

## **Mastery Professional Development**

### *Number, Addition and Subtraction*



## 1.5 Additive structures: introduction to aggregation and partitioning

Teacher guide | Year 1

### **Teaching point 1:**

Combining two or more parts to make a whole is called aggregation; the addition symbol,  $+$ , can be used to represent aggregation.

### **Teaching point 2:**

The equals symbol,  $=$ , can be used to show equivalence between the whole and the sum of the parts.

### **Teaching point 3:**

Each addend represents a part, and these are combined to form the whole/sum; we can find the value of the whole by adding the parts. We can represent problems with missing parts using an addition equation with a missing addend.

### **Teaching point 4:**

Breaking a whole down into two or more parts is called partitioning; the subtraction symbol,  $-$ , can be used to represent partitioning.

### Overview of learning

In this segment children will:

- use the + symbol to represent combining two parts to make a whole (aggregation)
- use the = symbol to represent the equivalence between a whole and the sum of its parts
- use the – symbol to represent the process of finding an unknown part.

The part–part–whole structure was introduced in segment 1.2 *Introducing 'whole' and 'parts': part–part–whole* and built upon in segments 1.3 *Composition of numbers: 0–5* and 1.4 *Composition of numbers: 6–10*. These segments focused on:

- developing understanding that a whole can be partitioned in different ways, and that the resulting parts can be recombined to form the whole
- developing factual fluency in partitioning the numbers to ten.

The purpose of this segment is to introduce abstract notation as a way of representing the part–part–whole structure. In this way, the addition and subtraction structures of aggregation and partitioning are introduced by building on children’s existing experience of finding wholes and missing parts.

The segment begins with the introduction of the addition symbol, which is used to write expressions such as  $3 + 2$ . Once this understanding has been established, the equals sign is introduced to produce equations, such as  $5 = 3 + 2$ . The equals sign should be read as ‘is equal to’, and represents the equality between the sum of the parts and the whole. Once children are familiar with the equation representation, they will begin to solve aggregation problems. Aggregation is the structure of addition in which two quantities are combined and addition is used to determine the sum, for example, ‘*There are two red flowers and eight yellow flowers. How many flowers are there altogether?*’ This structure of addition is different from the augmentation structure in which a given quantity is increased by a certain amount, for example, ‘*Charlotte has four flowers and then Ben gives her two more. How many flowers does Charlotte have now?*’ Augmentation is covered in segment 1.6 *Additive structures: introduction to augmentation and reduction*.

Children will already be familiar with finding a missing part when the whole and the other part are known, for example, ‘*There are five flowers. Two are red and the rest are yellow. How many yellow flowers are there?*’ In this segment, teaching progresses to the representation of such problems using the subtraction symbol. This is the partitioning structure of subtraction; this should not be confused with the more familiar ‘take away’ (reduction) structure in which a given quantity is reduced by a certain amount, for example, ‘*There are five flowers in a vase. Charlotte gives three away. How many flowers are left?*’ Reduction is covered in segment 1.6. Introducing subtraction for the first time using the partitioning structure is different from the usual way of beginning work on subtraction (taking away), and allows children to more easily apply any factual fluency they have already gained to solve subtraction problems, rather than relying on ‘counting back’ methods. Partitioning represents the inverse of aggregation.

Throughout the segment, children will practise writing equations to represent mathematical stories. It is essential for children to develop an understanding of what each symbol represents; the suggested stem sentences play a key role in this and children should be encouraged to relate each symbol to the context.

The *structures* rather than the *solutions* should remain the focus of the segment. To support this, teachers should plan using contexts for quantities within ten, to minimise focus on calculation. This will further negate any need to tell children to ‘*put the bigger number in your head and count on/back*’, which would draw attention away from the structures.

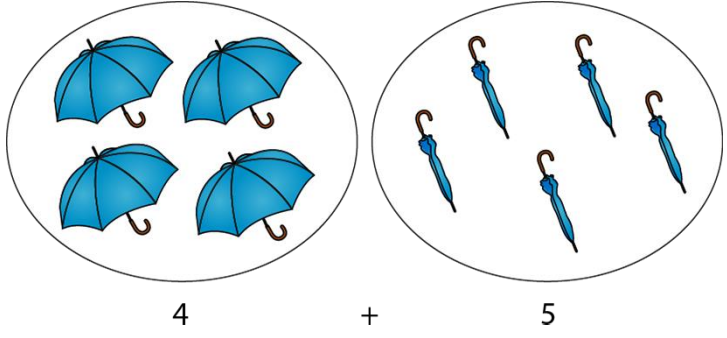
## 1.5 Aggregation and partitioning

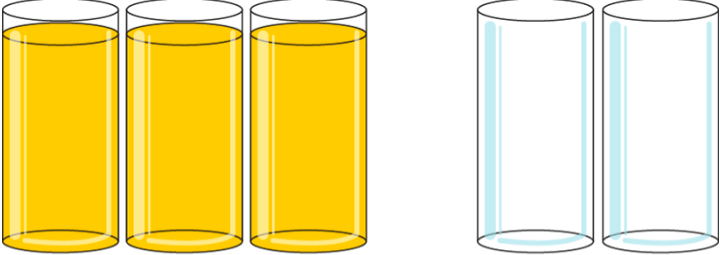
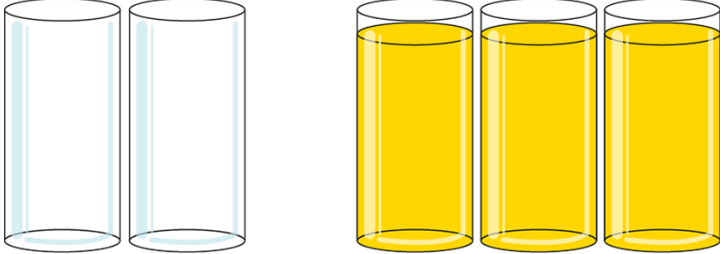
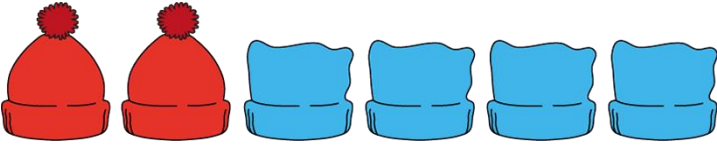
*An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: [www.ncetm.org.uk/primarympdpodcast](http://www.ncetm.org.uk/primarympdpodcast). The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks*

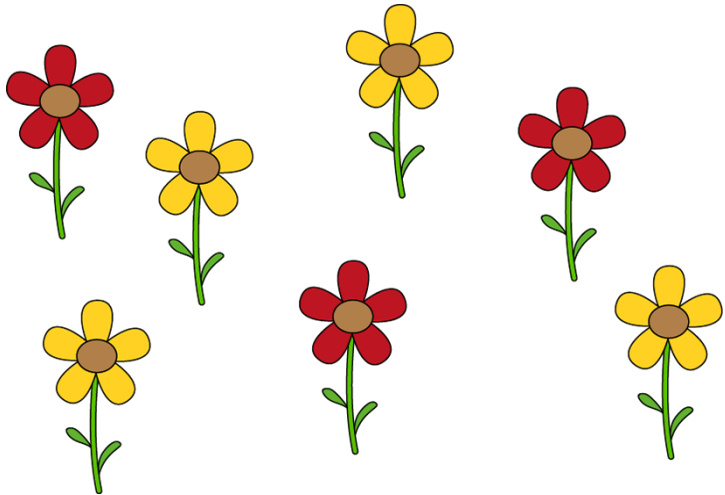
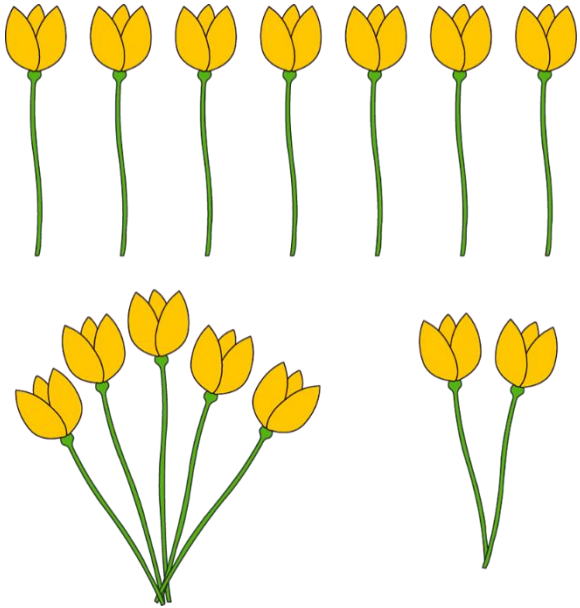
**Teaching point 1:**

Combining two or more parts to make a whole is called aggregation; the addition symbol, +, can be used to represent aggregation.

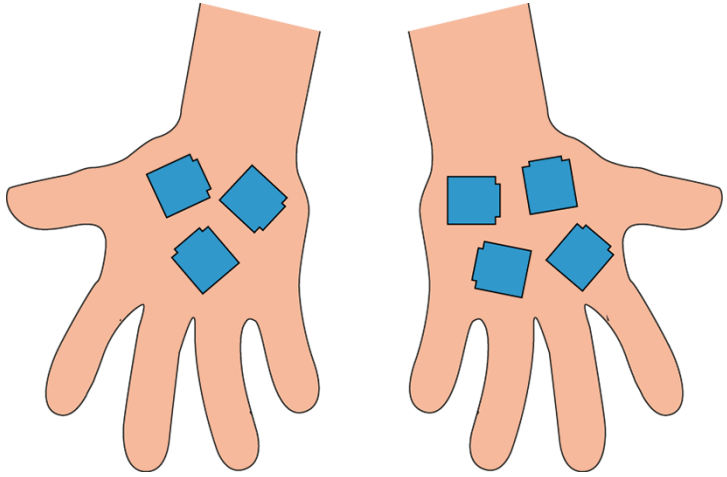
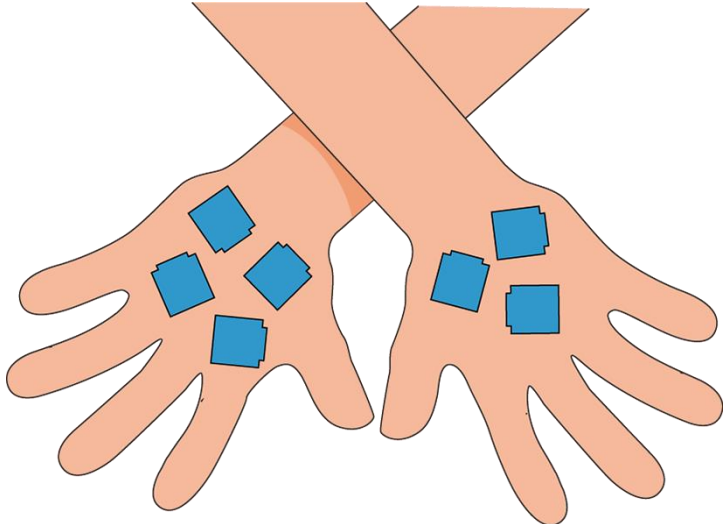
**Steps in learning**

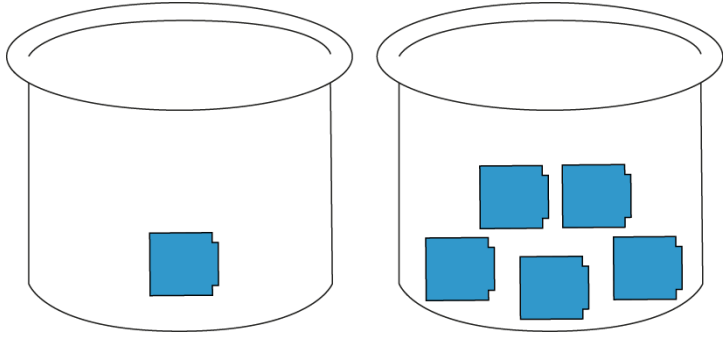
	<b>Guidance</b>	<b>Representations</b>
<b>1:1</b>	<p>Start by presenting children with concrete or pictorial contexts where the whole group is divided into two, easily distinguishable parts (for example, black sheep and white sheep, open and closed umbrellas). You may need to organise objects such that children's attention is drawn to the parts (for example, by moving all the open umbrellas together). You could also draw rings (pictorial) or place hoops (concrete) around the two parts in preparation for using the part-part-whole cherry representation in <i>Teaching point 2</i>.</p> <p>Children should be familiar with such contexts, but now you can begin to write expressions where the two parts are identified and combined using the addition symbol. At this stage do not refer to the whole (sum).</p> <p><math>3 + 2</math> is an expression, rather than an equation, because we have not yet equated it to anything (the equals sign will be introduced in <i>Teaching point 2</i>). For now, ensure that the focus is on recognising that parts can be added together, and that this can be represented in an abstract way: addend + addend. You can read '+' as either 'plus' or 'add' – we will use 'plus' throughout this segment. As well as presenting the expressions, provide opportunities for children to write the expressions themselves.</p> <p>Throughout, encourage children to link the concrete/pictorial and abstract representations by</p>	 <p style="text-align: center;">4                      +                      5</p> <ul style="list-style-type: none"> <li>• 'There are four open umbrellas and five closed umbrellas.'</li> <li>• 'We can write this as four plus five.'</li> </ul> <p style="text-align: center;"><math>4 + 5</math></p> <ul style="list-style-type: none"> <li>• 'The 4 represents the four open umbrellas.'</li> <li>• 'The 5 represents the five closed umbrellas.'</li> </ul>

	<p>describing the contexts and expressions in full sentences, using the following stem sentences:</p> <ul style="list-style-type: none"> <li>• <b>'There are... and...'</b></li> <li>• <b>'We can write this as ___ plus ___.'</b></li> <li>• <b>'The ___ represents the...'</b></li> <li>• <b>'The ___ represents the...'</b></li> </ul>
<p><b>1:2</b> Now explore the idea that the addends in these aggregation contexts can be expressed in either order, for example as <math>3 + 2</math> or as <math>2 + 3</math>. This forms an introduction to the commutative law which will be further developed in segment 1.7 <i>Addition and subtraction: strategies within 10</i>. Continue to use concrete and pictorial contexts with two easily distinguishable parts, but now write both expressions for each context.</p> <p>Use examples that take children through the following progression:</p> <ul style="list-style-type: none"> <li>• The two parts are clearly grouped together and the objects are rearranged to scaffold the rearrangement of the expression (for example, the empty/full glasses shown here). Show both arrangements and ask children <i>'What's the same?' and 'What's different?'</i></li> <li>• The two parts are clearly grouped together, but now the objects are <i>not</i> rearranged to scaffold the rearrangement of the expression (for example, the hats shown here). Ask children to write <i>both</i> expressions from a single arrangement.</li> <li>• The two parts are <i>not</i> clearly grouped together (for example, the red and yellow flowers); children now need to recognise and enumerate the two groups, and write both expressions.</li> </ul>	<p>Two parts clearly grouped – scaffolded:</p>  <p style="text-align: center;">3                      +                      2</p> <p><i>'There are three full glasses and two empty glasses. We can write this as three plus two.'</i></p> <p style="text-align: center;"><math>3 + 2</math></p>  <p style="text-align: center;">2                      +                      3</p> <p><i>'There are two empty glasses and three full glasses. We can write this as two plus three.'</i></p> <p style="text-align: center;"><math>2 + 3</math></p> <p>Two parts clearly grouped – unscaffolded:</p>  <ul style="list-style-type: none"> <li>• <i>'There are two red hats and four blue hats. We can write this as two plus four.'</i></li> </ul> <p style="text-align: center;"><math>2 + 4</math></p>

<p>Children should continue to describe the contexts in full sentences.</p>	<ul style="list-style-type: none"> <li>• <i>There are four blue hats and two red hats. We can write this as four plus two.'</i> <math>4 + 2</math></li> </ul> <p>Two parts <i>not</i> clearly grouped:</p>  <ul style="list-style-type: none"> <li>• <i>There are four yellow flowers and three red flowers. We can write this as four plus three.'</i> <math>4 + 3</math></li> <li>• <i>There are three red flowers and four yellow flowers. We can write this as three plus four.'</i> <math>3 + 4</math></li> </ul>
<p><b>1:3</b> Now progress to contexts for which the whole group is <i>not</i> divided into two easily distinguishable parts. Invite children to partition the group into two parts in different ways; for each distinct partitioning, they should write <i>both</i> expressions, for example <math>2 + 5</math> and <math>5 + 2</math>.</p> <p>This follows similar work in segment 1.2 <i>Introducing 'whole' and 'parts': part-part-whole</i>, where the children identified the whole and partitioned it in different ways. Now children have progressed to using an addition symbol to show that the two parts can be combined (aggregated).</p>	<p><i>There are seven flowers. How many ways can you find to make two bunches? Write addition expressions to match the number of flowers in each bunch.'</i></p> 

## 1.5 Aggregation and partitioning

		<p><i>'There are five flowers in one bunch and two flowers in the other. We can write this as five plus two or as two plus five.'</i></p> <p><math>5 + 2</math></p> <p><math>2 + 5</math></p>
<p><b>1:4</b></p> <p>Introduce general representations, such as cubes and counters:</p> <ul style="list-style-type: none"> <li>Put a different quantity of cubes in each hand, making sure they are visible; children write the expression to match (e.g. <math>3 + 4</math>). Then cross over your hands; children write the new expression (e.g. <math>4 + 3</math>).</li> <li>Place a different quantity of cubes in each of two pots; children write <i>both</i> expressions that can represent the partitioning of the cubes.</li> <li>Use double-sided counters for examples where a given number can be partitioned in more than one way.</li> </ul> <p>Continue to encourage children to describe the contexts in full sentences.</p>		<p>Generalised representation – scaffolded rearrangement of the expression:</p>  <p><i>'There are three cubes in this hand.'</i></p> <p><i>'There are four cubes in this hand.'</i></p> <p><i>'We can write this as three plus four.'</i></p> <p><math>3 + 4</math></p>  <p><i>'There are four cubes in this hand.'</i></p> <p><i>'There are three cubes in this hand.'</i></p> <p><i>'We can write this as four plus three.'</i></p> <p><math>4 + 3</math></p>

		<p>Generalised representation – unscaffolded rearrangement of the expression:</p>  <p><i>'There is one cube in this pot.'</i></p> <p><i>'There are five cubes in this pot.'</i></p> <p><i>'We can write this as one plus five or five plus one.'</i></p> <p><math>1 + 5</math></p> <p><math>5 + 1</math></p>
<p><b>1:5</b></p>	<p>Throughout, make sure you include examples where one of the 'parts' is zero, for example:</p> <ul style="list-style-type: none"> <li><i>'There are five counters red-side-up, and no counters yellow-side-up.'</i></li> <li><i>'We can write this as five plus zero: <math>5 + 0</math>.'</i></li> <li><i>'We can write this as zero plus five: <math>0 + 5</math>.'</i></li> </ul> <p>Note – this is an unscaffolded example, since we do not swap the order of the counters over in the sentence (or corresponding picture/manipulatives) before swapping the order of the addends in the expression.</p>	
<p><b>1:6</b></p>	<p>You can use a <i>dòng nǎo jīn</i> question to assess and promote depth of understanding. The example shown here requires children to think about both partitioning in more than one way and swapping the order of the addends in the expression.</p>	



Dòng nǎo jīn:

'Here are some school bags.'

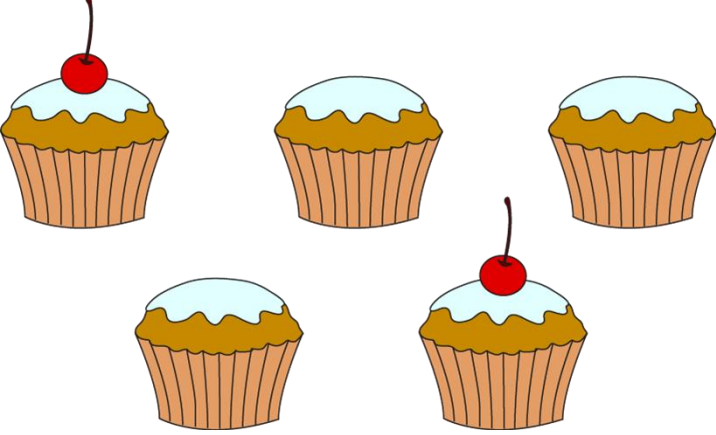


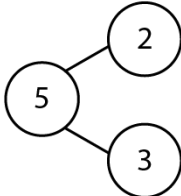
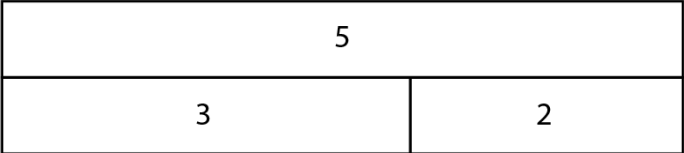
- 'Jay writes down the following expression to describe the bags:  
 $5 + 2$   
Sally writes  $4 + 3$ .  
Joel writes  $2 + 5$ .  
Who is right?'
- 'Can you write down another expression to describe the bags?'

**Teaching point 2:**

The equals symbol, =, can be used to show equivalence between the whole and the sum of the parts.

**Steps in learning**

	<b>Guidance</b>	<b>Representations</b>
<b>2:1</b>	<p>Once children are secure with writing expressions using the addition symbol, introduce the equals sign to show the equivalence between the sum of the two parts and the whole group. Children should already be familiar with the concept that two parts combine to equal the whole (see segment 1.2 <i>Introducing 'whole' and 'parts': part-part-whole</i>). In this teaching point, focus on the use of the equals sign to write an equation of the form:</p> <p style="padding-left: 20px;">addend + addend = sum</p> <p>Emphasise that the = symbol represents 'is equal to'.</p> <p>As in steps 1:1 and 1:2 above, use concrete or pictorial representations of contexts where the whole group is divided into two, easily distinguishable parts. For each example:</p> <ul style="list-style-type: none"> <li>• begin by referring to the whole and model how it can be split into two parts, for example, <math>5 = 2 + 3</math>.</li> <li>• show how four different equations can be written, for example:            <math>5 = 2 + 3</math>  <math>5 = 3 + 2</math>  <math>2 + 3 = 5</math>  <math>3 + 2 = 5</math> </li> </ul> <p>Note that we are now writing equations rather than expressions, since we are including the equals symbol to equate the combined parts to the whole.</p> <p>Throughout, ensure that:</p>	 <p>• <i>'There are five cakes. There are two cakes with cherries and three cakes without cherries.'</i></p> <p>• <i>'We can write this as five is equal to two plus three.'</i></p> <p style="padding-left: 20px;"><math>5 = 2 + 3</math></p> <p>• <i>'We can write this as five is equal to three plus two.'</i></p> <p style="padding-left: 20px;"><math>5 = 3 + 2</math></p> <p>• <i>'We can write this as two plus three is equal to five.'</i></p> <p style="padding-left: 20px;"><math>2 + 3 = 5</math></p> <p>• <i>'We can write this as three plus two is equal to five.'</i></p> <p style="padding-left: 20px;"><math>3 + 2 = 5</math></p> <p>• <i>'Two is an addend, three is an addend, and five is the sum.'</i></p>

	<ul style="list-style-type: none"> <li>• children explain, in full sentences, what each number/symbol in the equation represents</li> <li>• you model the use of the words 'addend' and 'sum' to refer to the different parts of the equation.</li> </ul>	
<p><b>2:2</b></p>	<p>You can build on children's previous experience of the part-part-whole model, using either the cherry representation or bar model to represent the contexts. Make sure that children:</p> <ul style="list-style-type: none"> <li>• describe how the numbers in the abstract representation relate to the context (for example, '<i>The 5 represents the whole group of cakes; the 3 represents the cakes with no cherries; the 2 represents the cakes with cherries.</i>')</li> <li>• explain which numbers represent parts and which number represents the whole (for example, '<i>Five is the whole; three is a part; two is a part.</i>')</li> <li>• continue to write the equation in all four ways</li> <li>• continue to use the words 'addend' and 'sum'.</li> </ul> <p>If, after moving to these representations, you find that children mistakenly interpret, for example, <math>5 = 3 + 2</math> to mean, for example, 'five plus three plus two', give them more time using concrete resources to ensure that they fully master the meaning of the equations before moving on.</p>	<p>Cherry representation:</p>  <p>Bar model:</p> 
<p><b>2:3</b></p>	<p>As with step 1:3, now consider contexts where the whole group is a number of identical objects that can be partitioned in different ways. For each example, ask children to:</p> <ul style="list-style-type: none"> <li>• draw a part-part-whole model to represent each of the different</li> </ul>	

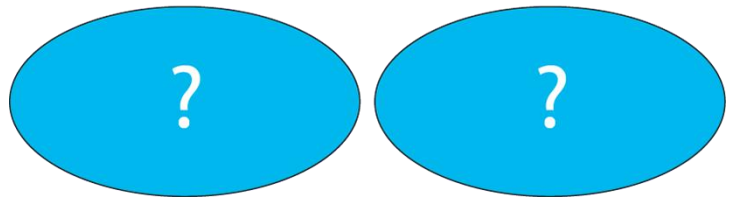
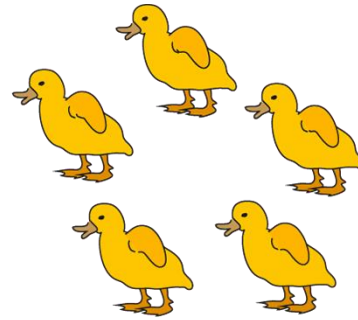
outcomes, and describe how the model represents the outcome

- write all four equations to represent each of the different outcomes, and describe how the equations represent the outcome.

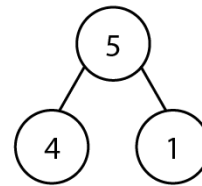
By this point, children should be confidently using the following stem sentences:

- '    is equal to     plus    .'
- '    plus     is equal to    .'
- '    and     are the addends.'
- '    is the sum.'

'Mother duck is in the park with her five ducklings. There are two ponds. How many ducklings could be playing in each pond?'



Example outcome:



5	
4	1

$$5 = 4 + 1$$

$$5 = 4 + 1$$

$$5 = 1 + 4$$

$$5 = 1 + 4$$

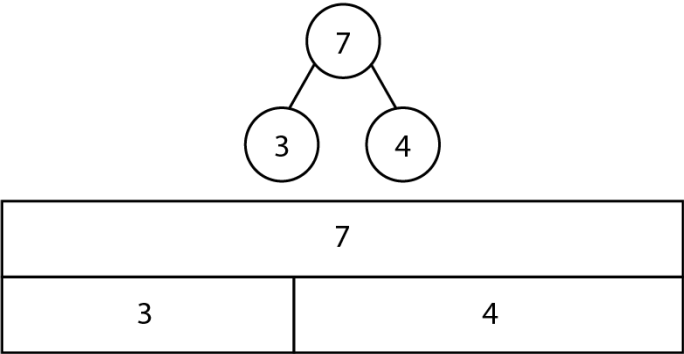

**2:4**

As with step 1:4, you can use generalised representations, such as double-sided counters, without a story context. Children could explore different partitioning and number sentences, then create their own stories to go with each. They should explain how the manipulatives, part-part-whole model, and equations represent their story.



- 'There are seven animals; three are cats and four are dogs.'
- 'The seven counters represent the seven animals; the three blue counters represent the three cats; the four red counters represent the four dogs.'

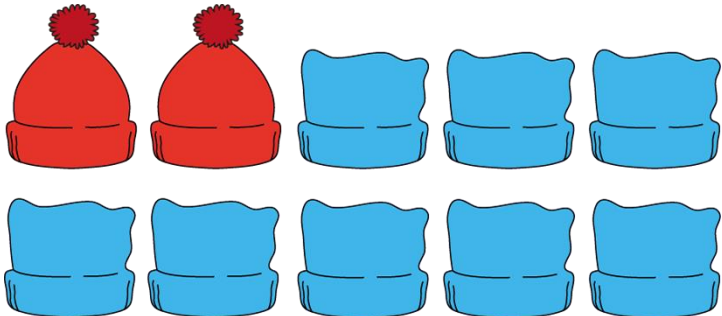
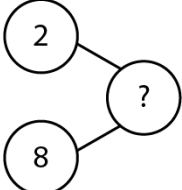
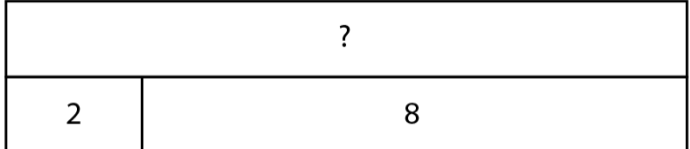
## 1.5 Aggregation and partitioning

		 <p>'Seven is the whole; three is a part; four is a part.'</p> $7 = 3 + 4$ $7 = 4 + 3$ $3 + 4 = 7$ $4 + 3 = 7$
<p><b>2:5</b></p>	<p>Again, include contexts for which one of the addends is zero. You could use the following problem to encourage children to 'remember' zero:</p> <p><i>'Dan has ten pennies and two pockets. Sarah says, "The most pennies that Dan can have in one pocket is nine." Is she correct?'</i></p> <p>As discussed in segment 1.2 <i>Introducing 'whole' and 'parts': part-part-whole</i>, remember not to refer to zero as a 'part'. Zero <i>can</i> be represented on the cherry model, but referring to it as a 'part' implies that it has a value, which may lead to misconceptions when children are using the bar model.</p>	
<p><b>2:6</b></p>	<p>Use a <i>dòng não jīn</i> question similar to that presented in step 1:6 above – the main difference now is that children should be looking at the full equation, and not just an expression of the sum of the two addends.</p>	<p><i>'Ilsan and Walid look at this picture.'</i></p>  <p><i>'Ilsan writes <math>9 = 3 + 6</math>.</i></p> <p><i>Walid writes <math>4 + 5 = 9</math>.</i></p> <p><i>Who is correct? Explain.'</i></p>

## Teaching point 3:

Each addend represents a part, and these are combined to form the whole/sum; we can find the value of the whole by adding the parts. We can represent problems with missing parts using an addition equation with a missing addend.

### Steps in learning

	Guidance	Representations
<p><b>3:1</b></p>	<p>Up until now you have been presenting children with the values of both the parts/addends and the whole/sum, or providing the whole and asking children to partition into parts themselves. Now progress to problems where children must <i>find</i> the whole/sum – i.e. problems where the whole is unknown.</p> <p>As usual, begin with concrete/pictorial contexts in which there are two clearly distinguishable parts. Children should:</p> <ul style="list-style-type: none"> <li>• represent the story using a part-part-whole diagram (cherry or bar model); this gives a good indication of whether their understanding of the structure is secure</li> <li>• find the sum</li> <li>• write the corresponding equations.</li> </ul> <p>Continue to ensure variation of the order of the addends and the position of the equals sign. Children should also continue to describe the contexts/equations.</p>	<p><i>'There are two red hats and eight blue hats. How many hats are there altogether?'</i></p>    <ul style="list-style-type: none"> <li>• <i>'There are two red hats and eight blue hats; there are ten hats altogether.'</i></li> <li>• <i>'Two plus eight is equal to ten.'</i> <math>2 + 8 = 10</math></li> <li>• <i>'Eight plus two is equal to ten.'</i> <math>8 + 2 = 10</math></li> <li>• <i>'Ten is equal to two plus eight.'</i> <math>10 = 2 + 8</math></li> <li>• <i>'Ten is equal to eight plus two.'</i> <math>10 = 8 + 2</math></li> <li>• <i>'Two is an addend, eight is an addend, and ten is the sum.'</i></li> </ul>

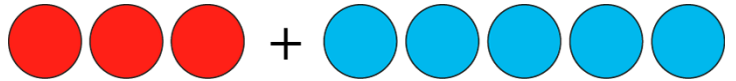
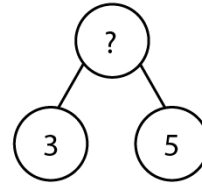
3:2

Progress to generalised representations, such as double sided counters or multilink – initially use them alongside the concrete/pictorial, then, as children gain confidence, remove the concrete/pictorial. Check understanding by asking children what each number in the equation represents, and encourage the use of precise language.

When using multilink, ensure that children *join* the two addends to make the sum. A potential misconception arises when children show an additional row of cubes; in the example opposite, this would give a total of 16 cubes instead of 8.

Counters and cherry model:

*There are three ducks in one pond and five ducks in the other pond. How many ducks are there altogether?*



$$3 + 5 = 8$$

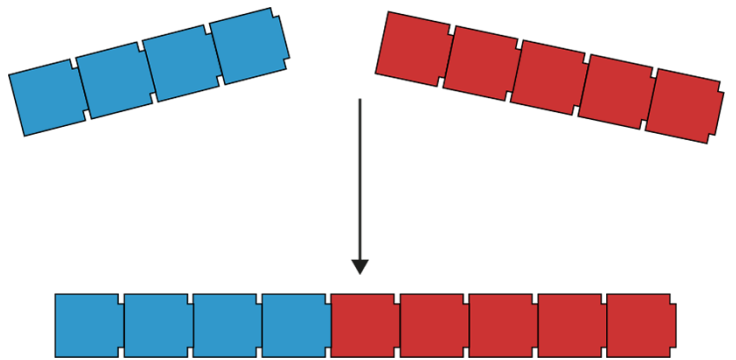
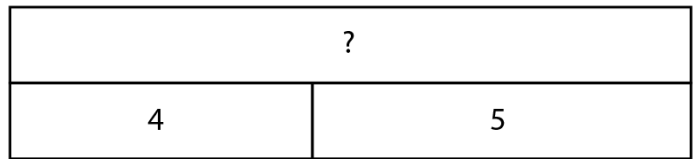
$$5 + 3 = 8$$

$$8 = 3 + 5$$

$$8 = 5 + 3$$

Multilink and bar model:

$$4 + 5 = \square$$

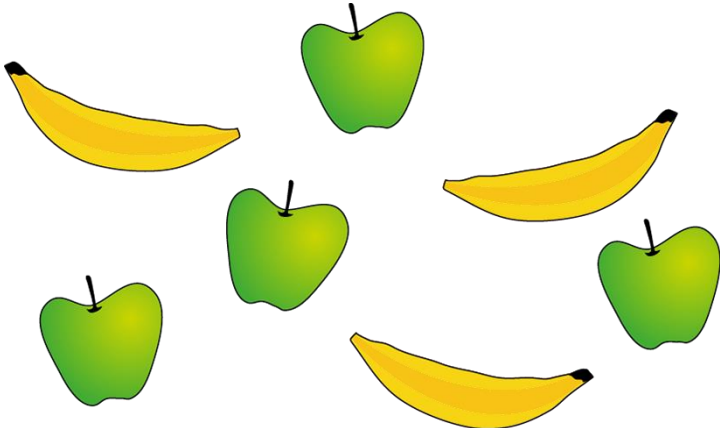


$$4 + 5 = 9$$


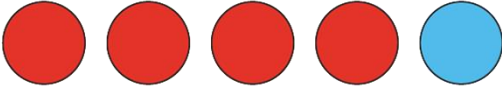
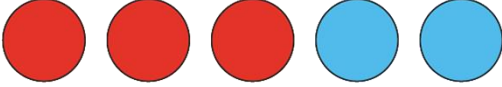
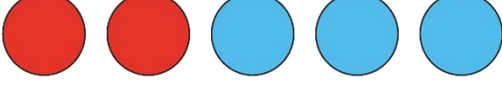

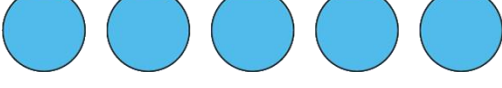
$$5 + 4 = 9$$

$$9 = 4 + 5$$

$$9 = 5 + 4$$

<p><b>3:3</b> Provide varied practice:</p> <ul style="list-style-type: none"> <li>• Present children with a story and/or a part-part-whole diagram with the whole missing (cherry or bar model), and ask them to: <ul style="list-style-type: none"> <li>• identify the correct equation from a selection, explaining their choice and why the other equations can't be right</li> <li>• complete equations with missing sums</li> <li>• write the full equations from scratch.</li> </ul> </li> <li>• Present children with an equation and ask them to: <ul style="list-style-type: none"> <li>• represent it using concrete resources or by drawing a picture</li> <li>• tell a story to go with the equation.</li> </ul> </li> </ul> <p>Remember to include examples where one of the addends is zero.</p> <p>To ensure that children can correctly use the symbols, you could provide true-or-false style questions with incorrectly formulated equations, for example:</p> <p><math>4 + 3 + 7 \times</math></p> <p><math>7 = 4 = 3 \times</math></p> <p><math>4 = 3 + 7 \times</math></p> <p>Use a <i>dòng não jīn</i> problem with three parts/addends to provide further challenge, for example:</p> <p><i>'Fill in the missing number and draw a cherry model to represent the equation.'</i></p> <p><math>1 + 2 + 4 = \square</math></p>	 <ul style="list-style-type: none"> <li>• 'Which equation matches the picture?' <ul style="list-style-type: none"> <li><math>3 + 3 = 6</math></li> <li><math>8 = 4 + 3</math></li> <li><math>5 = 4 + 1</math></li> <li><math>4 + 3 = 7</math></li> </ul> </li> <li>• 'Can you explain your choice?'</li> <li>• 'Can you explain why the others can't be right?'</li> </ul>
<p><b>3:4</b> In steps 3:1–3:3, the parts (addends) were given and the unknown value was the whole (sum). Now move on to using this structure to present problems where the whole is known, but the value of one of the parts is</p>	

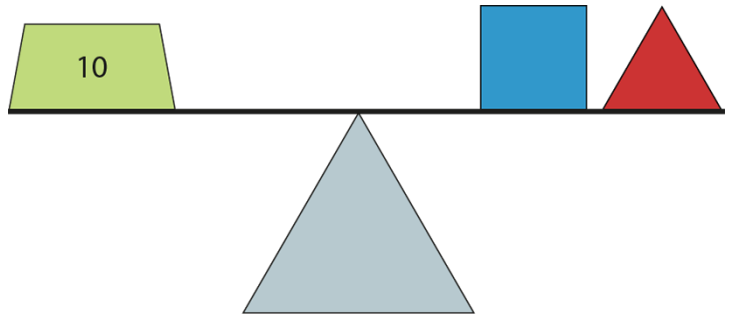


	<p>unknown, representing the problems with missing addend equations.</p> <p>Begin by using structured representations which clearly show both parts, such as lines of double-sided counters. Children should use this as a scaffold to complete the missing addend equations. The example shown here models systematic working – children should already be familiar with this from their work in segments <i>1.3 Composition of numbers: 0–5</i> and <i>1.4 Composition of numbers: 6–10</i>. Now they have progressed to writing equations to match the representations.</p> <p>By now children will be developing factual fluency in partitioning quantities within ten; some children may be able to complete these equations without using the scaffold, but at this stage it should still be provided.</p>	<p><i>'Fill in the missing numbers.'</i></p> <p>5 = 5 + 0 </p> <p>5 = 4 + <input type="text"/> </p> <p>5 = 3 + <input type="text"/> </p> <p>5 = 2 + <input type="text"/> </p> <p>5 = 1 + <input type="text"/> </p> <p>5 = 0 + <input type="text"/> </p>
<p><b>3:5</b></p>	<p>In step 3:3, children practised completing equations with an unknown part alongside a supporting visual image, which showed the cardinality (size) of the number. Now remove the visual scaffolding, asking children to simply complete the equations. As shown here, you can change the position of the equals sign to provide variation.</p>	<p><i>'Fill in the missing numbers.'</i></p> <p>7 + <input type="text"/> = 7</p> <p>6 + <input type="text"/> = 7</p> <p>5 + <input type="text"/> = 7</p> <p>4 + <input type="text"/> = 7</p> <p>3 + <input type="text"/> = 7</p> <p>2 + <input type="text"/> = 7</p> <p>1 + <input type="text"/> = 7</p> <p>0 + <input type="text"/> = 7</p>

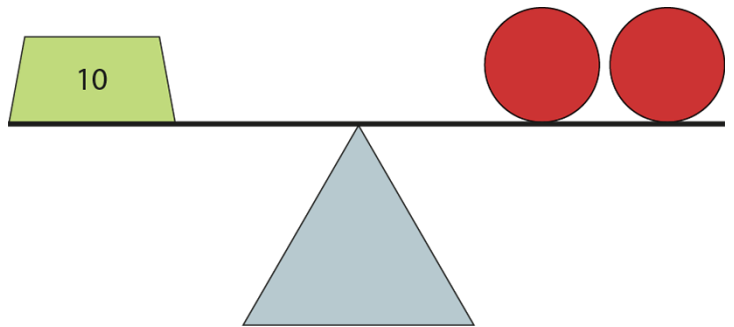
**3:6**

To promote further depth, use a dòng não jīn challenge which provides further abstraction. The example shown here requires children to reason and explain their answers.

*'Ten is balanced by a square and a triangle. So ten is equal to square plus triangle. Square and triangle are different numbers. What numbers do you think they are? Can you find more than one answer?'*



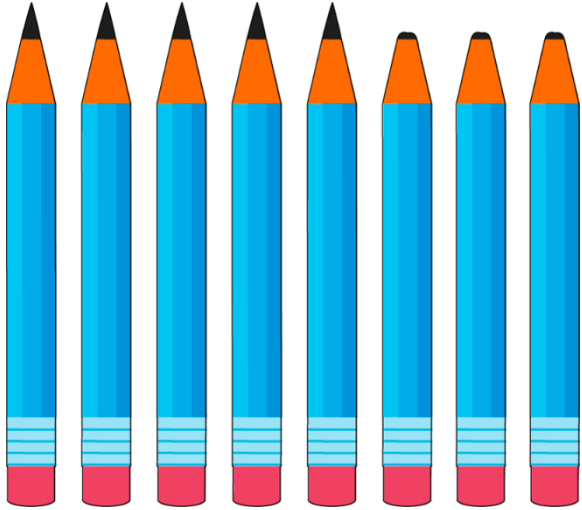
*'Ten is balanced by the two circles. So ten is equal to circle plus circle. What number do you think a circle represents? Why?'*



**Teaching point 4:**

Breaking a whole down into two or more parts is called partitioning; the subtraction symbol,  $-$ , can be used to represent partitioning.

**Steps in learning**

	<b>Guidance</b>	<b>Representations</b>
<b>4:1</b>	<p>Having experienced finding an unknown part represented within an addition equation, children can progress to finding the unknown part using a subtraction structure. This is the partitioning structure of subtraction: there are two distinct parts, one of which is unknown (it can also be thought of as the 'not' structure – see the examples below). This is different from the reduction structure, which will be covered in segment 1.6 <i>Additive structures: introduction to augmentation and reduction</i>, where one part is removed, or taken away from the whole. The reduction structure is the one that is most commonly taught, although in everyday life partitioning is actually a very common subtraction situation. Examples of the partitioning structure of subtraction include:</p> <ul style="list-style-type: none"> <li>• <i>'There are six children. Two have put their coats on. How many have not put their coats on?'</i></li> <li>• <i>'There are eight pencils. Five have been sharpened. How many have not been sharpened?'</i></li> <li>• <i>'There are five windows. Three are open. How many are closed?'</i></li> <li>• <i>'There are seven children. Six of them are having packed lunch. How many are not having packed lunch?'</i></li> </ul> <p>Present children with contextual examples like these, for numbers within ten.</p>	<p><i>'There are eight pencils. Five have been sharpened. How many have not been sharpened?'</i></p> 

4:2

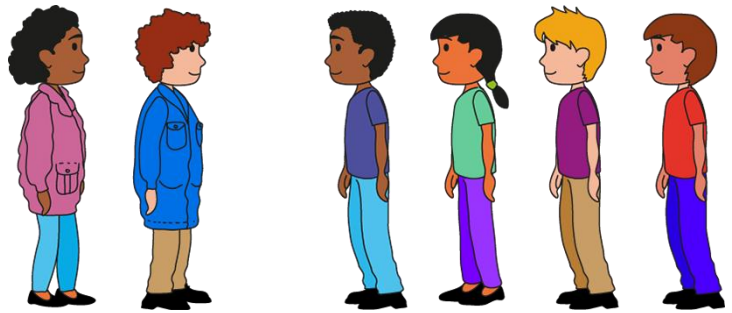
Now work towards introducing children to the subtraction symbol; although the concept of finding an unknown part will be very familiar to them by now, this will be the first time they encounter the subtraction symbol.

Use one of the contexts from step 4:1 (here we will use the example of children and coats), following these steps:

- Present the context pictorially, along with this question: *'There are six children. Two of them have put their coats on. How many have not put their coats on?'*
- Ask a child to circle the children who have put their coats on.
- Ask the children *'How many children have not put their coats on?'*
- Allow children to discuss and justify their answer.
- Having solved the problem, tell the children that this story can be written as a subtraction equation of the form:  
minuend – subtrahend = difference  
Show the equation  $6 - 2 = 4$ .
- Ask children what they notice about the equation and draw attention to the subtraction symbol which in this context means 'separate', 'split' or 'partition'.
- Then ask children to describe, in full sentences, what each number/symbol represents:
  - *'The 6 represents all of the children.'*
  - *'The minus 2 represents the children who have put their coats on.'*
  - *'The 4 represents the children who have not put their coats on.'* (The solution to the problem.)

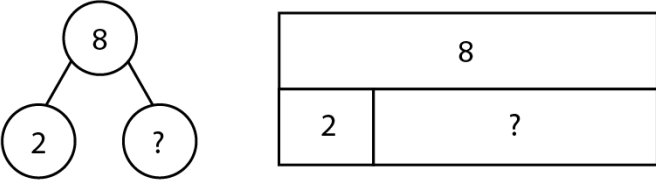
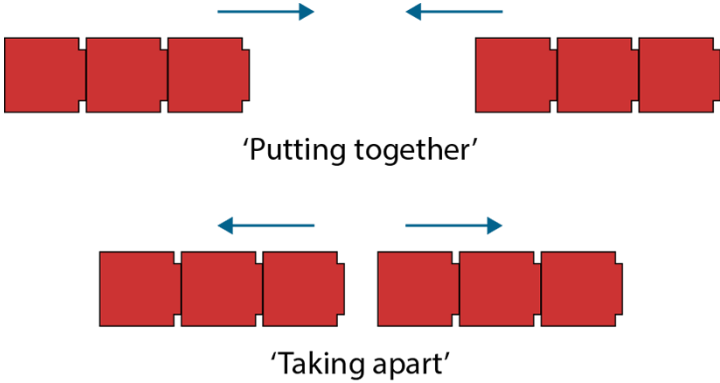
Throughout, model the use of the words 'minuend', 'subtrahend' and 'difference' to refer to the different parts of the equation.

*'There are six children. Two of them have put their coats on. How many have not put their coats on?'*



$$6 - 2 = 4$$

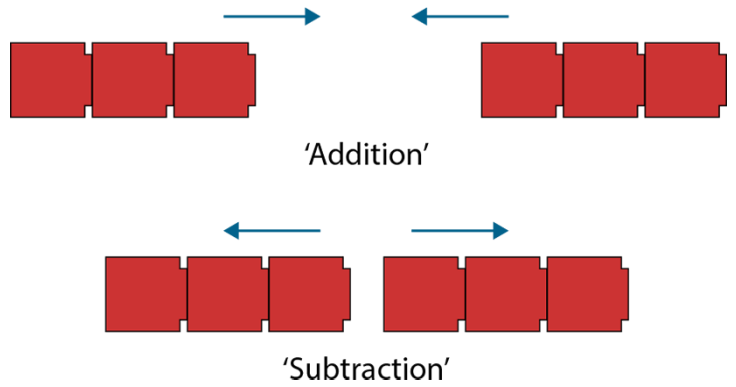
- *'The 6 represents all of the children.'*
- *'The minus 2 represents the children who have put their coats on.'*
- *'The 4 represents the children who have not put their coats on.'*

	<p>Explore a variety of contexts that represent the partitioning structure, for example:</p> <p><i>'The baker has six cookies. He gives four to Anya and the rest to Jake. How many cookies does Jake have?'</i></p> <p><math>6 - 4 = \square</math></p> <p><math>6 - 4 = 2</math></p> <p>You could extend this example into a dòng nǎo jīn problem: <i>'Write a subtraction equation that represents a story where Jake has the same number of cookies as Anya.'</i></p>	
<p><b>4:3</b></p>	<p>Now represent problems using part-part-whole diagrams (cherry or bar model). Children need to be able to interpret a problem and identify where each number fits within the structure of a part-part-whole relationship.</p> <p>As well as <i>presenting</i> problems using part-part-whole diagrams, ensure that children also practise representing subtraction problems themselves in this way. It is important that they identify the unknown part clearly by representing it with a question mark. Children can use this as an interim step before writing the equation and solving the problem.</p>	<p><i>'There are eight flowers. Two are red and the rest are yellow. How many are yellow?'</i></p> 
<p><b>4:4</b></p>	<p>Finally, bring together the two structures of addition and subtraction explored in this segment:</p> <ul style="list-style-type: none"> <li>• Give each child a stick of multilink cubes and practise moving them together and apart, whilst saying the words <i>'putting together'</i> and <i>'taking apart'</i>, and <i>'addition'</i> and <i>'subtraction'</i>.</li> <li>• Ask children to think of a story for each structure (aggregation and partitioning), reminding them of the types of stories explored earlier (quantities were put together to</li> </ul>	<p>Addition and subtraction:</p> 

## 1.5 Aggregation and partitioning

form the whole, or partitioned/separated to find an unknown part). Use generalised representations (such as cubes or counters) to illustrate the stories and enable children to clearly see the generalised structure of each.

- Use a scene like the one shown here to explore a variety of aggregation and partitioning contexts.



Aggregation and partitioning contexts:

