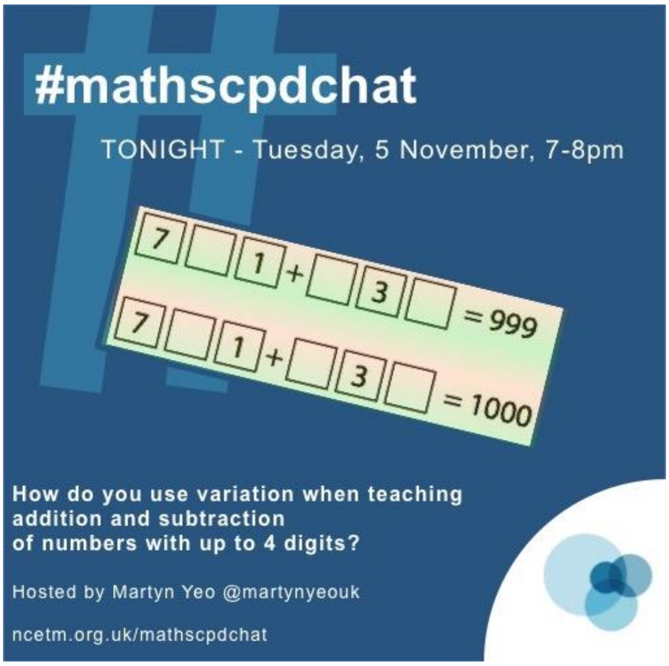
#mathscpdchat 5 November 2019

How do you use variation when teaching addition and subtraction of numbers with up to 4 digits?

Hosted by [Martyn Yeo]

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*



Some of the areas where discussion focussed were:

- that **to make sense of a sequence of varied examples pupils have to see patterns**, form expectations, express these as conjectures, **and generalise**, all of which are aspects of mathematical thinking that we want pupils to develop;
- that when creating a sequence of varied examples it is not easy to obtain the **right balance between too little and too much variation**;
- that in order to use variation effectively **it is necessary to make judgements about what is invariant** … for example, whether to keep both the tens and units invariant,

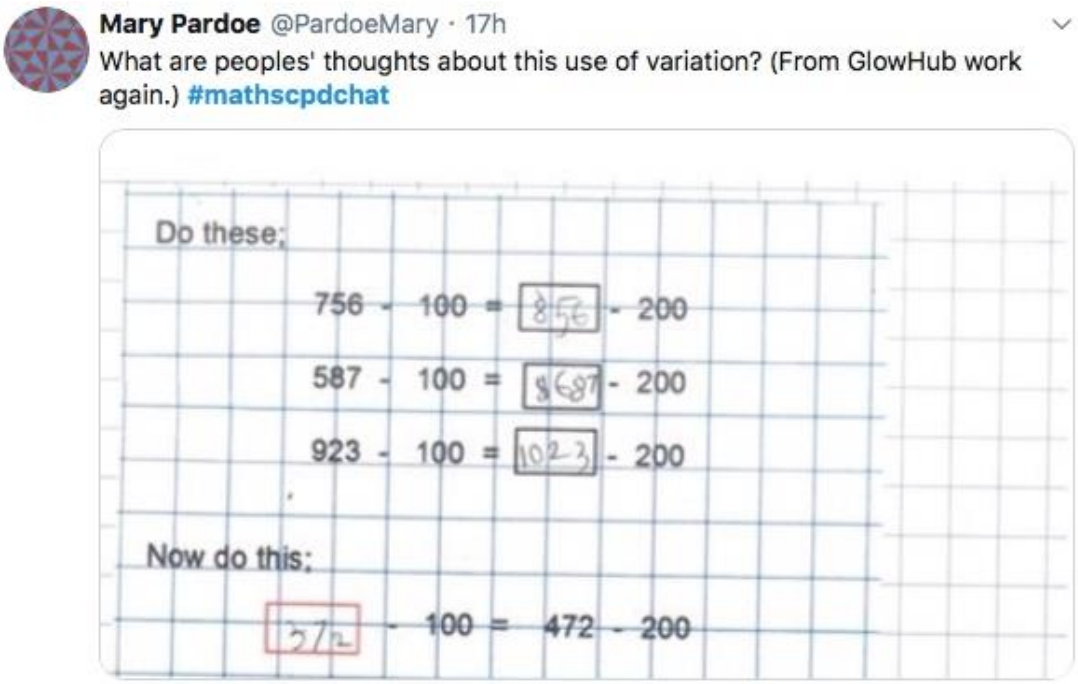
as in ‘827 − 100 = ☐ − 200, 727 − 100 = ☐ − 200, 627 − 100 = ☐ − 200, …’ or to have less invariance, as in ‘756 − 100 = ☐ − 200, 923 − 100 = ☐ − 200, 587 − 100 = ☐ − 200, …’ … whether by keeping too much in the examples invariant you make it less likely that pupils will make the conjectures, and reach the generalisations, that you want them to make and reach;

- being clear about **what can be reasoned from the pattern(s) that is/are discoverable** in a sequence of examples … exactly what the author hopes pupils will ‘see’ (**what they will conjecture**) as a result of working through a sequence of varied examples may not be clear … for example, was the purpose of pupils working through a particular sequence of examples that they conclude that generally A + n − (B + n) = A − B, or was it intended that, more generally, they find ways to simplify ‘awkward’ calculations;
- that in order to use variation to good effect the teacher must rise to the challenge ‘to **work out the mathematical flow of what we are trying to teach … to know where the next steps go**’;
- **comparing the learning-value of two quite similar sequences of varied examples** … deciding what makes the use of variation in sequence A better or worse than the way the examples in sequence B have been varied;
- that thinking about a sequence of varied examples intended to generate a particular conjecture may **prompt a teacher to create a different sequence of examples (with a different purpose that came to the teacher’s mind by his seeing the first sequence)** … for example, a sequence of examples intended to enable pupils to see that A + n − (B + n) = A − B prompted a teacher to create a new sequence likely to generate the ‘general-seeing’ that (A − B) − C is equivalent to A − (B + C) …(the example sequence given during the chat was ‘130 − 30 = ☐, 100 − 20 = ☐, 130 − 50 = ☐ ⋯’);
- that a way to create variation is to **invite pupils to write their own examples** ‘and move on from there’ … whether pupils might create examples unlikely to generate the desired learning … starting from ‘where they are’ and then ‘throwing-in examples to challenge their thinking’ … **challenging pupils to create ‘the hardest example that they can ‘do’**’ (for example, if their ‘hardest’ 4-digit addition is 9999 + 9999, challenge them to think of an ‘easy’ way to ‘see’ the-result-of-the-calculation, such as 20000 − 2), thus introducing discussion to support deeper understanding;
- **whether non-examples always support learning about addition and subtraction**, or whether they can sometimes create confusion and generate ‘hard-to-undo’ misconceptions;

- that **variation of examples can support learning at A level** … that variation of examples involving relatively complex ideas may suggest interesting extensions of those ideas;
- whether it is helpful (makes sense) to state of a sequence of examples that 'procedural' variation of the examples leads to 'conceptual' variation of them … **what do we mean by "moving deeper from 'procedural' to 'conceptual' variation" (?)**;
- whether an effective way to incorporate variation into teaching addition and subtraction is to **provide opportunities for pupils to use a variety of manipulatives to represent the same operation**;
- whether teachers use **multiple pictorial representations of addition and subtraction** operations, and, if so, what they are … the host provided an example of four different pictorial representations of '3 + 2 = 5' … there was no response to this question.

In what follows, click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

This is part of a 'conversation' of tweets that was stimulated by a contributor showing an example of a short sequence of varied examples. It was about what to keep invariant from one example to the next in relation to what the variation is for (what it is hoped pupils will learn from it). The conversation was generated by this tweet from [Mary Pardoe]:



and included these from [Martyn Yeo], [Sam Webster] and [Mary Pardoe]:

Martyn @martynyeouk · 17h
Replying to @PardoeMary
Much deeper way of thinking about addition and subtraction and getting children to see the balance.
#mathscpdchat

Sam Webster @WebsterMaths · 17h
Replying to @PardoeMary
I think the first numbers need to have less variation in them, 827 - 100; 727 - 100; 627 - 100. Then maybe mix things up a bit more: 617 - 100; 617 - 10; 607 - 10... #mathsCPDChat

Mary Pardoe @PardoeMary · 17h
Would you move to 617 - 10 = ... - 20 then? #mathscpdchat What idea are you wanting them 'see'?

Sam Webster @WebsterMaths · 17h
Yes, that would make sense at first, but you could then throw in a 617 - 10 = ... - 200. To hopefully trip them up, that poses the question "why were the previous problems easier?" And the connection between the place value in the calculations observed.

and these from Sam Webster and Richard Perring:

Richard Perring @LearningMaths · 17h
Replying to @WebsterMaths and @PardoeMary
Surely it depends what the variation is for? If the intention is to draw attention to the way that it's only the 100s digit that changes then I don't know why you wouldn't vary the other digits? #mathscpdchat

Sam Webster @WebsterMaths · 17h
It confuses the issue. VT is about keeping things invariant, so that you can make meaning from the effect of the changes that are made.

Richard Perring @LearningMaths · 17h
But the teacher has a role too? To help students understand the structure that underpins the changes? Comparing the minuend with the difference in each case would do that.
I think that I'd worry that by not changing the other digits, students wouldn't get enough to generalise.

Sam Webster @WebsterMaths · 17h
I would change the other digits - eventually. That way the students get to see that the other place value columns aren't affected, perhaps 617 - 100 is followed by 671 - 100 etc. But the important thing is that the initial understanding is there first.

Sam Webster @WebsterMaths · 17h
The benefit of tasks like this, with minimal difference between the questions, is that students can benefit from the self-teaching effect

(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

Variation and Mathematical Structure which is an article from *Mathematics Teaching* by Anne Watson and John Mason in which the authors explore how variation and invariance within examples and exercises can engage learners with mathematical structure. It was shared by Mary Pardoe

The Effects of Variation on Children's Learning which is a booklet produced by the GLOW Maths Hub, by teachers for teachers. The teachers started from pupils' work and then looked for ways to describe the variation involved and its effects. It was shared by Mary Pardoe

Primary Mastery PD: Number, Addition and Subtraction Year 4 which is a combination of detailed teacher guidance, and images, presented as animated PowerPoint slides, to further enhance teacher knowledge, and which can be used in the classroom. It was shared by Martyn Yeo

Improving Mathematics in Key Stages Two and Three which is a Guidance Report from the Education Endowment Foundation. It was shared by Martyn Yeo