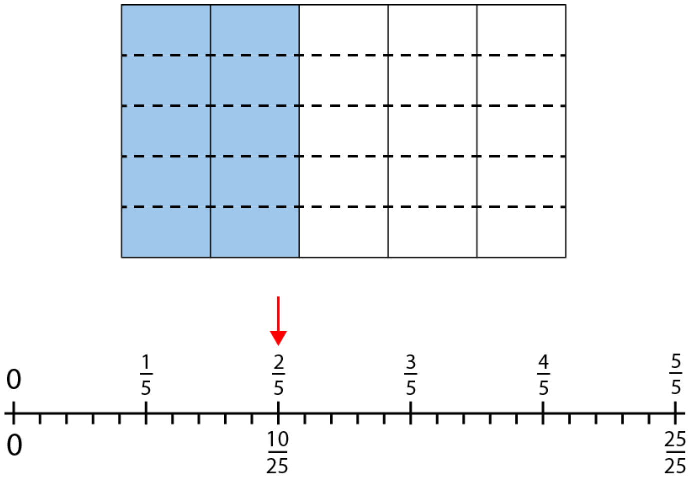
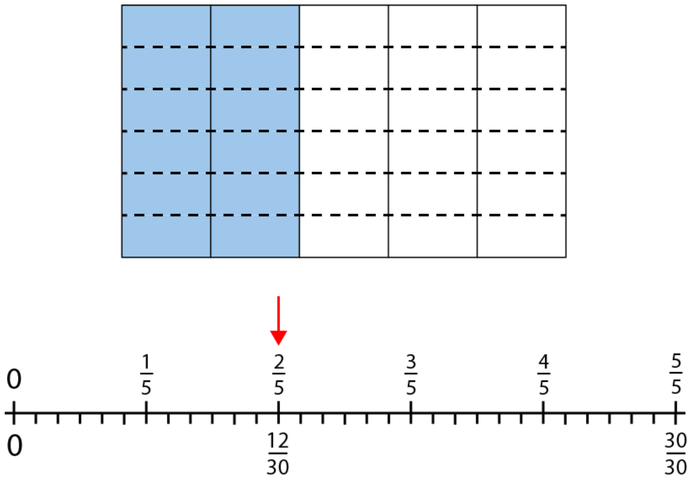


$\frac{2}{5} = \frac{10}{25}$ ‘$\frac{2}{5}$ *is equivalent to* $\frac{10}{25}$.’



$\frac{2}{5} = \frac{12}{30}$ ‘$\frac{2}{5}$ *is equivalent to* $\frac{12}{30}$.’
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2:18
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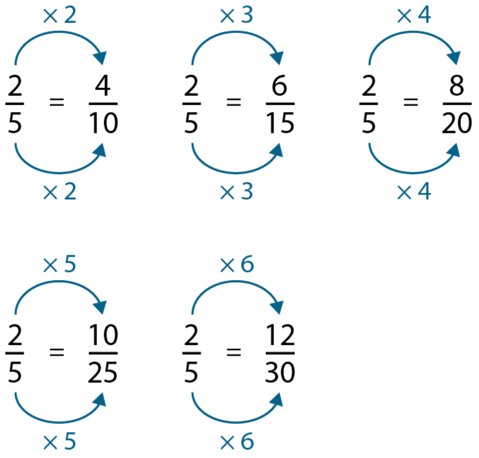
Look at the series of fractions that are equivalent to $\frac{2}{5}$. The vertical relationship is more difficult to identify this time, but the numerators and denominators are still in proportion to each other; two forms the same part of five as four does of ten. All the denominators are two-and-a-half times the numerator.

Display the fractions equivalent to $\frac{2}{5}$, as shown opposite, and examine the horizontal relationships, as you did with the previous example. In this case, the
</td><td>

$\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25} = \frac{12}{30}$



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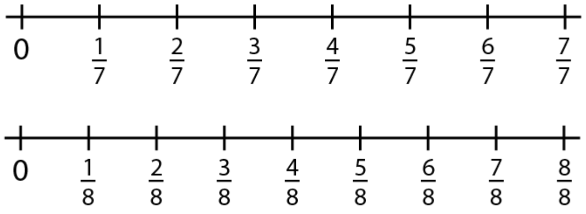
	horizontal relationships are much easier to recognise than the vertical relationship.	
2:19	Look back at the three sequences of fractions focused on so far. Once children are confident recognising these fraction sequences using images, progress to identifying missing numbers in fractions. Provide examples where they need to find *either* the missing numerator *or* the missing denominator. It is important that children reason using the language of scaling in order to reinforce what is happening each time the numerator or denominator is scaled up or down by the same amount. Encourage them in this.	$\frac{1}{5}=\frac{2}{10}=\frac{3}{15}=\frac{4}{20}=\frac{5}{25}=\frac{6}{30}$ $\frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\frac{4}{12}=\frac{5}{15}=\frac{6}{18}$ $\frac{2}{5}=\frac{4}{10}=\frac{6}{15}=\frac{8}{20}=\frac{10}{25}=\frac{12}{30}$
2:20	At this point, provide guided practice in writing *'families'* of equivalent fractions, along the lines of the examples opposite. Each time an equivalent fraction is found, ask children to explain how they know it is equivalent. Make sure they go beyond simply 'continuing the pattern' of adding one to the previous numerator and eight to the previous denominator. For unit fraction examples, e.g. $\frac{1}{7}$, children can look at the vertical proportional relationship in each fraction. For example, *'I know it must be $\frac{4}{28}$, because the denominator is seven times the numerator.'.* For both examples, children should justify their choices by reference to the first fraction in the chain. For example, *'$\frac{3}{8}$ is equal $\frac{12}{32}$ because both the numerator and denominator have been scaled by a factor of four'.* Again, really scrutinise children's understanding that these all have the	*'Complete these 'families' of equivalent fractions.'* $\frac{1}{7}=\frac{\square}{14}=\frac{\square}{21}=\frac{4}{\square}=\frac{5}{\square}=\frac{\square}{42}=\frac{7}{\square}$ $\frac{3}{8}=\frac{\square}{16}=\frac{\square}{24}=\frac{\square}{32}=\frac{\square}{40}=\frac{28}{\square}=\frac{21}{\square}$ 

	same value. Ask questions such as *'Which of these is the largest number?'* ensuring that children are comfortable with the fact that, although some fractions have larger numerators and denominators than others, the resulting fractions are all equal. Show a number line for each of your equivalent fraction families. For the examples given on the previous page, the number lines are marked in sevenths for the first family and eighths for the second family. Ask children to position each of the fractions on the relevant number line. Note that we don't need to add any additional marks to the number line in order to do this. Because all of the numbers in the first family are equal in value to $\frac{1}{7}$, they all sit at the $\frac{1}{7}$ mark on the number line. Because all of the numbers in the second family are equal in value to $\frac{3}{8}$, they all sit at the $\frac{3}{8}$ point on the number line. Again, you can really exaggerate this, for example '$\frac{21}{56}$ *sounds like an enormous number. That surely must go right over to the far end of the number line, shouldn't it?'* The children will take great delight in correcting you.	
2:21	At this point, you may find it useful to share a multiplication square, such as the example shown on the next page, as this can help emphasise the scaling aspect of equivalence. This may well give rise to a discussion about what happens if we keep on going. You may like to give an example such as *'I can scale my numerator and denominator as much as I want.* $\frac{1{,}000}{7{,}000}$ *and* $\frac{1{,}000{,}000}{7{,}000{,}000}$ *are both fractions that are equivalent to* $\frac{1}{7}$.'	

You may now find it useful to introduce the following generalisation: ***'When the numerator and denominator are multiplied or divided by the same number, the value of the fraction remains the same.'***

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	**14**	**21**	**28**	**35**	**42**	**49**	**56**	**63**	**70**	**77**	**84**
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	**6**	**9**	**12**	**15**	**18**	**21**	**24**	**27**	**30**	**33**	**36**
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	**16**	**24**	**32**	**40**	**48**	**56**	**64**	**72**	**80**	**88**	**96**
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

2:22 Repeat the sequence from step *2:20*, introducing different strings of equivalent fractions, such as the examples shown opposite. When presenting your own examples, you might like to mix them up to make them more challenging. For instance:

- Present fractions in no particular order, so that children can't just follow a pattern such as *'Add one to the top, add five to the bottom'* in order to complete them.

$$\frac{9}{81} = \frac{8}{\square} = \frac{7}{\square} = \frac{\square}{54} = \frac{\square}{45} = \frac{4}{\square} = \frac{3}{\square} = \frac{2}{\square} = \frac{1}{9}$$

$$\frac{2}{7} = \frac{\square}{14} = \frac{\square}{21} = \frac{8}{\square} = \frac{10}{\square} = \frac{\square}{42} = \frac{\square}{\square} =$$

$$\frac{2}{5} = \frac{16}{\square} = \frac{\square}{15} = \frac{\square}{35} = \frac{28}{\square} = \frac{32}{\square} = \frac{\square}{75} = \frac{92}{\square}$$

	- Repeat fractions. Note the repeated fraction in the middle of the last example on the previous page, derived once from the denominator (35) and once from the numerator (28). - Extend children's knowledge. The last two fractions require children to go beyond the scope of their times table knowledge. For example, in order to work out what four has been multiplied by to get to 92, children need to identify that they should calculate *92 ÷ 4*. Be aware that some children will find it much harder to identify what they need to do here than in examples where they can just apply times table knowledge. It is therefore suggested that you spend some time working through and discussing these.	

2:23	Provide varied practice. You might want to focus on this practice on:
	- finding missing numerators and missing denominators. It is important to vary the questions provided, to include:
	  - changing the position of the missing numerator/denominator
	  - problems that require multiplication of the numerator and denominator
	  - problems that require division.
	- matching fractions to their equivalents*
	- placing equivalent fractions on a number line.
	To deepen and consolidate children's learning, present them with dòng nǎo jīn problems, such as shown on page 34. In the first example question, we want children to reason that if the difference is zero, then the subtrahend must have the same value as the
		Missing-number problems:
		- *'Complete these equivalent fractions.'*
		$\frac{1}{3} = \frac{\boxed{}}{6} = \frac{\boxed{}}{9}$
		$\frac{1}{4} = \frac{\boxed{}}{8}$
		$\frac{1}{5} = \frac{\boxed{}}{10}$
		$\frac{6}{12} = \frac{1}{\boxed{}}$
		$\frac{\boxed{}}{10} = \frac{2}{20}$
		$\frac{3}{\boxed{}} = \frac{1}{5}$

minuend. As $\frac{22}{28} = \frac{11}{14}$, the subtrahend must be $\frac{11}{14}$.

* Note: use plausible distractors to identify children who may have some lingering misconceptions. For example, some children may think they can make an equivalent fraction by adding the same number to both the numerator and the denominator.

- *'Fill in the missing digits.'*

$\frac{4}{8} = \frac{12}{\square}$ $\frac{3}{5} = \frac{\square}{40}$

$\frac{3}{\square} = \frac{21}{63}$ $\frac{20}{32} = \frac{8}{\square}$

- *'What could the missing digits be? Explain your answer.'*

$\frac{\square}{11} = \frac{\square}{88}$ $\frac{3}{\square} = \frac{6}{\square}$

Matching:

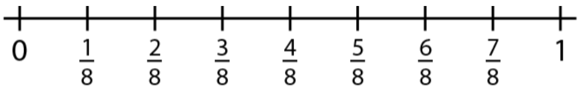
'Circle the fraction that is equivalent to the fraction card.'

$\boxed{\frac{4}{7}}$ $\frac{7}{4}$ $\frac{5}{8}$ $\frac{8}{14}$ $\frac{6}{9}$

Placing a number on a number line:

'Show where each fraction is positioned on the number line by converting to eigths.'

$\frac{9}{24}$ $\frac{36}{48}$ $\frac{12}{16}$ $\frac{10}{40}$ $\frac{9}{72}$



Dòng nǎo jīn:

- *'Fill in the missing digits.'*

$$\frac{22}{28} - \frac{\square}{14} = 0$$

$$\frac{2}{14} - \frac{1}{7} = \square$$

$$\frac{6}{12} - \frac{\square}{6} = 0$$

$$0 = \frac{4}{\square} - \frac{1}{5}$$

- *'Use the following number cards to complete the chain of equivalent fractions.'*

$$\boxed{3} \quad \boxed{24} \quad \boxed{8} \quad \boxed{1}$$

$$\frac{2}{6} = \frac{\square}{\square} = \frac{\square}{\square}$$

- *'Look at the two pairs of equivalent fractions. All the fractions are proper fractions. How many different ways can each equation be completed?'*

$$\frac{4}{\square} = \frac{\square}{4} \qquad \frac{\square}{7} = \frac{7}{\square}$$

Teaching point 3:

Fractions can be simplified by dividing both the numerator and denominator by a common factor.

Steps in learning

	Guidance	**Representations**
3:1	Once children understand the concept of equivalent fractions and are confident in finding equivalent fractions by exploring the vertical and horizontal relationship, scaling up and down, they are then ready to learn about the simplest form of fractions. So far, we have always converted a fraction to an equivalent fraction where the numerator or denominator is given, and therefore the factor used to scale the numerators and denominators up or down was already fixed. In this teaching point, children will need to look at the numerator and denominator within a fraction and identify a *common factor* they can scale it by. As such, learning here will build on prior learning of factors and multiples, as explored in *Spine 2: Multiplication and Division*, segment *2.21*. Start by checking that children are confident and accurate with using the language of factors and multiples, and in finding common factors. You may wish to ask probing questions, such as: • *'Which of these is a factor of eight: three, four, five or six?'* • *'Which number is a common factor of both twelve and fifteen: three, four or six?'* • *'Find the common factor of eighteen and twenty-four. Which is the highest common factor?'*	

3:2

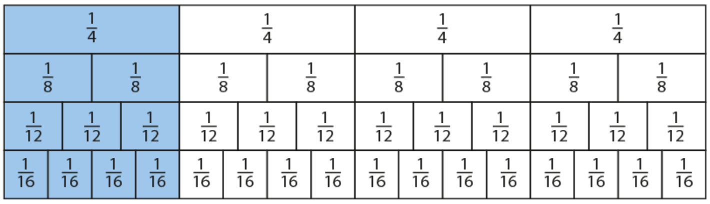
You may find it easiest to begin by returning to some of the families of fractions the children worked with previously. Explain to children that, while all of the fractions in *Example 1* are equivalent, we say that the *simplest form* in this family of fractions is $\frac{1}{4}$. In the family of fractions in *Example 2*, the simplest form of the fraction is $\frac{1}{7}$. In the family of fractions in *Example 3*, the simplest form is $\frac{1}{3}$.

Ask the children to discuss with a partner what we can conclude about what the *simplest form* of the fraction means from these examples. Some of their responses may include:

- *'In the simplest form, we have scaled the numerator and denominator to make them as small as we can.'*
- *'In the simplest form, we have made the fraction as small as possible.'**
- *'In the simplest form, we make the numerator one.'* (Based on the examples given here, this would be a reasonable generalisation. Non-unit fractions are covered in the next step.)

*Note: be precise with language. Someone will almost certainly say, *'We are making the fractions as small as possible.'* This statement needs discussion. We are not making the *fraction* as small as possible. We are keeping the fraction exactly the same size; because they are equivalent fractions, they all have the same value. The numerator and denominator are just made as small as possible, whilst keeping them in proportion.

Example 1:

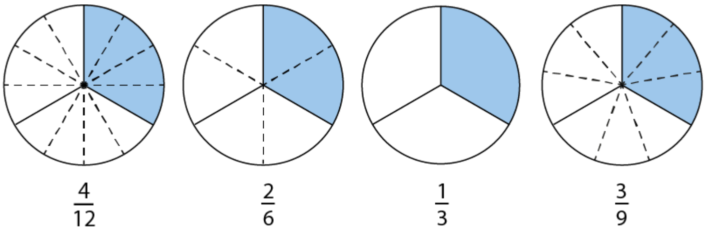


$$\frac{1}{4}=\frac{2}{8}=\frac{3}{12}=\frac{4}{16}$$

Example 2:

$$\frac{1}{7}=\frac{2}{14}=\frac{3}{21}=\frac{4}{28}=\frac{5}{35}=\frac{6}{42}=\frac{7}{49}$$

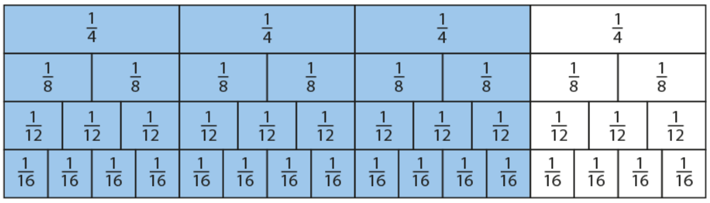
Example 3:



3:3

You may want to acknowledge that in the examples in the previous step, when the fraction was in its simplest form the numerator was always one. Now look at some examples where this isn't the case. Again, discuss with the children what the simplest form is in each of these groups of equivalent fractions. ($\frac{3}{4}$, $\frac{5}{7}$ and $\frac{2}{3}$.)

Example 1:

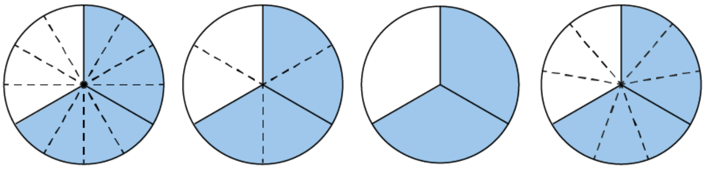


$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16}$$

Example 2:

$$\frac{5}{7} = \frac{10}{14} = \frac{15}{21} = \frac{20}{28} = \frac{25}{35} = \frac{30}{42} = \frac{35}{49}$$

Example 3:



$\frac{8}{12}$ $\frac{4}{6}$ $\frac{2}{3}$ $\frac{6}{9}$

3:4

Now look at how any fraction can be converted into its simplest form.

Show the children a fraction such as $\frac{4}{12}$.

We know from *step 3:2* that the simplest form of $\frac{4}{12}$ is $\frac{1}{3}$. The children can use their knowledge from the previous teaching point to identify that both the numerator and denominator have been divided by four. Four was chosen because both the numerator and denominator can be divided by it. It is both a factor of four *and* a factor of twelve. As mentioned in step *3:1*, this is called a 'common factor'.

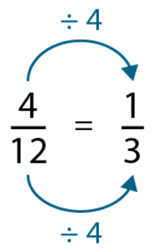
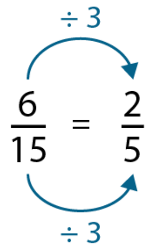
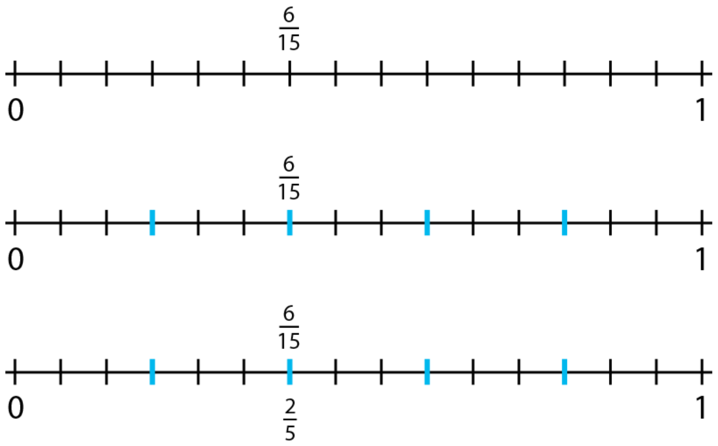
Ask the class to identify other common factors of four and twelve (one and two). Examine why these common factors haven't been used to convert $\frac{4}{12}$ to its simplest form. Dividing the numerator and denominator by one is

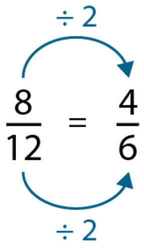
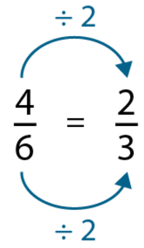
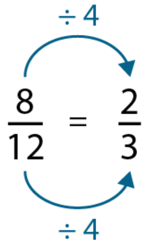
Finding the highest common factor:

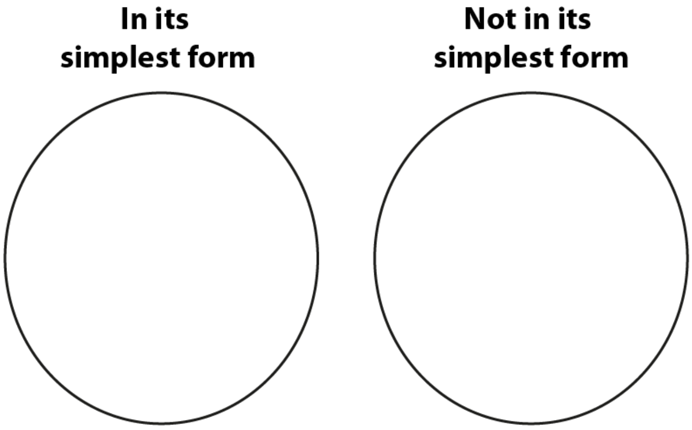
$$\frac{4}{12} \overset{\div 4}{\underset{\div 4}{=}} \frac{1}{3}$$

$$\frac{4}{12} \overset{\div 1}{\underset{\div 1}{=}} \frac{4}{12}$$

$$\frac{4}{12} \overset{\div 2}{\underset{\div 2}{=}} \frac{2}{6} \overset{\div 2}{\underset{\div 2}{=}} \frac{1}{3}$$

	not helpful because the fraction will stay in the same form. Dividing the numerator and denominator by two gives $\frac{2}{6}$, which itself can be further simplified to $\frac{1}{3}$. Four is the *highest common factor* of four and twelve. We divided both the numerator and the denominator by the highest common factor to express $\frac{4}{12}$ in its simplest form.	 Highest common factor = 4
3:5	Present a different example where the numerator is not the highest common factor, for example $\frac{6}{15}$. Show the children how to check whether the numerator is a factor of the denominator (if it is, this is always the highest common factor). Six is not a factor of fifteen, so other common factors need to be identified. Conclude that the highest common factor of six and fifteen is three, so both the numerator and denominator are divided by three in order to express the fraction in its simplest form. At this point, you may like to introduce the following generalisations: - ***'A fraction can be simplified when the numerator and denominator have a common factor other than one.'*** - ***'To write a fraction in its simplest form, divide both the numerator and denominator by their highest common factor.'***	Highest common factor = 3  

3:6	Show the children another fraction, such as $\frac{8}{12}$, and present the following scenarios: - *'Simran is simplifying the fraction $\frac{8}{12}$. She divides both the numerator and the denominator by 2 to get $\frac{4}{6}$. Then she divides both the denominator and the numerator by 2 again. Simran has $\frac{2}{3}$ as her simplified fraction.'* - *'Sam simplifies the same fraction. He divides both the numerator and the denominator by 4. Sam has $\frac{2}{3}$ as his simplified fraction.'* Ask children to compare Simran's and Sam's methods. - *'What is the same?'* - *'What is different?'* Explain to children that we need to find the highest common factor for the numerator and denominator. However, if we are not sure, we can check and simplify again until we reach the fraction in its simplest form.	- *'Simran is simplifying the fraction $\frac{8}{12}$. She divides both the numerator and the denominator by 2 to get $\frac{4}{6}$. Then she divides both the denominator and the numerator by 2 again. Simran has $\frac{2}{3}$ as her simplified fraction.'*   - *'Sam simplifies the same fraction. He divides both the numerator and the denominator by 4. Sam has $\frac{2}{3}$ as his simplified fraction.'* 
3:7	Work through some other examples, first as a class, and then as independent practice, until children are confident finding common factors and expressing fractions in their simplest form. Discuss that in order to do this, they will need to: - identify the highest common factor of the numerator and denominator - divide both the numerator and denominator by the highest common factor - look at the resulting fraction to check that it is in its simplest form and that the only remaining common factor is one.	

	Aim to include examples where the numerator is the highest common factor, and also where the numerator is *not* the highest common factor. Remind children what they learnt in step *3:4*: • Where the numerator is a factor of the denominator (and therefore also the highest common factor), such as $\frac{3}{9}$ or $\frac{7}{28}$, then the simplest form is a unit fraction. • Where the numerator is *not* a factor of the denominator, such as $\frac{4}{14}$ or $\frac{15}{20}$, then the simplest form is *not* a unit fraction.	
3:8	Once you are confident all children can simplify a fraction when told it is not in its simplest form, provide them with a selection of fractions to sort into sorting circles. They could sort them according to whether they are in their simplest form or not. Children are more likely to spot that fractions can be simplified if both the numerator and denominator are even. Therefore it is important to include fractions where the denominators and/or numerators are odd. Encourage children to justify their choices using the language of factors. Examples of this might include: • *'$\frac{4}{20}$ is not in its simplest form, because four is a common factor of four and twenty.'* • *'$\frac{23}{30}$ is in its simplest form, because one is the only common factor of twenty-three and thirty.'*	*'Sort the following numbers according to whether they are expressed in their simplest form or not.'* $\frac{3}{15}$ $\frac{2}{5}$ $\frac{4}{20}$ $\frac{25}{36}$ $\frac{1}{6}$ $\frac{7}{21}$ $\frac{18}{30}$ $\frac{9}{17}$ $\frac{5}{15}$ $\frac{11}{20}$ $\frac{23}{30}$ 

3:9	Provide varied practice for children to allow them to consolidate their understanding of fractions in their simplest form. Include dòng nǎo jīn problems, such as shown opposite.	Simplifying fractions: *'Simplify the following fractions.'* $\frac{5}{15}$ $\frac{3}{21}$ $\frac{15}{45}$ $\frac{12}{36}$ $\frac{16}{28}$ $\frac{13}{39}$ Missing-number problems: *'The following fractions have been simplified. Fill in the missing numbers.'* $\frac{20}{50}=\frac{\square}{5}$ $\frac{14}{49}=\frac{2}{\square}$ $\frac{\square}{56}=\frac{2}{7}$ $\frac{9}{\square}=\frac{1}{5}$ Dòng nǎo jīn: *'Order these numbers from smallest to largest by simplifying them to unit fractions.'* $\frac{3}{18}$ $\frac{5}{20}$ $\frac{4}{8}$ $\frac{2}{18}$ $\frac{4}{12}$ $\frac{6}{60}$
3:10	Once children have had ample opportunity to practise simplifying fractions, and they can identify fractions in their simplest form, these skills can now be applied to prior learning involving calculation with fractions. In segment *3.4 Adding and subtracting within one whole*, children learnt to add fractions with the same denominator. Now is an appropriate point to return to addition and subtraction of fractions and identify answers where the fraction is *not* in its simplest form (e.g. $\frac{3}{10}+\frac{3}{10}=\frac{6}{10}$). Model this example, as shown opposite, concluding that the answer of $\frac{6}{10}$ can be further simplified to $\frac{3}{5}$.	$\frac{3}{10}+\frac{3}{10}=\overset{\div 2}{\underset{\div 2}{\frac{6}{10}=\frac{3}{5}}}$

3:11	Provide practice with a mixture of calculations, some where the answer is already given in its simplest form and some where the answer requires simplification.	$\frac{2}{9}+\frac{4}{9}=$ $\quad\frac{3}{7}-\frac{1}{7}=$ $\quad\frac{4}{15}+\frac{2}{15}=$ $\frac{5}{12}+\frac{5}{12}-\frac{2}{12}=$ $\quad\frac{2}{13}+\frac{7}{13}-\frac{4}{13}=$
3:12	In segment *3.5 Working across one whole: improper fractions and mixed numbers*, children learnt to add and subtract mixed numbers and fractions with a value greater than one. This learning can also be further developed using simplification. Provide children with the calculation $\frac{9}{12}+\frac{11}{12}=\frac{20}{12}$ and ask them what they notice. '*Can this answer be simplified?*' Display methods 1 and 2, opposite and ask: - *'What is the same?'* - *'What is different?'* Work through each method and arrive at the same answer. Do they prefer one method over the other? Repeat this process with a subtraction (e.g. $\frac{27}{6}-\frac{5}{6}=\frac{22}{6}$).	Method 1: $$\begin{tikzcd} \frac{20}{12} = \arrow[r, bend left, "\div 4"] \arrow[r, bend right, "\div 4"'] & \frac{5}{3} & = 1\frac{2}{3} \end{tikzcd}$$ Method 2: $$\begin{tikzcd} \frac{20}{12} = 1\frac{8}{12} \arrow[r, bend left, "\div 4"] \arrow[r, bend right, "\div 4"'] & = 1\frac{2}{3} \end{tikzcd}$$
3:13	Provide practice opportunities that allow the children to apply their preferred method, including problems without context and word problems with context. Include dòng nǎo jīn questions, such as shown on the next page. Challenge children's thinking with appropriate reasoning opportunities to determine their understanding of what has been learnt. For example, the problem opposite deals with a common misconception around simplification of fractions.	Problems without context: $\frac{4}{5}+\frac{4}{5}=$ $\quad\frac{7}{10}+\frac{5}{10}+\frac{3}{10}=$ $\quad\frac{8}{9}+\frac{8}{9}-\frac{1}{9}=$ Problems with context – word problems: *'Ahmed says, "To simplify a fraction, you just halve the numerator and halve the denominator." Is Ahmed's statement always true, sometimes true or never true? Explain your answer.'*

		Dòng nǎo jīn: *'Fill in the missing numbers.'* $\frac{11}{16} + \frac{\square}{16} = 1\frac{1}{2}$ $\square\frac{9}{10} + 3\frac{\square}{10} = 5\frac{3}{5}$
3:14	In segment *3.6 Multiplying whole numbers and fractions*, children learnt to multiply fractions and mixed numbers by whole numbers. Now is a good point to return to this type of fraction calculation and identify answers that could be simplified. Show the children a multiplication (e.g. $\frac{2}{10} \times 4 = \frac{8}{10}$). Ask them: • *'What do you notice about the answer?'* • *'Is this answer in its simplest form?'* Demonstrate how $\frac{8}{10}$ can be simplified to $\frac{4}{5}$. Repeat this process with further examples, similar to those shown opposite. Also display mixed numbers multiplied by whole numbers, using an example where the answer can be simplified.	$\frac{2}{10} \times 4 = \frac{8}{10}$ $6 \times \frac{2}{9}$ $1\frac{1}{6} \times 3 = 3\frac{3}{6} = 3\frac{1}{2}$
3:15	Children can now revisit the many different fractions calculations from the previous segments and extend them to include simplification. Practice examples should also include exemplars that do not require simplification, so that children gain experience in recognising when it should be used. A small sample of the examples are provided opposite. Include dòng nǎo jīn questions to deepen their understanding.	$3\frac{7}{10} + 2\frac{9}{10} =$ $\frac{13}{8} + \frac{11}{8} =$ $7\frac{1}{6} - 1\frac{2}{6} =$ $\frac{17}{3} - \frac{5}{3} =$ $4\frac{1}{9} \times 3 =$ $10 \times \frac{5}{7} =$

		Dòng nǎo jīn: $\frac{3}{10} + \frac{\square}{10} = \frac{4}{5}$ $\frac{5}{6} \times \square = \square\frac{1}{3}$ $\frac{\square}{12} \times 5 = 4\frac{\square}{6}$