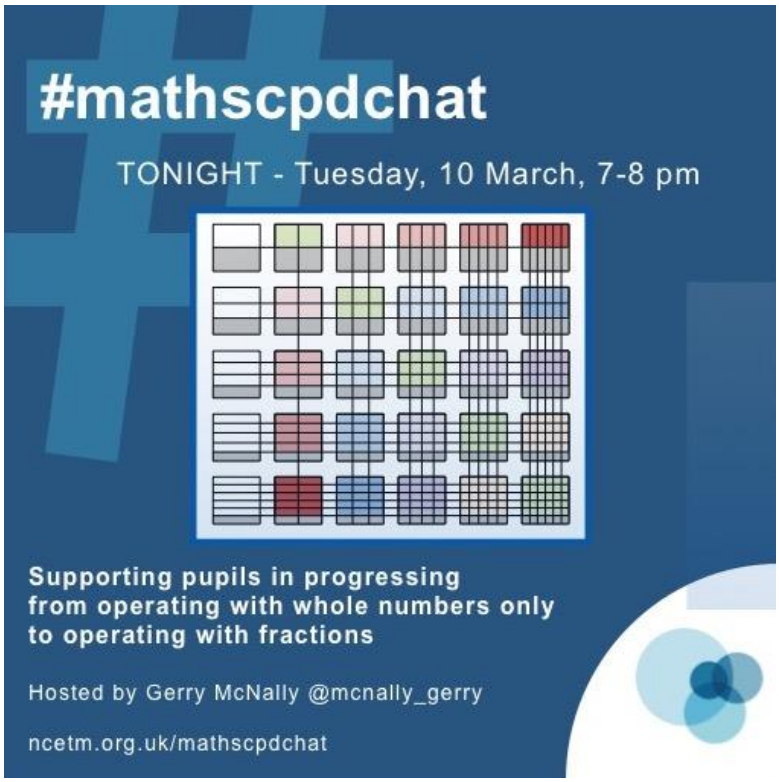
#mathscpdchat 10 March 2020

Supporting pupils in progressing from operating with whole numbers only to operating with fractions.
Hosted by Gerry McNally

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*



Some of the areas where discussion focussed were:

preferred teaching approaches and order of addressing the four operations

- that **the order of teaching, and the approach adopted**, is a consequence of whether the planner's (teacher's) thinking about how people learn to calculate with fractions is **dominated by an image of a hierarchy of procedures or by an image**

of connections between ideas … i.e. is the planning determined by procedures that the pupils will be shown or by the nature of, and relationships between, the ideas that they will need to understand?

- that **in many 'high-attaining countries' learning about fractions is addressed in the following order**: part-whole (e.g. using 2-D images or Cuisenaire™ rods), measure (e.g. half an hour, three-tenths of a metre), quotient (e.g. the result of dividing 10 by 15), operator (e.g. 'half of', +, −, ×, ÷), ratio (e.g. scale-factor in context of similarity);
- spending a lot of time **establishing pupils' understanding of 'fractions of an amount' and of equivalent fractions**, before starting to add and subtract fractions … finding equivalent fractions by multiplying by 1 expressed as n/n;
- **teaching multiplication first in finding fractions of whole numbers** … distinguishing between fractions as operators and as numbers;
- that the **related ideas of '1/n of P' and 'P lots of 1/n' can probably be understood** by learners **well before** (for example during Year 2) **the stage at which they are usually introduced** … possibly during a 'multiplication of fractions' topic;
- that **very young pupils (e.g. in Year 1) are adding fractions to make wholes** (for example adding 3/5 and 2/5 to make 5/5) … that these young pupils are using pictures and number-lines to create and explore such additions … that care needs to be taken to **build on early understanding,** rather than allowing it to be 'smothered' at a later stage by 'blind' reliance on procedures that are not properly understood;
- representing, then comparing, then operating with, fractions by **drawing rectangles on square-grid-paper**; (link to 'Fraction Arithmetic on Grid Paper' below)
- various **ways of representing fractions** (mainly in images) when using examples to **focus pupils' attention on the commutativity of multiplicatio**n (a/b × c/d = c/d × a/b), and also on the fact that a/b × c/d = a/d × c/b … that such representations can also illustrate **how numerators in products of fractions can be 'dis-associated' from denominators and 're-associated' in any (useful) way** … … … for example 2/3 × 6/7 × 7/8 = 2 × 6 × 7 × 1/3 × 1/7 × 1/8 = 6/3 × 7/7 × 2/8 = 2 × 1 × 1/4 = 2/4 =1/2;
- that by thinking about **'how many there are in'** (possibly aided by helpful images) **when dividing BY fractions**, pupils can learn to 'see' the value of numerical expressions such as '3/4 ÷ 1/8' without having to use (follow) a formal procedure;
- that teaching about operating with fractions should enable pupils to achieve a **true understanding of reciprocal** … … … … … i.e. they should understand that because a/b × b/a = ab/ab = 1, therefore b/a = 1 ÷ a/b and a/b = 1 ÷ b/a … … and therefore

N ÷ a/b = (N × 1) ÷ a/b = N × (1 ÷ a/b) = N × b/a;

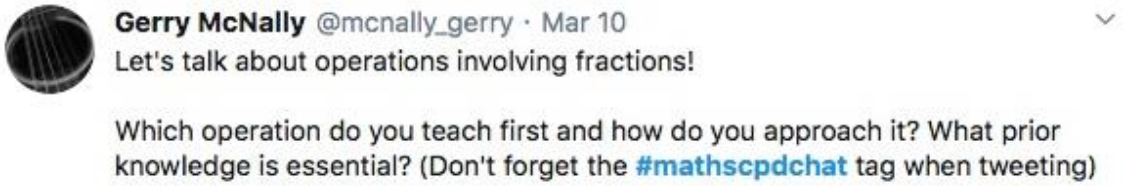
- the need to take care that pupils do not **acquire misconceptions from their use of images to represent operations with and on fractions** … for example by themselves drawing a shape and then 'shading' half of it, then drawing another copy of the same shape and 'shading' a quarter of that might result in their thinking (and saying) 'a half plus a quarter makes three-eighths' … that care needs to be taken when representing 'marks in a test' as fractions (for example, a mark of 6/12 in one test followed by a mark of 3/12 in another test might be represented misleadingly as 6/12 + 3/12 = 9/24);
- using a **number-line as a visual aid** when pupils are learning to add fractions;

helping pupils become competent and confident when operating with fractions:

- using related addition-facts (such as 1/2 + 1/3 = 5/6 and 1/3 + 1/4 = 7/12) as **starting-points to prompt inquiry** … pupils **making conjectures** as a result of exploring more examples … **generalising** (e.g. 1/n + 1/(n + 1) = (2n + 1)/(n(n + 1)));
- using Yohaku puzzles as an interesting way to practise adding and subtracting fractions (link below);
- teaching pupils, all at one time, to calculate with different 'types of number' … that is, **addressing addition and subtraction with fractions, decimals and integers together**, then addressing multiplication and division with all three 'types' of number;
- **devising examples** with the aim of **helping pupils learn how to add and subtract fractions with unlike denominators** (e.g. challenging pupils to work out 2/3 − 1/5 having just worked out 10/15 + 3/15).

In what follows, click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

This is a part of a conversation in which contributors mentioned how pupils may first encounter multiplication by fractions, and then discussed the consequences of using some kinds of visual representation of operations with fractions. The conversation was generated by this tweet from Gerry McNally:

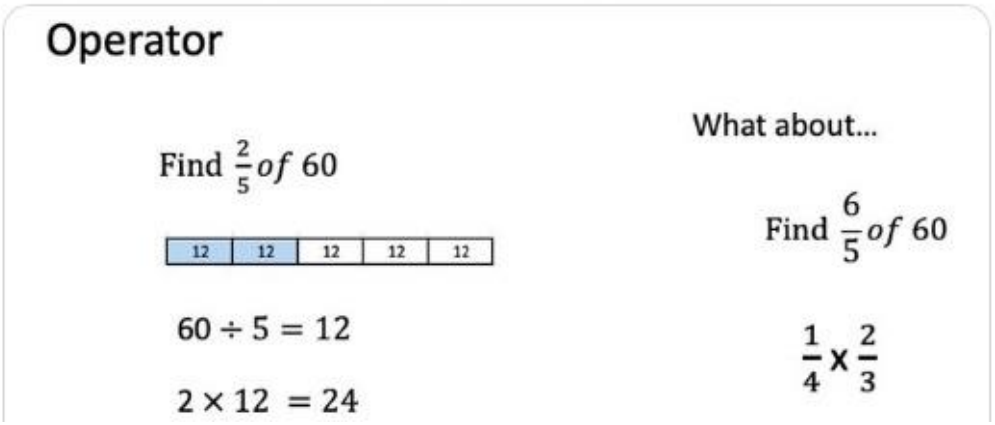


and included these from Chris McGrane, Gerry McNally and Sharon Malley:

Chris McGrane @ChrisMcGrane84 · Mar 10

Replying to @mcnally_gerry

I think there is an argument that most of us teach multiplication first (fraction as an operator), even if we don't do it explicitly... #mathscpdchat



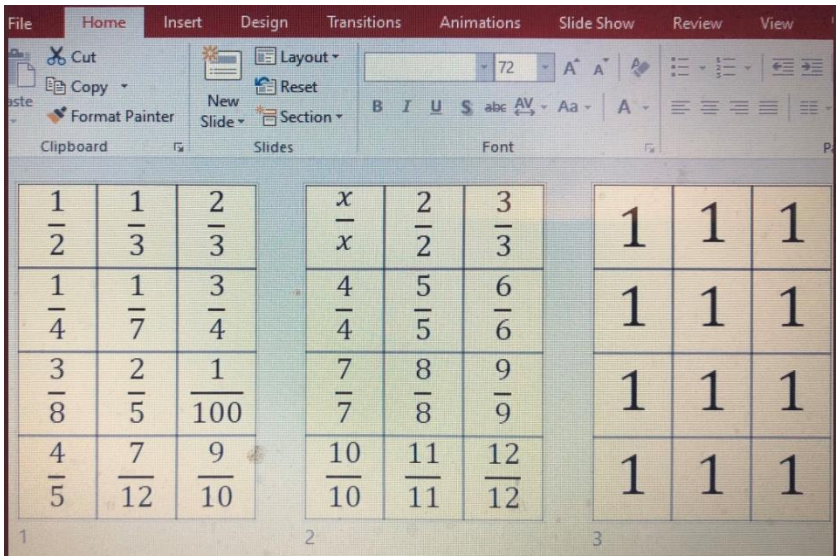
Gerry McNally @mcnally_gerry · Mar 10

Replying to @ChrisMcGrane84

Agreed, Chris.
What about 2/5 = (2 x 2)/(5 x 2) = 4/10?
'The way we teach making equivalent fractions is, as @mathsmumof2 pointed out, essentially multiplying two fractions together
2/5 x 2/2, ie 2/5 x 1.

Sharon Malley @mathsmumof2 · Mar 10

I'm working with LPA yr 7 students on equivalent fractions, today we used calculators to 'find' equivalent fractions to 3/5. Tomorrow we are going to investigate selecting a fraction card and a double-sided '1' card & multiplying together to find the equivalent



these from David MacKenzie, Gerry McNally, and Mary Pardoe:

David MacKenzie @DavidMa10376024 · Mar 10

Replying to @ChrisMcGrane84 and @mcnally_gerry

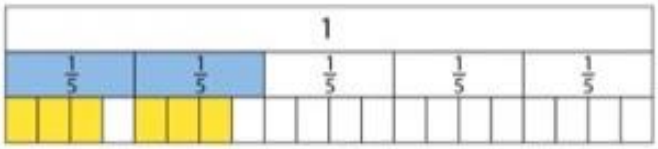
Could learners benefit from being shown at an earlier stage that 1/3 of 12 is equivalent to 1/3 x 12 and so is equivalent to 12 x 1/3.12 lots of 1/3 could probably be understood by learners earlier than most of us would usually introduce multiplication of fractions #mathscpdchat

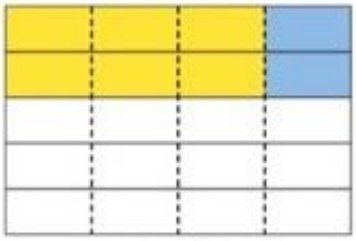
Gerry McNally @mcnally_gerry · Mar 10

Great point, David. This touches on my idea that operating with fractions is not so different to operating with "ordinary" numbers (positive integers). Multiplication is still commutative! #mathscpdchat

Mary Pardoe @PardoeMary · Mar 10

Yes! Many useful images re commutativity in the NCETM material (ncetm.org.uk/resources/53653) #mathscpdchat ... e.g.





$$\frac{3}{4} \times \frac{2}{5}$$

$$\frac{2}{5} \times \frac{3}{4}$$

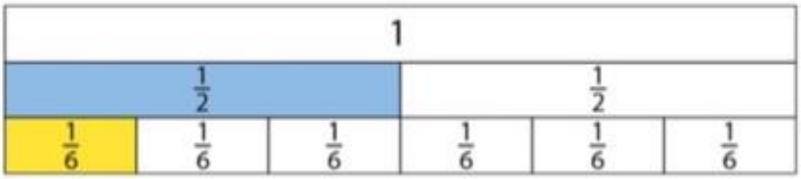
and these from Mary Pardoe and David Connell:

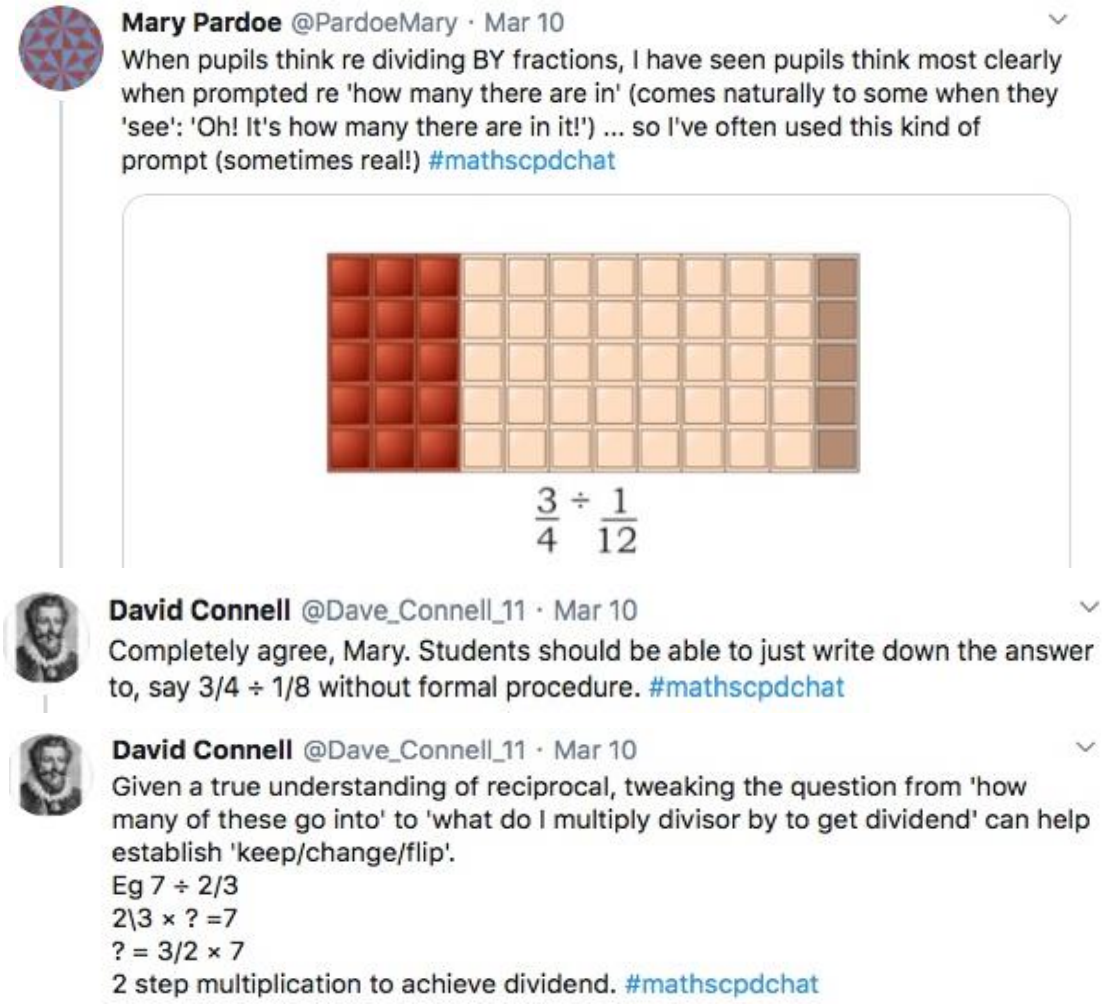
Mary Pardoe @PardoeMary · Mar 10

Replying to @mcnally_gerry @DavidMa10376024 and @ChrisMcGrane84

The NCETM material has 'similar' images for the start of thinking about division #mathscpdchat ...

$$\frac{1}{2} \div 3 = \frac{1}{6} \rightarrow \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$





(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

Fraction Talks which is a website containing very many attractive images designed to generate thought and talk by students about fractions and relations between them, and consequent action. This resource, if used carefully and creatively by teachers, can provide varied opportunities for students to reason, and to articulate their reasoning. It was shared by Ashling Dolan

Fraction Arithmetic on Grid Paper which is a resource by Henry Picciotto in which he clearly and thoughtfully describes and demonstrates how the areas of rectangles drawn on square-grid paper may be used effectively to help pupils understand and operate with fractions. It was shared by Henry Picciotto

Visual Fractions which is a website that provides opportunities for pupils to identify, rename and compare fractions represented in visual images, and to add, subtract, multiply and divide fractions aided by visual images. It was shared by Sharon Malley

Yohaku-like fraction addition puzzles which are Yohaku fraction puzzles presented by Don Steward that provide opportunities for pupils to practise adding and subtracting fractions. You can download a PowerPoint file that suggests ways of tackling these particular fraction puzzles. It was shared by Sharon Malley

Multiplying fractions and dividing fractions by a whole number which is section 3.9 of the NCETM Mastery Professional Development material that provides detailed teaching guidance and support for teachers. It was shared by Mary Pardoe