Secondary Magazine 106

National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 106 of the Secondary Magazine

In this issue of the Secondary Magazine, New Year's Resolutions and the opportunities they provide for fresh starts are considered alongside the second of our series of articles in the Key Ideas in Teaching Mathematics series and the last of the thought-provoking articles focusing on the Numbers Count programme. Even if outside is wet and wintry, enjoy the mathematical comfort provided in this issue.

Contents

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From the editor: New Year's Resolutions

The start of a new year can be a trigger to prompt reflection on the achievements from the last year and a chance to set new goals or targets for the year ahead, although if you read Tales from the classroom in this issue, you might begin to think of everyday as the start of a new year and find endless opportunities for review and projection! To enter into the spirit of the season, here are some possible New Year's Resolutions for mathematics teachers…

Read the NCETM Secondary Magazine! There is an archive of previous issues, some of which include the Up2d8 Maths resources (you might like to try this one which considers the factors surrounding the probability of getting a double yolk in an egg). You can also subscribe to the Secondary Magazine RSS feed which means you will never miss an issue. And of course, you can find us on Facebook and Twitter too.

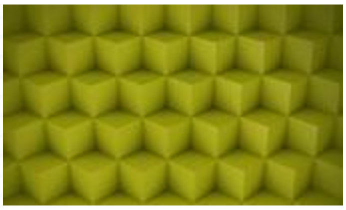
Find an area of the curriculum that pupils don't readily understand and collect or make some resources to support their understanding. For example, if you chose simultaneous equations you might draw upon

- the NCETM Departmental Workshop Simultaneous equations
- Simultaneous biscuits (from the NCETM microsite, What makes a good resource)
- the NRICH problem Matchless
- Simultaneous Equations Inquiry (from Inquiry maths, accessed via the National STEM Centre eLibrary).

Find a piece of IT software that would enhance your mathematics teaching then build up your skills and resources to use this programme in your classroom. You might find some inspiration from

- FMSP Live online professional development, specifically Using Geogebra to enhance teaching in GCSE Mathematics
- ICT and digital technology used in mathematics teaching. This NCETM microsite has lesson reports of how teachers have used ICT (you could start by reading this account of using Grid Algebra in the classroom) and case studies. It also includes some self-evaluation materials and links to useful online resources.

Do share your mathematical New Year's Resolutions with us.



Key Ideas in Teaching Mathematics – geometric and spatial reasoning

In this and subsequent issues, the Secondary Magazine will feature a set of six articles, written by Keith Jones, Dave Pratt and Anne Watson, the authors of the recent publication Key Ideas in Teaching Mathematics. While not replicating the text of this publication, the articles will follow the themes of the chapters and are intended to stimulate thought and discussion, as mathematics teachers begin to consider the implications of the changes to the National Curriculum. This article is the second in the series and focusses on Geometric and spatial reasoning in Key Stage 3. Future articles will feature Statistical reasoning, Place Value, Algebra and Probabilistic Reasoning. The previous article focussed on similarity, ratio and trigonometry in Key Stage 3.

The geometry curriculum for Key Stage 3 makes use of the word “derive” quite often. Examples include “derive and apply formulae to calculate and solve problems involving perimeter and area”, “derive and use the standard ruler and compass constructions”, “derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures”, “derive and use the sum of angles in a triangle”, and so on. Here the word “derive” means to obtain something by some process of reasoning. This chimes with the overarching curriculum statement that students learn to “reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language”. In particular, the curriculum signals that it is at Key Stage 3 that students “begin to reason deductively in geometry” (as well as in number and algebra).

When teaching about perimeter and area, available research suggests that overly focusing on rectilinear shapes drawn on squared paper and on working on unconnected area formulae doesn’t necessarily help students to develop long-term understanding. With such an approach to teaching, students may not move much beyond the procedural counting of squares with the result that many students can end up confusing perimeter and area.

Geometric constructions have been in the curriculum for some time, but research indicates that developing mechanical competence with handing the relevant instruments (be it ruler and compass or the equivalent computer software) takes time. Throughout such teaching the focus can be on the accuracy of the construction, which, while quite proper, can mean that the students do not get to understand the link between a construction that they have been asked to perform and a related proof. In terms of properties of shapes and the use of theorems such as the angle sum in a triangle, research confirms that students can be satisfied by empirical arguments and not appreciate the need for anything beyond that.

All this points to the issue that teaching students to reason deductively in geometry is, as the available research consistently shows, not at all straightforward. The scale of the issue is reflected in what can be a lack of clarity in referring to measures. In any situation in the physical world, any measure has an error. This applies to ruler and compass constructions just as it does to the equivalent computer software. In contrast, when using geometric properties, or theorems, when we say that the circumference of a circle is πD, we do not mean approximately πD; we mean precisely πD. When we refer to a unit square, we do not mean that each side is approximately one unit; likewise, when we refer to a right-angled triangle we do not mean that the right-angle is approximately 90 degrees. In the first cases, we mean precisely one unit and, in the second, we mean precisely 90 degrees. The key point is that geometric objects such as unit squares and right-angled triangles are figural concepts in that they combine figural and conceptual aspects. As such, in teaching and learning geometry, *attention has to be paid to both*. By that I mean the spatial aspects, and the aspects that relate to reasoning with geometrical properties and theorems.

These two aspects, the spatial aspects, and the aspects that relate to reasoning, are not separate, they are entwined; they are the *yin and yang* of geometry teaching and learning. This means that when attempting to derive and use some geometric property or theorem, a secondary school student might move from making conjectures using measures taken from a geometrical drawing, to using definitions and theorems, then go back to the drawing, and so on. This moving back and forth between what might be termed 'spatio-graphic geometry' and 'theoretical geometry' is what is behind the challenge of teaching geometric ideas at the secondary school level (just as it's behind the challenge of specifying a suitable curriculum).

A further challenge is the breadth of approaches to geometry that need to be incorporated into teaching. Attention has to be paid not only to a range of key geometric ideas, including symmetry, invariance, transformation, similarity, congruence and so on, but also to various approaches to geometry, including synthetic, transformation, and analytic geometry.

With the curriculum set out as a set of bullet points, it could be tempting to treat each one in isolation. An alternative is to keep in mind the need to focus on the spatial aspects of geometry as well as the aspects that relate to reasoning. The following guidelines for successful teaching are adapted from research:

- geometrical situations selected for the classroom should, as far as possible, be chosen to be useful, interesting and/or surprising to students;
- classroom tasks should expect students to explain, justify or reason and provide opportunities for them to be critical of their own, and their classmates', explanations;
- tasks should provide opportunities for pupils to develop problem solving skills and to engage in problem posing;
- the forms of reasoning expected should be examples of local deduction, where pupils can utilise geometrical properties that they already know in order to deduce or explain other facts or results.
- in order to build on learners' prior experience, classroom tasks should involve the properties of 2D and 3D shapes, aspects of position and direction, and the use of transformation-based arguments that are about the geometrical situation being studied (rather than being about the transformations themselves solely as mathematical operations);
- while measures are important in mathematics, and can play a part in the building of conjectures, the generating of data in the form of measurements should not necessarily be an end point to learners' geometrical activity. Indeed, where sensible and in order to build geometric reasoning and counter possibly deep-seated reliance on empirical verification, it is worth considering classroom tasks where measurements (or other forms of data), or purely perceptual reasoning, are not generated.

Paying attention to the two closely entwined aspects of geometry, the spatial aspects, and the aspects that relate to reasoning with geometrical theory, across both 2D (plane) and 3D (solid) geometry, takes time; perhaps more time than is currently allocated to geometry within the typical school scheme of work. As with other key aspects of mathematics teaching at secondary school level, effective teaching of geometry needs coordination of the development of various components of geometry across school years and between teachers.

Keith Jones, Dave Pratt and Anne Watson

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A resource for the classroom – geometry problems

This issue of the magazine has an article linked to the recent publication, Key Ideas in Teaching Mathematics. There is a website that accompanies the book which provides links to some relevant resources. Our article this month is related to spatial and geometric reasoning so the resources for the classroom are a suite of problems which have been selected to develop spatial and geometric reasoning. Some of these problems may be familiar whilst others may be new to you; all have been chosen to develop and deepen understanding.

The website considers two aspects of geometry and states

> *Geometry education at the school level should attend to two closely-entwined aspects of geometry: the spatial aspects and the aspects that relate to reasoning with geometrical theory. These twin aspects of geometry, the spatial and the deductive, are not separate; but interlocked. Just as geometry and algebra have been called the "twin pillars" of mathematics, spatial and geometric reasoning are the yin-yang of geometry education. Both are interconnected and inter-dependent in such a way that each gives rise to the other.*

The individual problems are:

- Designing Logos
- Packaging
- Playground Snapshot
- Tiling Patterns
- Pentakite
- Where Are They?
- Pythagoras Proofs
- Proximity
- Rotating Triangle.

What will you do now?

You could:

- select a problem and try it out with a particular class
- select a problem and work with a colleague to consider how you can use the problem to develop understanding for a group of pupils
- include some of these problems in your scheme of work
- consider how these problems develop the powerful aspects of the curriculum and the links between them.

Do tell us what you find out...

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Focus on…Every Child Counts

In this article, the last in the series, Andy Tynemouth from Edge Hill University and National Adviser for Every Child Counts *discusses the problems of supporting learners who struggle to make sense of mathematics*

This is the last in a series of four articles and as such represents the culmination of a series of reasonably dense arguments and descriptions. In order to best equip yourself with the tools to follow its content you would be well advised to read the preceding three. These can be found in Issues 103, 104 and 105 respectively. In these articles I describe how the Numbers Count (NC)[1] programme, originally developed to support the lowest attaining leaners in Y2, has been extended to support learners as old as Y7 or Y8. In particular, I have described and discussed the development of the Learning Map. This is a device used to support the learning of those who find mathematics particularly difficult, usually those whose attainment is in the bottom 5% or 6%.

The Learning Map is a kind of expression of the constructivist principles embodied in all Every Child Counts (ECC) programme, but particularly in NC (for more detail please see the first article in Issue 103). It supports learners in recognising and employing their mathematical resources – that is their pre-existing skills, knowledge and understanding – in order to further develop as mathematicians in particular and as learners in general.

As described in Issue 105, the Learning Map essentially works by providing the learner and the teacher with a platform for acknowledging the learner's resources and needs – dialogue regarding these can then be conducted. More than this, the Learning Map gives a kind of corporeal form to an essentially intangible quality: learning. What compounds the difficulties that it is possible for learners to encounter when tackling mathematics, is not only that learning itself is intangible but the content of mathematics is intangible, and abstract in every way. As I have suggested in the previous articles, for (far too) many learners the intangibility of learning mathematics leads to the development of an inhibiting fear of the subject and to what Carol Dweck describes as a fixed mindset where the learner's lack of belief in their capacity to learn places an artificial, self-imposed cap on their potential. In the last article I suggested that there were three reasons why the Learning Map had such a positive impact with the learners and teachers who engage with it. Those three reasons were:

- it makes the learner's prior and current learning clear to them;
- it supports the development of a 'growth mindset'
- it provides a platform to maintain a positive and effective educational dialogue between the teacher and the learner

I also discussed the first two reasons. In this article I will complete my exposition by discussing the third, but first a preamble.

Many of us, as teachers, will have come across the name of the Russian psychologist Lev Vygotsky. His work has had an enormous impact on teachers and their trainers. I suspect that there is not a teacher training course in the land that doesn't include reference to his core ideas. Vygotsky's essential thesis – as outlined in two seminal works *The Mind in Society* and *Thought and Language* – is that learning (the development of the higher cognitive functions) takes place within a cultural context. In fact, not only is learning shaped by the culture of the learner, but cannot take place outside of a clearly defined culture. There is evidence to support this notion outside of psychology. Feral children, that is children raised in the wild by wolves or apes, who are later introduced to human society are often seen to be incapable of acquiring a natural language such as English or Chinese. Beyond a rudimentary 'home sign' used to communicate basic needs and desires they remain incapable of interaction: their cognitive development permanently arrested. Similar, although often not as pronounced, problems can occur in the deaf children of hearing parents when the failure to develop a natural language and the reliance on a home sign leaves them excluded from the world of learning[2]. Neurologists believe that the human brain goes through a series of critical developmental phases between birth and early maturity perhaps resulting the incapacity to acquire key cultural tools such as language beyond a given age[3]. So what? I cite these examples to illustrate the fundamental importance of the participation in a culture in order to develop the cultural tools and higher cognitive functions described by Vygotsky: such as mathematics.

The quality and quantity of such participation is of particular importance in mathematics. Why? Because mathematics is peculiarly abstract, and in a way that is not necessarily apparent to those proficient in it. Consider how a young child learns to count. Three distinct phases can be described.

- numbers as recitation – at this stage the child simply knows the number rhyme *'1,2,3,4…'* and so on. They are not aware that the numbers are individuated.
- numbers as adjectives – as children learn to count groups of objects they acquire the ability to use one to one correspondence, achieve a stable order (i.e. reliably say the right words in the right order) and recognise that the last number name said yields the number of objects in a given group (known as cardinality). This gives the number names a new and useful function and meaning as now the counting can be used to describe the quantities of counted groups *'1,2,3,4…there are four teddies in the box'*.
- numbers as nouns – this happens when the number names are reified, that is to say when they are treated in the language of mathematics ***as if*** they were real objects. Compare the two sentences *'3 and 2 make 5'* and *'Andy and Mikey make a mess'*. Clearly the last describes observable objects in the real world whilst the first treats mathematical 'objects' as if they were real observable objects: which they are not. Only examples of them can be observed, not the numbers themselves.

In fact the closer you begin to look at this quandary the more complex it becomes. Anna Sfard[4] suggests that the apparently simple statement 3+4=7 conceals a mass of undisclosed complexity. In fact what 3+4=7 is saying should go something like this:

- *if I have a set that whenever I count the elements I stop at the word three,*
- *and I have yet another set such that whenever I count its elements I stop at the word four*
- *and* if I put these two sets together

then

- *if I count the elements of the new set I will always stop at seven.*

This is certainly not what most people conceive when uttering the words '3 and 4 make 7'. Sfard suggests that this compaction of meaning, which she calls ontological collapse, is simultaneously what allows mathematics to be so powerful and so troublesome to learn. Powerful because such a short statement as 3+7=10 can say so much. Troublesome to learn because this is not obvious to either teachers or to learners.

In order to compensate for this masked complexity with struggling learners it is essential to engage in a constructive dialogue – and I mean constructive in the literal sense i.e. consciously constructing understanding *together*. It is precisely this that a device like the Learning Map can help teachers and learners to do. Its detailed representation of the learner's mathematical resources and targets allow the teacher and the learner to locate and inhabit what Vygotsky referred to as the zone of proximal development. To take the learner on from what they can securely do unsupported and into new territory which can be explored and made sense of together. This is my version of educational dialogue. As a teacher I hold the belief that mathematics is more than a bunch of rules and procedures to be learned, remembered and churned out when required. Mathematics is a discourse regarding human thought about how the world works and the learner has to construct their own, compatible, understanding of it. They do this by participating in the dialogue, not by simply listening to the teacher telling them what to do and think.

Of course, as any who have attempted to write a short series of episodes articulating their passion for learning and teaching will know, it is not an easy task. I do not say this as any form of excuse, but perhaps to better explain my intent. My guess is that not everyone will see eye to eye with the content of these articles – nor all of the ideas I've espoused. My hope though is that they will have prompted thought amongst those, who like me, care deeply about the well-being of the learners in their charge. In particular for those at greatest risk of losing sight of the value and beauty of mathematics. If I have achieved that in one soul then I am content. It is my fervent belief that the majority of those who struggle with mathematics do not need to, and it is my conviction that their struggle can lead to serious and undesirable consequences for both society in general and, in particular, the individuals afflicted. It is to those individuals that we owe our greatest duty of care.

Footnotes

[1] NC is one of the Every Child Counts suite of programmes developed and rolled out nationally by Edge Hill University. It is a teacher-led, one to one mathematics intervention aimed at the lowest 6% of attainers. These are learners for whom nothing else will work. Originally conceived and designed to target Year 2 its success has led to its development to address the needs of learners up to, and including, Year 8. Learners on NC make an average of 16.5 months Number Age gain in around 40, half-hour NC lessons over three or four months. For more information please see the Every Child Counts website.

[2] The neuropsychologist Oliver Sacks (1990) wrote a passionate account of the importance of supporting deaf people to develop full fluency in a natural language. He argues powerfully that the best way to do this is to focus the teaching of deaf people on sign language rather than signed speech or lip reading. Whilst he acknowledges that both are important he believes that for deaf people participation in the signing community is both more natural and easier to achieve than participation in the speaking community.

[3] Current thinking in neuroscience is that the brain develops very quickly in infants forming something like 1,000 trillion synapses by the time they are three. After this period the brain undergoes pruning until maturity. This pruning supports the keeping of regularly used synapses and allows unused ones to atrophy. So if a child has learned Mongolian until they were three and then never uses it again the synaptic connections dealing with speaking and understanding Mongolian will decay away. This may be the mechanism that underpins the critical period hypothesis that children not exposed to natural language until maturity will never develop a fluent natural language and therefore never develop the higher psychological functions described by Vygotsky. It is interesting to note that some neuroscientists have suggested that different aspects of the brain develop at different rates and it is the very fact that the pre-frontal lobe develops more slowly than the rest of the brain that allows infants their language learning genius (Thompson-Schill et al, 2009). Some interesting further reading can be found on the (American) Child Welfare Information Gateway website.

[4] Anna Sfard's contention in her inspirational book 'Thinking As Communicating' (2008) is that there is no way to sensibly talk about thinking without reference to its linguistic basis. Or, as the philosopher Ludwig Wittgenstein put it: 'between a thought and its expression'

References

Sacks, O. (1990) *Seeing Voices* London: Picador

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Thompson-Schill, S.; Ramscar, M.; Chrysikou, M. (2009). "Cognition without control: When a little frontal lobe goes a long way". *Current Directions in Psychological Science* 8 (5): 259–263

Vygotsky, L. (1978) *The Mind In Society* London: HUP

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5 things to do

Read the latest Charlie's Angles blog from NCETM Director Charlie Stripp in which he reflects upon PISA results for mathematics: how can they help us?

Book your place for the BCME8 conference which will be held 14-17 April 2014 in Nottingham. As the website says:

> *BCME is the largest mathematics and mathematics education conference in the UK, attracting delegates from all sectors. It brings together teachers from early years to higher education, researchers, teacher educators, CPD providers, consultants, policy makers, examiners and professional and academic mathematicians. Wherever you work you will, as a delegate at BCME8, have an unrivalled opportunity to mix with colleagues in your own specialist area and broaden your horizons by making connections with those working in other specialist areas. You will be able to compile your individual programme by selecting from over 250 general sessions and research paper presentations. You will be able to choose between standard and en-suite student rooms, on-site hotel accommodation and non-residential registration.*

Catch up on the Royal Institution Christmas Lectures 2013 The Life Fantastic, or those from previous years.

Enjoy this beautiful visualisation for pi.

And does this make you laugh?

> *A mathematician, a physicist and an economist from London are on the train on the way to a conference in Edinburgh. Just after they cross the border they look out of the window and see a black sheep. The economist says "Look, the sheep in Scotland are black!" The physicist says "No, we only know that some of the sheep in Scotland are black." The mathematician sighs and says "There is at least one sheep in Scotland, at least half of which is black."*

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Tales from the classroom: Mocks

I've never been one for New Year resolutions. For me every day is the start of a new year. I still find it exasperating that my students just don't get that every day is the start of a New Year, so they can "resolve" daily should they wish to. I've tried the idea of going along the lines of the beginnings of a mathematical proof - that there is always a day that will be 365 days forward from today, and that constitutes (in the calendar we currently use) a year. Hence today is the start of a New Year - and so is tomorrow - and the next…but it doesn't seem to wash.

Anyway, a big part of the 'New' Year for me, is the feedback of Mocks from the 'Old' Year. I'm now much more careful of how I go about that than I used to be. I've written before about managing expectation and esteem as being perhaps more important for me than the actual teaching of mathematical concepts; getting back a mock paper is probably one of the biggest 'home-truths' a student will have to accept during the run-up to exams and likely to have a considerable impact on their self-esteem. I think I will be especially careful this year as a result of a conversation I had with one of my NQTs on the penultimate day of the Autumn term.

I was venting about one of our most behaviourally challenging students whose initial response to sitting the mock had been to behave so poorly that despite all of our ignoring in the exam hall, he had to be removed. (He had picked up his exam table, strategically placed by our examinations officer to be at the very front, turned it through 180° and then sat back down to face his entire peer group. Unperturbed, most students continued, but when the poor soul sat directly in front of him could no longer ignore his taunts it was time for him to go. Roll on Christmas!) He "completed" his paper in isolation with his student support worker…a relationship that often works for us all. However, on marking his script, he had kindly and carefully answered all the questions with "dunno blud". After having corrected his spelling, I soon felt my anger rising.

I had seen bizarre answers before, and vividly remember the hilarity in our staff room over a decade ago when a student called Charlotte had tackled the question in our Year 7 baseline test. Her response to the question asking to name angles will remain with me forever. We were expecting the answers, obtuse, right and reflex, yet to our great amusement were provided with the names "Charlotte, Alan and Shearer". No doubt all perfectly reasonable names, but another case of the football star having greater influence than the maths teacher!

The response I had had from our troubled soul this year was different in its nature. Whereas Charlotte's had been an attempt to do well and succeed, the "dunno blud" appeared downright dismissive and reckless. As I continued to rant about the student's apparent stupidity, my NQT joined in with me, saying

"Yeah, I had one of those too, but mine was more polite - a "Don't know Miss. Sorry""

Definitely more polite than mine and, to rub the salt in, spelt correctly too. My NQT then tailed off with "Sad really"

And that was when I suddenly got it. The tone in her voice said it all. The annoying, red mist forming, "Dunno, you haven't taught us this" is not always, and indeed perhaps rarely is, a goad or challenge for action. More it is an expression of insecurity, embarrassment, disappointment and, albeit masked, a plea for help.

So, although it was 19 December my New Year resolution (my year starting on the 20 December and thereby finishing on the 19 December) was evidently before me. If I am going to give mock papers I need to make sure I am ready to deal with the emotions with which the students answer in addition to the numbers they write. They know it is me that will mark them. They would not write that in a real exam. So they write it for me to let me know they want to know it, but don't want to have to ask, or be seen to ask.

So for this year, I will not see the ridiculous "dunno" type answer as a challenge and a chance to chide. I will just say "OK, I can help with that topic, let's see what we need to do." When I hand back the papers, I won't talk about who got what and what was needed etc. I will just say, OK so well done, now we know we need to learn about...

And in a year from now, or 365 days I'll have a think and see if anything has changed. And as tomorrow is the start of a "New Year", I can have another "New Year" resolution. That's what is so good about my style of New Year resolutions – you are only limited to 365 of them – unless each second is the start of another new year…but that is beginning to sound like I could be introducing differentiation.

The author is a mathematics subject leader and assistant principal working in the South West