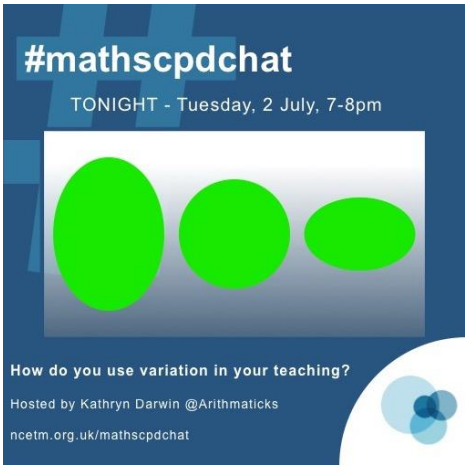
#mathscpdchat 2 July 2019

How do you use variation in your teaching?
Hosted by Kathryn Darwin

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*



Some of the areas where discussion focussed were:

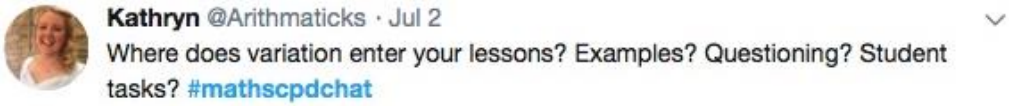
- that the words **'variation' and 'variety' have different meanings** (particularly when used in the context of mathematics education);
- that in mathematics education, **variation-to-facilitate-learning is achieved by changing something particular in an example (e.g. changing the value of a variable in a numerical or algebraic expression) while everything else stays the same** … that it may be unhelpful to change several variables at the same time … that there is more to effective variation than just constructing 'minimally different' examples;
- that when variation is intended to facilitate learning, the **aims go beyond teaching pupils just to spot and extend patterns**;

- knowing whether/when/how **mathematical structure can be clarified using variation** … knowing which 'bits-of-mathematics' can be made available for pupils to notice by their seeing differences in carefully varied examples … **choosing examples that differ in ways that make something mathematical (e.g. a property of a mathematical object, or a relationship between objects) likely to be noticed by pupils** … e.g. when exploring factors of whole numbers, well-chosen examples may make it likely that pupils will notice that square numbers have an odd number of factors, so that at least some of the pupils seek naturally for an explanation … that **non-examples** also often help to facilitate learning;
- drawing pupils' attention to **what can be varied in an example … the range of permissible change** in which it remains an example of what is being exemplified;
- that **variation may be more or less useful-for-learning depending on what is varied** … e.g. choosing what to vary in a polygon when the aim is for pupils to understand how enlargement affects the area enclosed by a 2-dimensional shape … choosing what to vary in examples when the aim is for pupils to understand how simultaneous equations may be solved;
- that **just varying the 'level of difficulty' of examples may not facilitate learning** … that varying the 'level of difficulty' may distract attention from what might be learnt by varying the examples in a different way;
- that when pupils are 'working through' a set of varied examples they may be trying to **'guess what's in the teacher's mind'**;
- **teachers working on pupil tasks** in order to sensitise themselves to something of what pupils experience, so that they may **avoid inappropriately telling pupils what to do** … **giving too much guidance** to pupils when opportunities exist for pupils to notice by themselves mathematical entities such as properties, relationships and connections … the danger of generalising **for** pupils … that sometimes **prompting-by-questioning** (in a way that enables pupils to learn gradually how to prompt themselves) may be necessary;
- teachers, when planning to use variation in their teaching, **being aware of pupils' relevant prior knowledge and their fluency in recalling and using relevant facts and procedures**;
- using **Structured Variation Grids** (link provided below) to provide opportunities for pupils to detect patterns, predict, conjecture, test conjectures, and generalise;
- **seeing an exercise as a single mathematical object** (links to articles provided below);
- that **variation in the mode of representation** of a mathematical 'object' or 'fact' may facilitate understanding-of/ability-to-use it;

- **pupils using variation effectively when generating their own examples** … e.g. pupils using interactive software to draw graphs of linear relationships, systematically changing numbers in the equations, and noticing what happens to the lines … pupils creating, and then expanding, their own algebraic expressions of the form (x +/- a)(x +/- b) where a and b are constants … pupils exploring graphs of functions f(x) = asinbx where a and b are each one of +2, -2, +½, -½;
- **pupils writing about structures and relationships that they notice as a consequence of seeing variation** in examples … sharing and discussing these noticings;
- **using variation** (in a sequence of questions) **when questioning pupils**;
- **what we mean by 'delving deeper'** in the context of pupils' mathematical actions and understandings … eg does it mean pupils looking into the structure underpinning mathematics that they 'can do' or 'are doing'?
- **observing what pupils are doing when they are extending variation patterns** (when they are going 'with the grain') … knowing that they might be 'diligently extending and extending the pattern and merely checking that LHS = RHS each time without homing in on structure' (that is, without trying to see what looking 'across the grain' reveals, without seeking to justify 'LHS = RHS' equalities);
- that **effective variation has been incorporated into good teaching for decades,** eg in the Open University 'Project Update' materials published in 1988 (link provided below);
- that the use of variation in teaching and learning mathematics is **considered-in/incorporated-into lesson studies and learning studies** (e.g. as used in Hong Kong).

In what follows, click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

This is part of a 'conversation' of tweets, about hoping/wanting to be less 'prescriptive' when using variation in examples, and wanting not to do the generalising *for* the pupils; becoming less prescriptive by letting pupils take control, create their own examples (possibly within given constraints) and 'delve deeper'. The conversation was generated by this tweet from Kathryn Darwin:



including these from Simon Ball, Clare James and Mike Ollerton:

Simon Ball @ballyzero · Jul 2

Replying to @Arithmaticks

It's more in examples for me at the moment, with tiny amounts in questioning and tasks. I know I've gone 'easiest first', but I'm hoping to use it much more in the future! #mathscpdchat

Clare James @clarerjames13 · Jul 2

Replying to @Arithmaticks @ballyzero

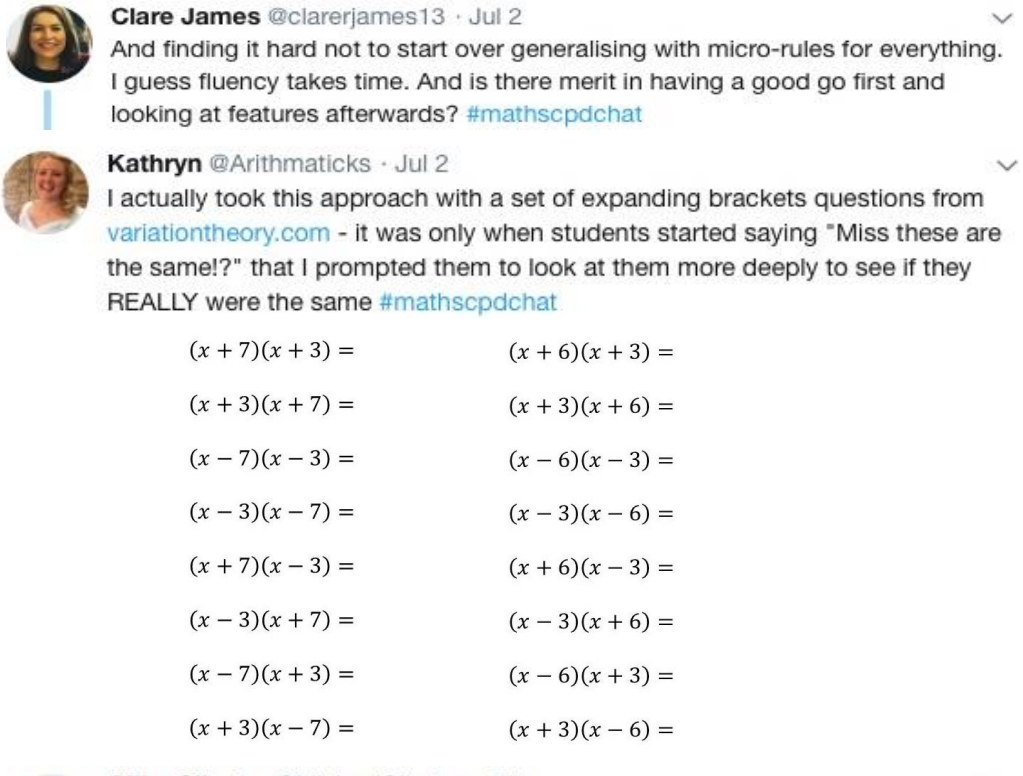
That's what's hard- feeling like I'm doing easiest first and being a bit prescriptive about what we are looking at. When to let go of the reins a bit. #mathscpdchat

Mike Ollerton @MichaelOllerton · 21h

Replying to @clarerjames13 @Arithmaticks @ballyzero

#mathscpdchat yes, this is what I think I referred to earlier about students creating their own examples within a given set of constraints, e.g. to explore graphs of functions f(x) = asinbx where a and b are of the form +/-2, +/-1/2

these from Clare James, Kathryn Darwin and Mike Ollerton:



Clare James @clarerjames13 · Jul 2

And finding it hard not to start over generalising with micro-rules for everything. I guess fluency takes time. And is there merit in having a good go first and looking at features afterwards? #mathscpdchat

Kathryn @Arithmaticks · Jul 2

I actually took this approach with a set of expanding brackets questions from variationtheory.com - it was only when students started saying "Miss these are the same!?" that I prompted them to look at them more deeply to see if they REALLY were the same #mathscpdchat

$(x+7)(x+3) =$ $(x+6)(x+3) =$

$(x+3)(x+7) =$ $(x+3)(x+6) =$

$(x-7)(x-3) =$ $(x-6)(x-3) =$

$(x-3)(x-7) =$ $(x-3)(x-6) =$

$(x+7)(x-3) =$ $(x+6)(x-3) =$

$(x-3)(x+7) =$ $(x-3)(x+6) =$

$(x-7)(x+3) =$ $(x-6)(x+3) =$

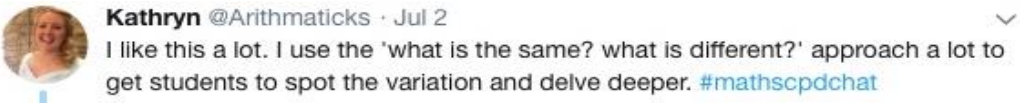
$(x+3)(x-7) =$ $(x+3)(x-6) =$

Mike Ollerton @MichaelOllerton · 21h

Replying to @Arithmaticks @clarerjames13 @ballyzero

#mathscpdchat is there any 'room' in variation theory for students to take more control where, instead of the teacher presenting students with a 'worksheet', she sets up a scenario such as: Create and expand expressions of the form (x +/- a) (x +/- b) where a and b are constants?

and these from Kathryn Darwin and Mike Ollerton:



Kathryn @Arithmaticks · Jul 2

I like this a lot. I use the 'what is the same? what is different?' approach a lot to get students to spot the variation and delve deeper. #mathscpdchat

(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

Variation in mathematics: A collection of writings from ATM Mathematics Teaching which is a book from the Association of Teachers of Mathematics (ATM) for anyone involved in teaching mathematics, at whatever level. Edited by Anne Watson, it is valuable collection of readings, that were originally published in *Mathematics Teaching* (the journal of the ATM), on the subject of Variation, and grounds current ideas in the wisdom and experience of four decades. It was shared by Teachers of Maths

Seeing an exercise as a single mathematical object: using variation to structure sense-making which is a paper by Anne Watson and John Mason (2006) in which they indicate how dimensions of possible variation inform the design and use of a well-planned exercise. It was shared by La Salle Education

Variation Theory which is a website from which you can download 'worksheets' of exercises, which, it is stated, provide 'Intelligent, varied maths practice from Craig Barton'. It was shared by Kathryn Darwin

Variation: analysing and designing tasks which is an article in *Mathematics Teaching 252* by Anne Watson with participants of a workshop that Anne ran at the ATM conference 2016. It addresses the role of variation in teaching mathematics, the effects of variation on mathematical experience, and the pedagogic uses of variation beyond an unclear surface distinction between 'procedural' and 'conceptual' variation. It was shared by Mary Pardoe

Evolution of a Task Domain which is an article by John Mason, published online (09 January 2019) in *Digital Experiences in Mathematics Education*. The author explains, with examples of tasks that the reader is advised to work on, how teachers can utilise variation in turning potentially rich tasks into a task used richly. It was shared by Mary Pardoe

Structured Variation Grids (SVGs) which are interactive two-dimensional grids of cells designed by John Mason, in which each cell has an upper(a calculation) and a lower(a result) part, so that, treating a single cell as a statement of equality, each cell gives a particular case of some general formula or relationship. Each grid can be thought of as a window onto an infinite grid extending in all directions. It was shared by Mary Pardoe

Graph Plotter: An Online Graphing Calculator which is a website from which you can download 'worksheets' of exercises, which, it is stated, provide 'Intelligent, varied maths practice from Craig Barton'. It was shared by Heather Scott

Don Steward: mathematics teaching 10 - 16 which is Don Steward's website from which you can obtain lovely ideas for teaching mathematics, with many interesting tasks and exercises, all of which are designed by Don Steward. It was shared by La Salle Education

What is made possible to learn when using the variation theory of learning in teaching mathematics? which is a paper by Angelika Kullberg in which the author describes 'the variation theory of learning, its underlying principals, and how it might be appropriated by teachers'. It was shared by La Salle Education

Project Mathematics Update was a collection of excellent materials produced by the Open University's Centre for Mathematics Education (CME) during the late 1980s. From this link you can download any of these ten (very-valuable-as-a-personal-resource) books that contain many useful examples, including the enlightening *Expressing Generality* by John Mason. It was shared by Peter Gates

The Approach of Learning Study: Its Origin and Implications which is a paper by Eric C.K. Cheng, Lo Mun Ling of the Hong Kong Institute of Education. It was shared by Geoff Wake

NonExamples.com is a website from which you can obtain material to use with pupils in mathematics lessons, including collections of examples and nonexamples of mathematical objects and relationships. It was shared by La Salle Education

Dynamic Worksheet Generation from Complete Mathematics which is a tool from *Complete Mathematics* 'which captures some of the 'teacher logic' inherent in worksheet design and have incorporated intelligent variation and sequencing in the questions generated'. It was shared by La Salle Education

Thinking Mathematics Lessons which is a website from which you can obtain detailed plans for mathematics lessons prepared by Michael Shayer and Munder Adhami. It was shared by Alex Black