

Mastery Professional Development

Mathematical representations



The Gattegno chart

Guidance document | Key Stage 3

	1 000 000	2 000 000	3 000 000	4 000 000	5 000 000	6 000 000	7 000 000	8 000 000	9 000 000	
	100 000	200 000	300 000	400 000	500 000	600 000	700 000	800 000	900 000	
	10 000	20 000	30 000	40 000	50 000	60 000	70 000	80 000	90 000	
	1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000	
	100	200	300	400	500	600	700	800	900	
	10	20	30	40	50	60	70	80	90	
	1	2	3	4	5	6	7	8	9	

10 times larger (x 10)

10 times smaller ($\div 10$)

Please note that these materials are principally for professional development purposes. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

What it is

The Gattegno chart is made up of rows and columns of numbers, with the rows organised according to successive powers of ten. For example, in the Gattegno chart below, the first row contains thousands, the second row hundreds, the third row tens, and so on. The chart can have fewer or more rows, including or excluding decimals. Each column of the chart is a multiple of the first column, with the multiplier increasing by one as you move from left to right, i.e. in the tens row, 1×10 is represented in the first column, followed by 2×10 in the next column, etc.

1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

The numbers in the chart are aligned so that the digit representing each power of ten is in the same position in the column in every row; for example, the digits representing the number of tens in **7 000**, **700** and **70** are all aligned. This alignment creates identifiable place-value columns within each column of numbers, and the resulting staggered effect is visible in the chart above.

Why it is important

The Gattegno chart is particularly useful for highlighting the structure of numbers, as well as allowing students to explore the inverse relationship between multiplication and division. While the organisation of numbers resembles, to some extent, the properties of a place-value chart, the Gattegno chart's focus is linguistic in nature. In the place-value chart, the emphasis is on the fact that a particular digit can have a number of different values, depending on where it is placed in the chart. In the Gattegno chart, the numbers are organised according to their ordinality, making the structure of numbers visible and supporting students in their understanding and use of the language of number.

A general way of working with the chart is to point and tap on the chart, with the whole class calling back a response in unison. Later on, students can do the pointing and tapping, or have their own chart to use. Tapping out a number to be read out loud, or asking students to point to a sequence of cells that represent a given number, can support students in moving between the Gattegno chart, written numerals and spoken number names.

Despite its simplicity, the Gattegno chart is not always easy to use effectively, but it is worth persevering in order to develop your confidence and skill in using it. One of the complexities of using the chart is the need to be flexible and responsive to students' needs and the level of their confidence on any particular task. One way of using the chart is to work with it for 5–10 minutes a day for one week, using a progression of tasks or challenges.

How it might be used

Structure of the number system

If students have previous experience of the Gattegno chart, then they will likely have used it to support counting up and down in different amounts and identifying patterns that the numbers follow. The Gattegno chart is a particularly useful representation for this, since it draws parallels between the numbers 1–9, 10–90, 100–900, and so on.

1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Rehearsing the naming of numbers (both integers and decimals) by pointing to their constituent parts on the Gattegno chart, helps students to appreciate the structure of numbers, and is especially useful when working with very large integers. By pointing to or highlighting different parts of a number, students can practise saying and writing the number in full.

The constituent parts of the number 6 265 973, for example, can be shown on the chart like this:

10 000 000	20 000 000	30 000 000	40 000 000	50 000 000	60 000 000	70 000 000	80 000 000	90 000 000
1 000 000	2 000 000	3 000 000	4 000 000	5 000 000	6 000 000	7 000 000	8 000 000	9 000 000
100 000	200 000	300 000	400 000	500 000	600 000	700 000	800 000	900 000
10 000	20 000	30 000	40 000	50 000	60 000	70 000	80 000	90 000
1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

The chart can support students in reading the number and writing it down, saying the number as six million, two hundred and sixty-five thousand, nine hundred and seventy-three, and recognising the partitioned form of the number as $6\,265\,973 = 6\,000\,000 + 200\,000 + 60\,000 + 5\,000 + 900 + 70 + 3$.

Students can also be given numbers to represent on the chart themselves. It is important to give students opportunities to recognise and create numbers that contain zeros; for example, representing 576 024 and recognising that no number needs to be highlighted in the hundreds row.

Multiplying and dividing by powers of ten

Recognising that moving (up or down) from one row to the next represents multiplying and dividing by ten, helps students to gain an awareness of the relationship between multiplication and division by powers of ten. Using the chart in this way helps students to establish that multiplication by ten is equivalent to moving up one row, multiplication by 100 is moving up two rows, and so on. Division by ten and 100 can be introduced as the reverse processes.

	1 000 000	2 000 000	3 000 000	4 000 000	5 000 000	6 000 000	7 000 000	8 000 000	9 000 000
	100 000	200 000	300 000	400 000	500 000	600 000	700 000	800 000	900 000
	10 000	20 000	30 000	40 000	50 000	60 000	70 000	80 000	90 000
	1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000
	100	200	300	400	500	600	700	800	900
	10	20	30	40	50	60	70	80	90
	1	2	3	4	5	6	7	8	9

10 times larger ($\times 10$)

10 times smaller ($\div 10$)

Students may have used the Gattegno chart at primary level to multiply or divide by ten from a given starting number. The chart can be used to develop this process further by encouraging students to examine combinations of multiplications and divisions. For example, multiplying a number by 100 and then dividing the result by 1 000 is the same as dividing by ten.

The Gattegno chart also provides an accessible support for working with and expressing numbers in standard form. The powers of ten can be made visible, either in the way that the rows of the chart are referenced (e.g. 10^2 to denote the hundreds row) or by writing the numbers within the chart as a number between one and ten, multiplied by a power of ten.

1×10^2	2×10^2	3×10^2	4×10^2	5×10^2	6×10^2	7×10^2	8×10^2	9×10^2
1×10^1	2×10^1	3×10^1	4×10^1	5×10^1	6×10^1	7×10^1	8×10^1	9×10^1
1×10^0	2×10^0	3×10^0	4×10^0	5×10^0	6×10^0	7×10^0	8×10^0	9×10^0
1×10^{-1}	2×10^{-1}	3×10^{-1}	4×10^{-1}	5×10^{-1}	6×10^{-1}	7×10^{-1}	8×10^{-1}	9×10^{-1}

Finding percentages

Students can use the Gattegno chart to explore finding a percentage of a number, by considering the structure of the percentage they are seeking to find and splitting it up into tens and ones. For example, students can calculate 31% of 80 by recognising that 30% can be split into three lots of 10% plus 1% and then use the chart to find 10% and 1% (as shown below). By identifying that dividing by ten is the same as finding 10% and that dividing by 100 is the same as finding 1%, the rows of the Gattegno chart can support students' understanding of the calculation to be carried out.

1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

$\div 10$ gives 10% $\div 100$ gives 1%

$$10\% \text{ of } 80 = 8$$

$$1\% \text{ of } 80 = 0.8$$

$$10\% \text{ of } 80 + 10\% \text{ of } 80 + 10\% \text{ of } 80 + 1\% \text{ of } 80 = 31\% \text{ of } 80 = 8 + 8 + 8 + 0.8 = 24.8$$

Students may recognise that dividing by ten and then dividing by ten again, is the same as dividing by 100 (finding 1%). Using the Gattegno chart to highlight these connections can support students' development of flexible and efficient calculation methods. Students may also notice ways to get 20% or 5% based on 10%, and this can similarly speed up the calculation process.

While the Gattegno chart is useful in supporting such informal methods, it is less effective when calculating percentages using a single multiplier (for example, calculating 31% by multiplying by 0.31). Here, the double number line is likely to provide a more helpful representation. It can help students to see that under this multiplier, 100 is mapped onto 31 and, therefore, 80 is mapped onto $80 \times 0.31 = 24.8$.

Further resources

Producing Gattegno charts, either to display to the whole class or to print for individual use, can be time consuming, and so the ability to produce custom, printable charts can be helpful.

See, for example:

MathsBot.com

<https://mathsbot.com/tools/gattegnoChart>

This weblink gives access to a Gattegno chart generator, which can be customised for powers of ten between -8 and 8 via the 'Min Power' and 'Max Power' dropdown menu options. Numbers increase as you move from top to bottom within the chart. There is the option to express numbers within the chart in standard form, with the ability to choose at which point this begins (from one digit, so all numbers are expressed in standard form, to six digits, so numbers in the hundred thousands row and greater (and the hundred thousandth row and smaller), are expressed using standard form notation). Clicking on a cell in the chart hides the number in that cell and the 'Random' button, when selected, hides numbers in the chart randomly. There is also the option to 'Hide All', making all entries in the chart disappear. When this feature has been selected, clicking on a cell reveals the number in that cell. Once the chart has been customised as required, it can be printed for use with students in the classroom.

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