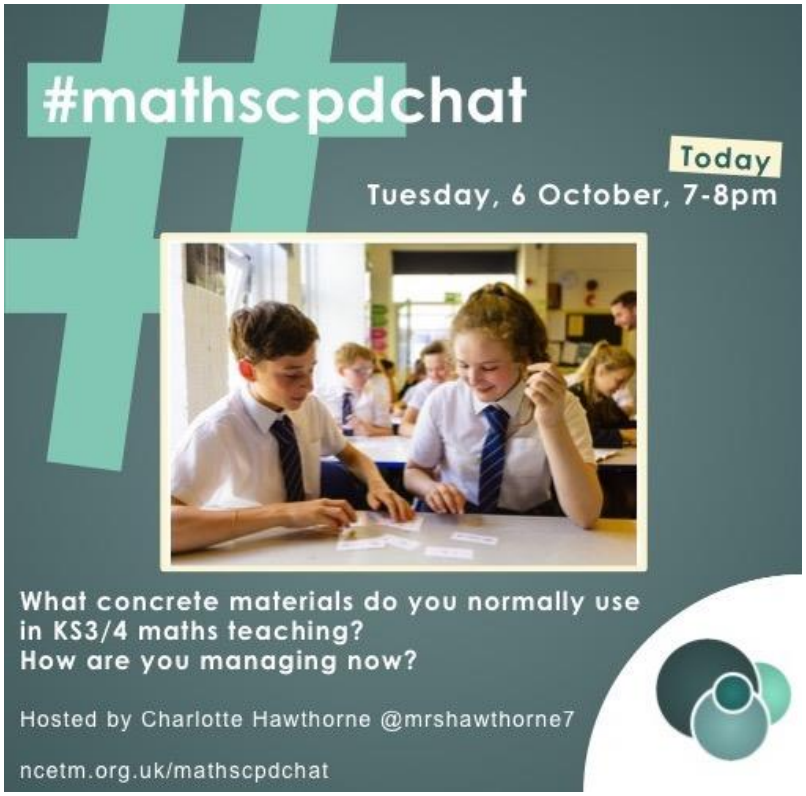
# #mathscpdchat 6 October 2020

**What concrete materials do you normally use in KS3/4 maths teaching? How are you managing now?**

Hosted by Charlotte Hawthorne

*This is a brief summary of the discussion – to see all the tweets, follow the hashtag* ***#mathscpdchat*** *in Twitter*




The **results of a poll,** tweeted by the host at the start of the chat, were:


The **results of a second poll,** tweeted by the host part-way through the chat, were:


- other 'favourite' manipulatives/concrete materials that were mentioned included … 2-D and 3-D pegboards, pattern blocks, prime factor counters and equivalent fraction tiles;
- a teacher commented that pattern blocks are helpful in teaching early geometrical ideas, and can be used to aid 'most kinds of multiplicative reasoning';

Some of the other areas where discussion focussed were:

**concrete materials that teachers are currently using**:

- owing to their present teaching environments many teachers are not using any concrete materials … they are relying on the use of **virtual manipulatives** to support both in-school and remote learning … some teachers are using physical/concrete manipulatives only on a

visualiser so that pupils do not touch them … for example 'I am using Cuisenaire® rods under the visualiser and really missing having the kids manipulating them too';

- some teachers are **using real double-sided (red-yellow) counters to help pupils understand directed number calculations** … for example each pair of pupils has ten such counters to work with in lessons … pupils are introduced to the idea of 'zero sum pairs' (a counter placed red-side-up (representing negative-1) together with another counter placed yellow-side-up (representing 1)) … to subtract negative-1 from any number, the red-side-up counter is removed from the 'zero sum pair' leaving one extra positive (yellow) counter, thus representing the fact that subtracting negative-1 is equivalent to adding 1 … the point was made that this builds on pupils' understanding of subtraction as 'take away' … it was suggested that the two sides of each counter might be 'hatched', rather than coloured, in different ways to make their use accessible to colour-blind pupils … the use of physical counters is presently limited owing to necessary anti-virus sanitising procedures … at least one teacher makes his own single-use paper counters;
- some teachers **use double-sided counters 'alongside a number line approach'** (possibly with vectors) … some teachers are concerned that introducing Year 7 or 8 pupils to the representation of directed number operations using double-sided counters might confuse those pupils who in Year 6 appeared to understand well number line representations … 'if I could only have one representation for number, the number-line would be it' … that it is effective to work with pupils using multiple representations 'so it leaves the door open for other approaches that can be generated by the students';
- some teachers are using physical **algebra tiles** with pupils who struggle to grasp demonstrations performed by the teacher using online virtual algebra tiles … pupils 'move on to drawing the tiles in their books', and finally to carrying out algebraic procedures without tiles;
- using virtual manipulatives in online resources created **using Geogebra** (link provided below);
- that **'Key Stage 4 topics, like completing the square, are so much easier with manipulatives'** … students liked the concrete representations 'Oh Miss I understand now why it's called completing the square' … the teacher 'felt guilty for not showing them earlier';

**'top tips' (of contributors to the chat) for teachers who are considering using manipulatives for the first time:**

- when you are using a particular set of manipulatives for the first time **make sure that you yourself clearly understand what you are trying to represent and how the objects will represent it** … if you have plenty of practice first (with colleagues if possible) of 'playing' with the manipulatives you not only increase your confidence in understanding their potential uses, but you may also anticipate pupils' possible 'stumbling blocks' … also

rehearse the ‘phasing out’ process (from using the manipulatives), knowing that you and/or your pupils can always return to using them if you want to;

- that concrete materials **can ‘loose impact’ if they are used only in a very prescriptive way**;
- make sure that you **allow students who are using manipulatives for the first time enough time to understand HOW the manipulatives are representing the ideas or procedures** … for example, students ‘took nearly a whole lesson’ to understand how double-sided counters were being used to represent directed number calculations;
- when using manipulatives with a class for the first time ‘try a small activity to start with, that **doesn’t have to use up the whole lesson**, or a small demonstration’;
- when **using geoboards with students for the first time** it is advisable to say that they are being trusted to use them sensibly (e.g. to not ‘twang’ the rubber bands) … practise all together how to form shapes on the board and ‘count/perceive/describe’ properties of shapes … some teachers start by following a ‘I go, we go, you go’ action routine in order to help pupils ‘see how it is done’ (with the teacher using concrete materials on a visualiser or working in an online virtual manipulative environment);
- that **manipulatives can be any physical objects** (such as Hula Hoops or A4 sheets of paper) … you don’t always necessarily need a purchased ‘set of this or that’;
- some teachers find that it is effective to **introduce manipulatives first as concrete-objects-representing-mathematical-objects** (for example using Hula Hoops to represent circles) before moving on to using manipulatives in more complex representations that aim to facilitate the grasping of procedures and relationships;
- some teachers have **used manipulatives with ‘older students’** … for example at least one teacher in the chat had ‘used them with postgrad students – some wanted to (and did) build towers. It did my nut in!’;
- that **if older students start by saying that using manipulatives is ‘childish’** the key is to let them ‘get the silliness out of their system first’;
- when using two-colour counters with Year 11 students the resistance (seeing the using of manipulatives as childish) was mild and did not last long … they loved them, and subsequently made far fewer mistakes in calculations with negative numbers … that **using concrete materials can support and enhance the learning of all pupils including the highest attainers of any age** … that it is tempting with high attainers to go ‘straight for the abstract because I knew they could handle it’;
- **give all pupils time to ‘play’ with the manipulatives**, and while they are ‘playing’ observe what they like doing so you can build tasks on the pupils’ first observations;
- that teachers **need not always ‘show’ pupils how to use manipulatives** … that it can be most effective to ‘relinquish control and allow children to construct their own meanings’;
- **count in and count out** the materials;

**contributors' favourite ways to use multi-link cubes or geoboards:**

- using **multilink cubes** to enhance learning to find the nth term of a sequence, and also in learning to create 2-D images of 3-D objects … that physically building 3-D objects helps students develop the ability to visualise 'views' of 3-D objects;
- square **geoboards** are helpful when learning about area and fractions, properties of quadrilaterals and transformations;

**what is holding back those teachers who would like to make more use of concrete materials in their teaching:**

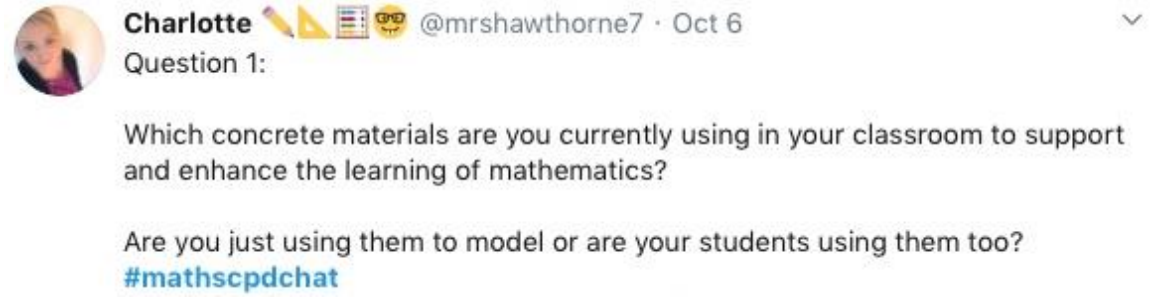
- not having a 'complete set' of manipulatives for each class;
- the practicality of having to move from room to room (in the present circumstances);
- having to wash the objects after every use;
- not feeling confident … feeling like a novice … lack of 'training';
- feeling that there is not enough teaching time;

**the aspects of their maths learning in which teachers believe students would benefit most from using concrete materials:**

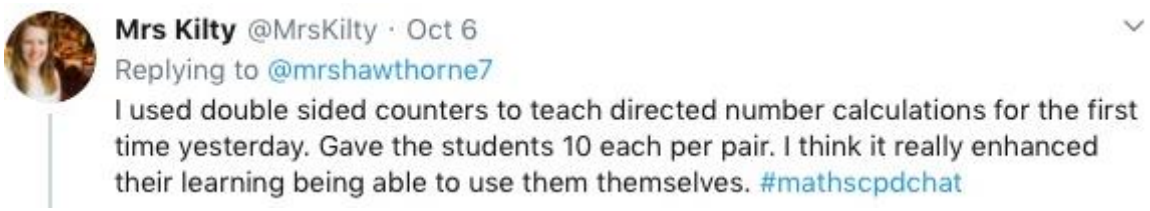
- in getting to understand and be able to work efficiently with place value … using a place value chart with digit tiles;
- understanding the distributive law and factorising … using unit tiles and Cuisenaire® rods;
- in 'sorting out the basics' of understanding and using algebra, including in solving equations.

In what follows, click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

This is a part of a conversation about using double-sided counters to support learners in calculating with directed numbers … how to represent the subtraction of a negative number in particular. The conversation was generated by this tweet from Charlotte Hawthorne:




and included these from Mrs Kilty and Charlotte Hawthorne:



**Charlotte** ✏️📐🗒️🤓 @mrshawthorne7 · Oct 6
That's really interesting! How did the students respond? #mathscpdchat

**Mrs Kilty** @MrsKilty · Oct 6
Replying to @mrshawthorne7
They really took to it. Although subtracting a negative still felt like a huge leap & like magic. **#mathscpdchat**

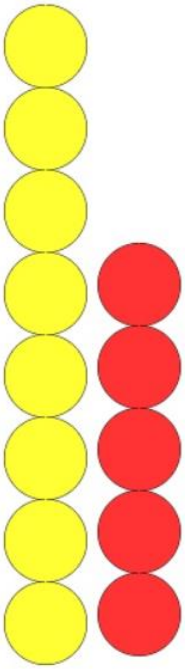
these from Heather Scott, Sam Webster and Rob Brown:

**Heather Scott** @MathsladyScott · Oct 6
Yes, would really like some guidance on how to model 3 - -4 for example #mathscpdchat 😀 I think if I can just get that sorted I would use them too 😀

**Sam Webster** @WebsterMaths · Oct 6
Replying to @MathsladyScott @mrshawthorne7 and @MrsKilty
In Year 7, as students already have a concept of subtraction as 'taking away' I would model this with 3 positive counters, discuss how we don't have any negative counters to remove, and that we can add in zero pairs until we can.

**Rob Brown** @mrbrownsays · Oct 6
Replying to @MathsladyScott @mrshawthorne7 and @MrsKilty
Can I try that? As much for my own CPD as yours.

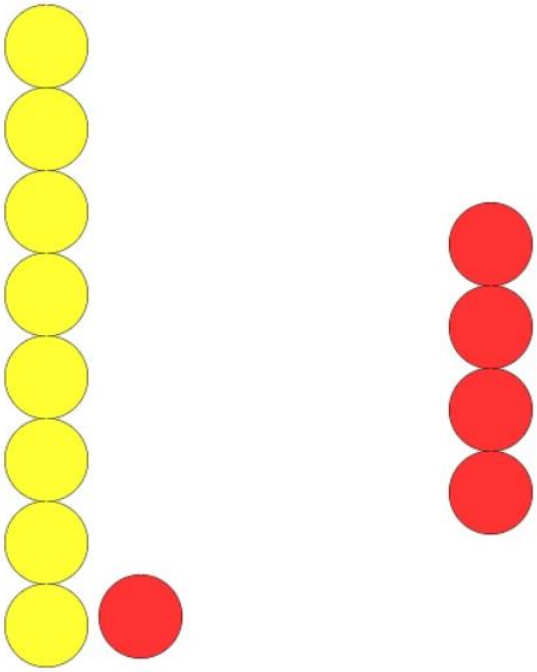
**Rob Brown** @mrbrownsays · Oct 6
Replying to @mrbrownsays @MathsladyScott and 2 others
First I would represent my 3 with plenty of zero pairs.



**Rob Brown** @mrbrownsays · Oct 6

Replying to @mrbrownsays @MathsladyScott and 2 others

I would then physically show the subtraction (removal) of 4 negative counters. It can be observed that the result is positive seven.



**Rob Brown** @mrbrownsays · Oct 6

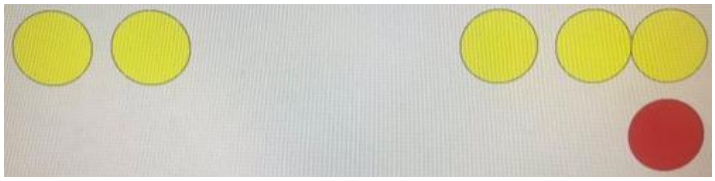
I deliberately used more than 4 zero pairs but not exactly 4. It is clear to us as mathematicians that we only need 4 zero pairs, but a student won't make that leap until we explain that.

these from Mrs Kilty:

**Mrs Kilty** @MrsKilty · Oct 6

Replying to @MrsKilty and @mrshawthorne7

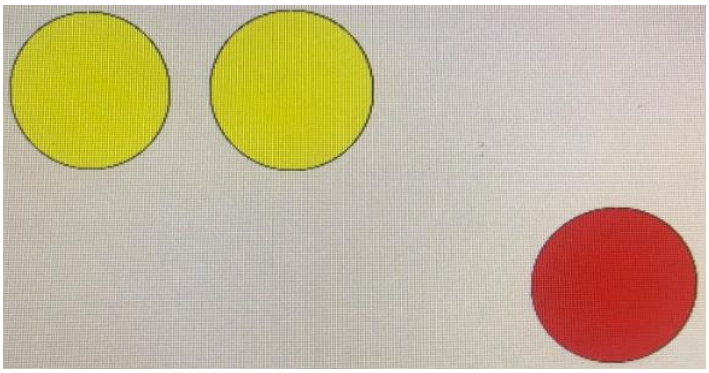
1) do some work on zero sum pairs and eg these are both representations of 2. 2) I can't show 2-(-1) with the left representation but with the right it's straight forward =3 #mathscpdchat



**Mrs Kilty** @MrsKilty · Oct 6

Replying to @MrsKilty and @mrshawthorne7

OR 2-(-1) . Start with two positive. Bringing in the red would be + -1 but we want - -1 which to be the opposite so flip the counter. End with 3 yellow #mathscpdchat



and these from Rob Brown, Peter Mattock, Heather Scott, and Tom Francome:


(to read the discussion sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

Mathsbot Virtual Manipulatives which is a very useful collection of an enormous range of virtual manipulatives. It was shared by Charlotte Hawthorne

Complete Mathematics Webinar CPD Programme which is a programme of webinars during October and November 2020. It includes a description of a webinar, which will be hosted by Jonathan Hall on 15 October, and will focus on the power of the geoboard in engaging and inspiring pupils. It was shared by Jonathan Hall

Pattern Blocks which is a new book by Simon Gregg from the Association of Teachers of Mathematics, which provides a fascinating exploration into using pattern blocks in the maths classroom. It was shared by Charlotte Hawthorne

Cyclic Quadrilaterals (printable dotty circles) which is an NRICH interactivity where you can download printable sheets of circles with (various numbers of) equally-spaced dots on their edges. It was shared by Heather Scott

Cuisenaire Environment which is an NRICH interactive environment where the user can manipulate virtual Cuisenaire® rods, either on their own or collaboratively with other users. It was shared by Heather Scott

Maths link cubes which is where you can buy packs of 100 maths link cubes, with each pack containing 10 sets of 10 cubes, with each set being a different colour. It was shared by Heather Scott

Geogebra Mathematics Classroom Resources which is the starting point from which you can reach many interactive Geogebra mathematical 'environments' (applets). It was shared by MrAdamsMaths

Three Fraction Bars which is a Geogebra applet in which the user can create and move 'fraction bars' representing fractions with denominators ranging from 2 to 12. It was shared by Atul Rana

Matholia tools which is a collection of interactive maths applets. It was shared by Nicola Stevenson

Exploring area and Fractions with Square Geoboards which is a book by Geoff Faux from the Association of Teachers of Mathematics. It provides a sequence of tasks that develop a deep conceptual understanding of area and fractions. It was shared by Mary Pardoe

Non-transitive Dice which is a video in which James Grime and David Spiegelhalter describe, explain and explore ways in which pupils can investigate the consequences of playing games with some very unusual dice! It was shared by Mary Pardoe