MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Multiplication and Division

2.14 Multiplication: partitioning leading to short multiplication

Teacher guide | Year 4

Teaching point 1:

The distributive law can be applied to multiply any two-digit number by a single-digit number, by partitioning the two-digit number into tens and ones, multiplying the parts by the single-digit number, then adding the partial products.

Teaching point 2:

Any two-digit number can be multiplied by a single-digit number using an algorithm called '*short multiplication*'; the digits of the factors must be aligned correctly; the algorithm is applied working from the least significant digit (on the right) to the most significant digit (on the left); if the product in any column is ten or greater, we must '*regroup*'.

Teaching point 3:

The distributive law can be applied to multiply any three-digit number by a single-digit number, by partitioning the three-digit number into hundreds, tens and ones, multiplying the parts by the single-digit number, then adding the partial products.

Teaching point 4:

Any three-digit number can be multiplied by a single-digit number using the short multiplication algorithm.

Overview of learning

In this segment children will:

- use informal written methods, based on the distributive law and supported by Dienes and unitising language, to multiply two-digit numbers by single-digit numbers, without regrouping, for example:

$2 \times 34 = 2 \times 30 + 2 \times 4$	↓ *partition and multiply*	2×3 tens $+ 2 \times 4$ ones $= 6$ tens $+ 8$ ones
$= 60 + 8$	↓ *add the partial products*	6 tens + 8 ones = 68
$= 68$		

- extend the use of Dienes and informal methods to understand the origin of regrouping when:
  - the partial product resulting from multiplication of the *ones* by the single-digit number is equal to ten or more, e.g.:

$3 \times 24 = 3 \times 20 + \mathbf{3 \times 4}$	↓ *partition and multiply*	3×2 tens + **3 × 4 ones** = 6 tens + **12 ones** **12 ones = 1 ten + 2 ones**
$= 60 + 12$	↓ *add the partial products*	6 tens + 1 ten + 2 ones = 7 tens + 2 ones = 72
$= 72$		

  - the partial product resulting from multiplication of the *tens* by the single-digit number is equal to 100 or more, e.g.:

$4 \times 32 = \mathbf{4 \times 30} + 4 \times 2$	↓ *partition and multiply*	**4 × 3 tens** + 4 × 2 ones = **12 tens** + 8 ones **12 tens = 1 hundred + 2 tens**
$= 120 + 8$	↓ *add the partial products*	1 hundred + 2 tens + 8 ones = 128
$= 128$		

- explore examples where regrouping of *both* the tens *and* ones is required (e.g. *4 × 26*)
- learn to apply the short multiplication algorithm for each of the above examples (no regrouping, regrouping of only the tens or the ones, and regrouping of both tens and ones), initially and briefly using an 'expanded' layout (in which the partial products are written separately), supported by Dienes and comparison with the informal written method, in order to understand the structure of the algorithm
- extend informal methods to three-digit × single-digit calculations, supported initially by Dienes (and then, as numbers become larger, by place-value counters) and unitising language
- extend the short multiplication algorithm to three-digit × single-digit calculations, including regrouping of the hundreds into thousands (e.g. *512 × 3*)
- use estimation to predict/check their answers, and apply the short multiplication algorithm to a range of practice problems.

For children to succeed with this segment, it is important for them to already have mastered:

- partitioning two-digit numbers into tens and ones (*Spine 1: Number, Addition and Subtraction*, segment *1.9*)
- partitioning three-digit numbers into hundreds, tens and ones (*Spine 1: Number, Addition and Subtraction*, segment *1.18*)
- multiplying a multiple of ten by a single-digit number (segment *2.13 Calculation: multiplying and dividing by 10 or 100*)
- applying their understanding of commutativity to write factors in either order (e.g. $2 \times 34 = 34 \times 2$)

Children should ultimately be able to use mental methods to multiply a two-digit number by a single-digit number. The short multiplication algorithm is applied to such calculations here for the purposes of developing children's understanding of how the algorithm works, and allowing them to build their confidence as they move towards *three-digit* × *single-digit* calculations. The overall aim is for children to be able to make sensible decisions about which is the most efficient method for a particular caclulation, including for *three-digit* × *single-digit* calculations; for example, mental methods are probably more efficient than short multiplication for a calculation such as 201×4.

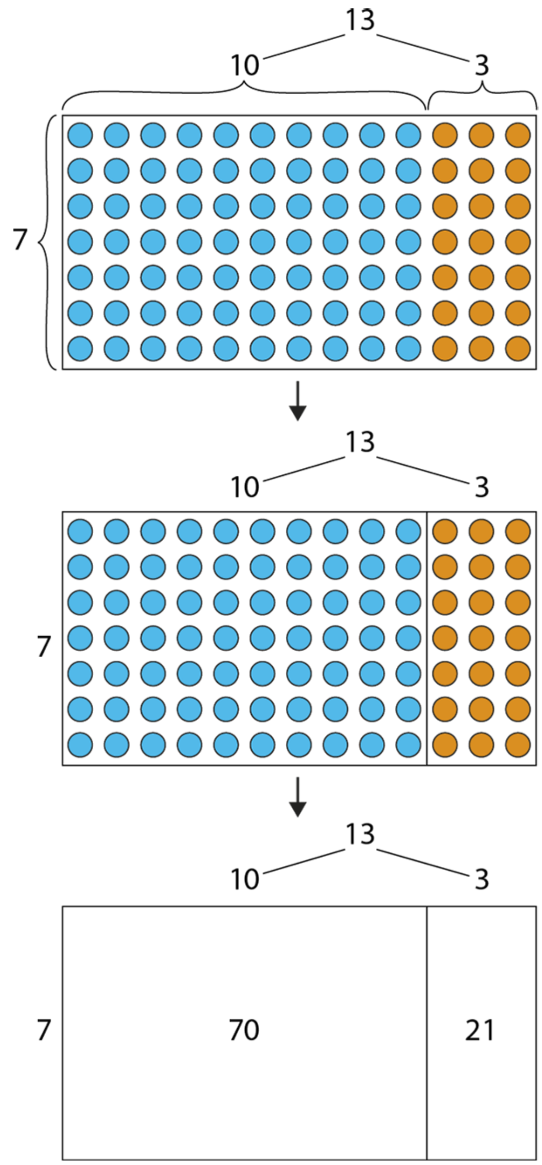
As with column addition and subtraction (*Spine 1: Number, Addition and Subtraction*, segments *1.20* and *1.21*), the use of squared paper will support children in correctly laying out their calculations.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

The distributive law can be applied to multiply any two-digit number by a single-digit number, by partitioning the two-digit number into tens and ones, multiplying the parts by the single-digit number, then adding the partial products.

Steps in learning

	Guidance	Representations
1:1	In this teaching point, children will apply their understanding of the distributive law and multiplying by ten from segments *2.10 Connecting multiplication and division, and the distributive law* and *2.13 Calculation: multiplying and dividing by 10 or 100*. Briefly review application of the distributive law to the multiplication of a teen number by a single-digit number (e.g. 13×7), using the array-to-grid representation as shown opposite (for more guidance, see segment *2.10*). Draw attention to the fact that the factors can be written in either order and the partial products will remain the same (supported by the understanding that the same array-to-grid representation can be used to represent both $13 \times 7 = 70 + 21$ and $7 \times 13 = 70 + 21$). Repeat for a few different examples, gradually removing the scaffold of the array-to-grid representation.	 $13 \times 7 = 10 \times 7 + 3 \times 7$ $= 70 + 21$ $= 91$ $7 \times 13 = 7 \times 10 + 7 \times 3$ $= 70 + 21$ $= 91$

1:2

Now progress to multiplying a non-teen two-digit number by a single-digit number. Dienes can be used to represent the numbers, in a similar way to *Spine 1: Number, Addition and Subtraction*, segments *1.15*, *1.16*, *1.20* and *1.21*.

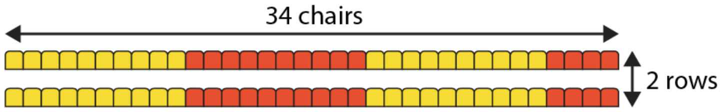
Present a problem, such as: '*There are two rows, each with thirty-four chairs. How many chairs are there altogether?'*

Note that, for now, numbers should be chosen to avoid the need to regroup.

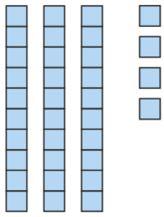
Ask children to partition the 34, describing it in terms of tens and ones, then represent it using Dienes. Then make another copy of the Dienes representation of 34, since we are multiplying by two. Finally, bring the tens together and the ones together, to represent the product. Record equations and use unitising language as you work through the steps, as shown opposite and on the next page.

Repeat the process for a range of *two-digit* × *single-digit* calculations, without regrouping, until children are confident. Use examples where the two-digit number is sometimes presented as the first factor and sometimes as the second factor.

'There are two rows, each with thirty-four chairs. How many chairs are there altogether?'



Step 1 – partition thirty-four:

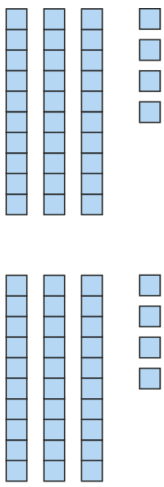


34 = 30 + 4

- *'Thirty-four is equal to three tens and four ones.'*

34 = 3 tens + 4 ones

Step 2 – make a copy of the representation (multiply by two):



34 × 2

- *'Three-tens-and-four-ones multiplied by two.'*

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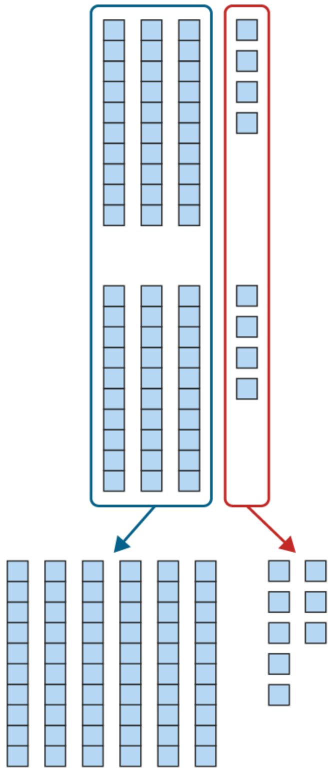
Step 3 – multiply the tens and ones and recombine:



$34 \times 2 = 30 \times 2 + 4 \times 2$
$= 60 + 8$
$= 68$

- *'Three-tens-and-four-ones multiplied by two is equal to three tens multiplied by two and four ones multiplied by two.'*

3 tens × 2 = 6 tens

4 ones × 2 = 8 ones
</td>
</tr>
</table>

1:3

Once children are comfortable with the representations, equations and language used in step *1:2*, move to an example that requires regrouping of ones into tens, for example, *'There are three rows, each with twenty-four chairs. How many chairs are there altogether?'*

Follow the same sequence as for step *1:2*, then regroup the ones (note that only the final two steps are shown opposite and on the next page).

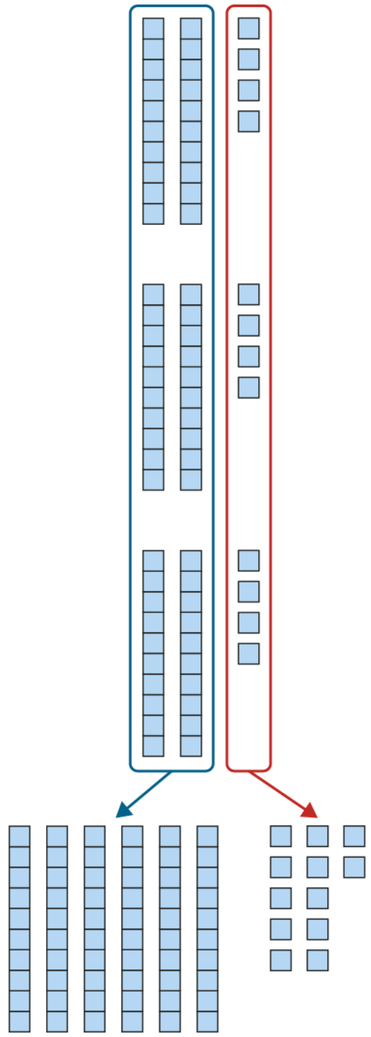
Work through several examples, working towards the generalisation: ***'If there are ten or more ones, we must regroup the ones into tens and ones.'***

Use examples where the two-digit number is sometimes presented as the first factor and sometimes as the second factor. Also, include examples for which the ones are regrouped into *more* than one ten (and some ones), but avoiding the need to regroup the tens into hundreds, for example:

$$28 \times 3 = 20 \times 3 + 8 \times 3$$
$$= 60 + 24$$

'There are three rows, each with twenty-four chairs. How many chairs are there altogether?'

Multiply the tens and ones and recombine:

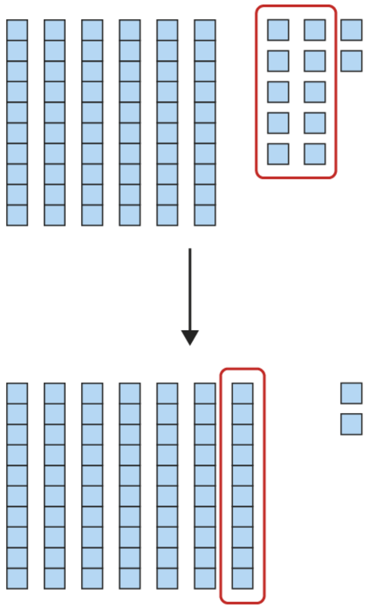


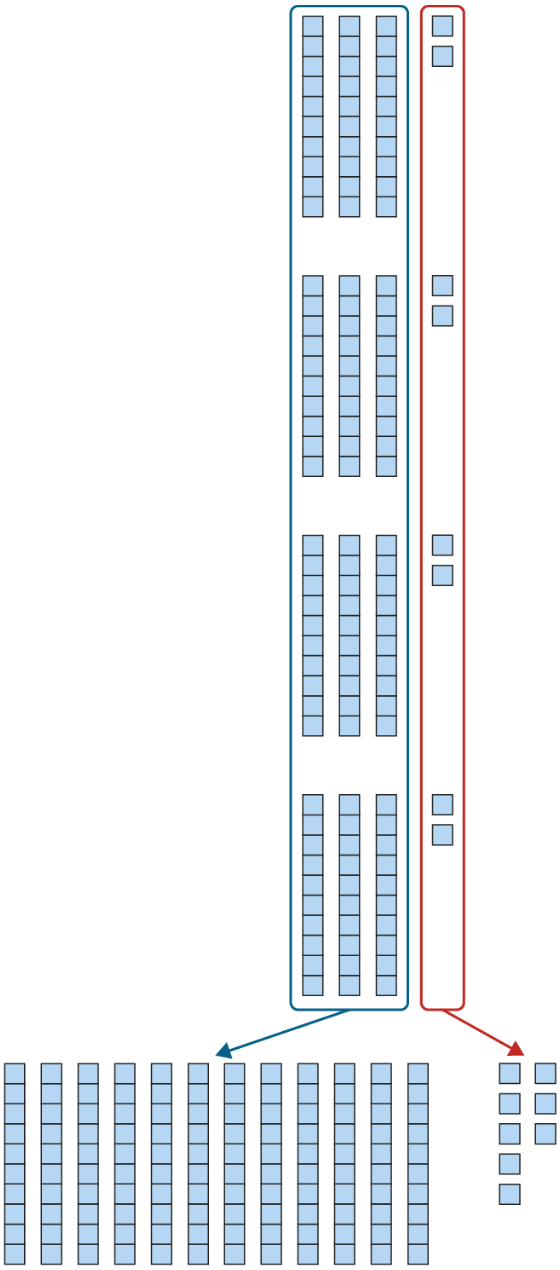
$$24 \times 3 = 20 \times 3 + 4 \times 3$$
$$= 60 + 12$$

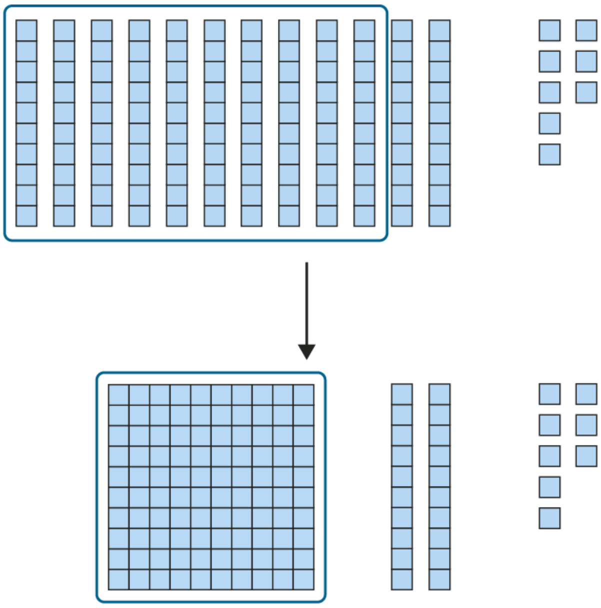
- *'Two-tens-and-four-ones multiplied by three is equal to two tens multiplied by three and four ones multiplied by three.'*

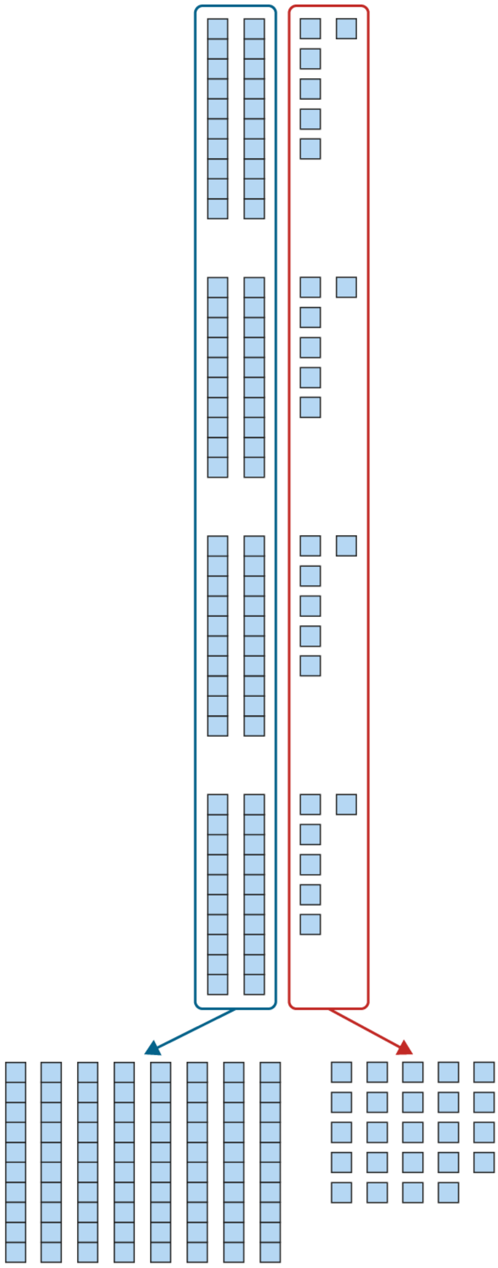
2 tens × 3 = 6 tens

4 ones × 3 = 12 ones

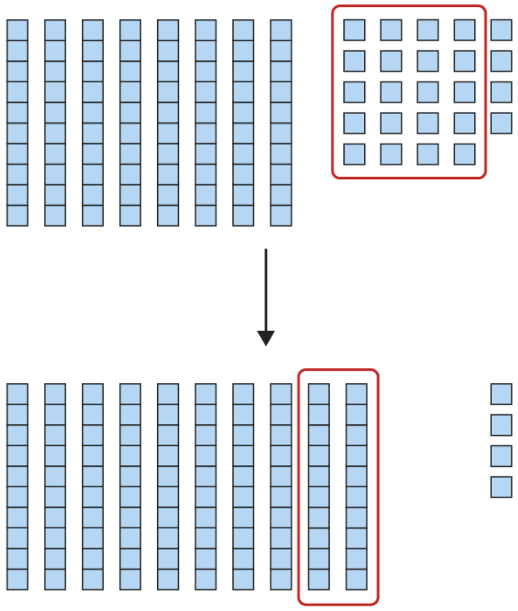
		Regroup the ones into tens and ones:  $24 \times 3 = 60 + 10 + 2$ $= 70 + 2$ $= 72$ • *'Twelve ones is equal to one ten and two ones.'* 12 ones = 1 ten + 2 ones

1:4	Now move on to an example that requires regrouping of tens into hundreds, for example, *'There are four rows, each with thirty-two chairs. How many chairs are there altogether?'* Follow the same sequence as for step *1:3*. Work through several examples, working towards the generalisation: ***'If there are ten or more tens, we must regroup the tens into hundreds and tens.'*** Continue to vary the order in which the factors are presented, and include examples for which the tens are regrouped into more than one hundred (and some tens), but avoiding the need to regroup the ones into tens; for example: $41 \times 6 = 40 \times 6 + 1 \times 6$ $= 240 + 6$	*'There are <u>four</u> rows, each with <u>thirty-two</u> chairs. How many chairs are there altogether?'* Multiply the tens and ones and recombine:  $32 \times 4 = 30 \times 4 + 2 \times 4$ $= 120 + 8$ • *'Three-tens-and-two-ones multiplied by four is equal to three tens multiplied by four and two ones multiplied by four.'* 3 tens × 4 = 12 tens 2 ones × 4 = 8 ones

		Regroup the tens into hundreds and tens:  $32 \times 4 = 120 + 8$ $= 128$ • *'Twelve tens is equal to one hundred and two tens.'* 12 tens = 1 hundred + 2 tens

1:5	Now work through an example where regrouping the ones creates the need to regroup the tens. In the example opposite, the product has a zero in the tens place, which can be useful for challenging any misconceptions regarding place value.	*'There are four rows, each with twenty-six chairs. How many chairs are there altogether?'* Multiply the tens and ones and recombine:  $26 \times 4 = 20 \times 4 + 6 \times 4$ $= 80 + 24$ • *'Two-tens-and-six-ones multiplied by four is equal to two tens multiplied by four and six ones multiplied by four.'* 2 tens × 4 = 8 tens 6 ones × 4 = 24 ones

Regroup the ones into tens and ones:



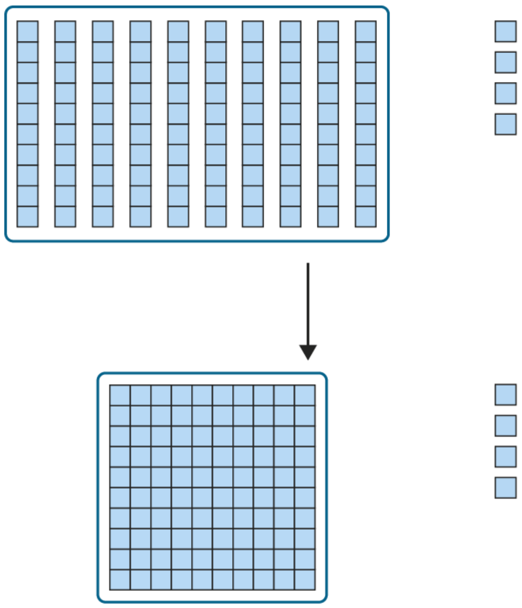
$26 \times 4 = 80 + 24$

$= 80 + 20 + 4$

- *'Twenty-four ones is equal to two tens and four ones.'*

24 ones = 2 tens + 4 ones

Regroup the tens into hundreds and tens:



$26 \times 4 = 80 + 20 + 4$

$= 104$

- *'Ten tens is equal to one hundred.'*

10 tens = 1 hundred

1:6	Finally, work through some examples without the support of Dienes, until children can confidently partition the two-digit number into tens and ones, multiply the resulting parts by the single-digit number and add the partial products. Two different methods of showing working are exemplified opposite; as long as children can confidently and correctly show their working (including correct use of the '=' sign), the method chosen is not important. Note that the regrouping is now 'hidden' within addition across tens/hundreds boundaries. Regrouping will be apparent again in *Teaching point 2*, when children use the short multiplication algorithm.	*'One pack of biscuits costs 84 p. How much do six packs cost?'* Example working 1: $84 \times 6 = 504$ (84 partitioned into 80 and 4) $80 \times 6 = 480$ $4 \times 6 = 24$ $480 + 24 = 504$ *'Six packs of biscuits cost 504 p.'* Example working 2: $84 \times 6 = 80 \times 6 + 4 \times 6$ $= 480 + 24$ $= 504$ *'Six packs of biscuits cost 504 p.'*
1:7	To complete this teaching point, provide children with practice multiplying two-digit numbers by single-digit numbers, using the informal written methods outlined above. Children can initially use Dienes for support but should progress to working with jottings/equations only. Continue to vary the order in which the factors are presented. Also include contextual problems that contain some superfluous information, to check that children can identify the required information and apply the method; for example: '*One pack of three peppers costs 92 p. How much do four packs cost?*'	Matching multiplication expressions with partial products: *'Draw a line to match each multiplication expression with the correct addition expression.'* 48×3 $120 + 18$ 6×23 $80 + 24$ 26×4 $120 + 24$ Missing-number problems: *'Fill in the missing numbers.'* $15 \times 4 = 10 \times 4 + 5 \times 4$ $= \square + \square$ $= \square$

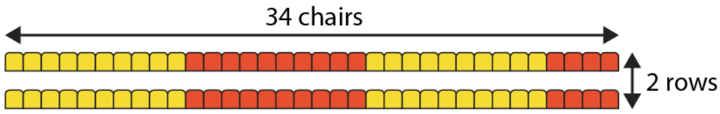
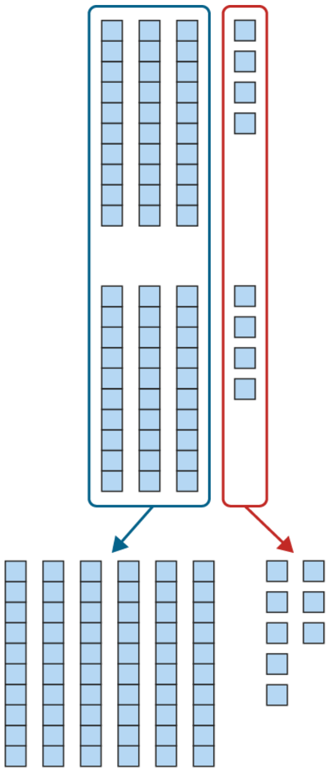
		$25 \times 4 = \square \times 4 + 5 \times 4$ $= \square + \square$ $= \square$ $35 \times 4 = \square \times 4 + \square \times 4$ $= \square + \square$ $= \square$ $6 \times 28 = 6 \times 20 + 6 \times 8$ $= \square + \square$ $= \square$ $6 \times 28 = 6 \times \square + 6 \times 8$ $= \square + \square$ $= \square$ $35 \times 4 = 6 \times \square + 6 \times \square$ $= \square + \square$ $= \square$ $42 \times 7 = \square$ $9 \times 72 = \square$ $3 \times 57 = \square$ $35 \times 8 = \square$

		Dòng nǎo jīn: *'Sam used the strategy of partitioning the two-digit number into tens and ones to do this multiplication:'* 25×7 *'Which of these calculations did he use to find the product? Explain your reasoning.'* $140 + 35$ $70 + 70 + 35$ $147 + 28$ $100 + 75$

Teaching point 2:

Any two-digit number can be multiplied by a single-digit number using an algorithm called *'short multiplication'*; the digits of the factors must be aligned correctly; the algorithm is applied working from the least significant digit (on the right) to the most significant digit (on the left); if the product in any column is ten or greater, we must *'regroup'*.

Steps in learning

	Guidance	Representations
2:1	This teaching point introduces the short multiplication algorithm as a consistent way to record and apply the principles covered in *Teaching point 1*. Begin with the example from step *1:2* above (*34* × *2*), which does not require any regrouping. This time, model how the calculation can be recorded vertically, at this stage keeping the partial products separate, as illustrated on the next page (we will refer to this as the 'expanded' layout). For now, include place-value headings to support children. Build up the calculation as you work through the problem using Dienes, and continue to use the unitising language introduced in *Teaching point 1*. This time, first multiply the *ones* digit (of the two-digit number) by the single-digit number, and then multiply by the *tens* digit by the single-digit number, in preparation for the convention of working through the algorithm from right to left (which will facilitate regrouping). Note that the expanded layout is used only to ensure that children understand how the algorithm works, and should only be used for a short time.	*'There are <u>two</u> rows, each with <u>thirty-four</u> chairs. How many chairs are there altogether?'*  Dienes representation: 

Multiplication algorithm – expanded layout:

Step 1 – write the factors:

	10s	1s
	3	4
×		2

Step 2 – multiply the single-digit number by the ones:

	10s	1s	
	3	4	
×		2	
		8	2 × 4 ones = 8 ones

Step 3 – multiply the single-digit number by the tens:

	10s	1s	
	3	4	
×		2	
		8	2 × 4 ones = 8 ones
	6	0	2 × 3 tens = 6 tens

Step 4 – add the partial products:

	10s	1s	
	3	4	
×		2	
		8	2 × 4 ones = 8 ones
	6	0	2 × 3 tens = 6 tens
	6	8	

2:2	Now compare the expanded algorithm with the informal written method used in *Teaching point 1*, asking children: • *'What's the same?'* • *'What's different?'* Draw attention to the following: • In both, we multiply the single-digit number separately by the ones and by the tens of the two-digit number. • This gives rise to the same partial products. • In both cases we then add the partial products to find the product. • The layout is different. Then look more closely at the expanded algorithm and point out that: • when the factors are recorded, the digits are aligned correctly (ones with ones), and this is similar to laying out a column addition/subtraction calculation • when the partial products are recorded, the digits are aligned correctly.	Comparing methods: **Informal written method:** $34 \times 2 = 30 \times 2 + 4 \times 2$ $= 60 + 8$ $= 68$ **Expanded multiplication algorithm:** 10s | 1s 3 | 4 × | 2 | 8 — 2×4 ones = 8 ones 6 | 0 — 2×3 tens = 6 tens 6 | 8
2:3	Now work through another example, keeping to factors that avoid the need to regroup. This time, present an abstract problem with the single-digit number written as the first factor (e.g. 3×32 rather than 32×3). Draw attention to the fact that we usually write the larger factor at the top in the algorithm. Remind children of the commutative property of multiplication which allows us to write the factors in the most convenient order, with the product remaining the same.	$3 \times 32 = ?$ 10s | 1s 3 | 2 × | 3 | 6 — 3×2 ones = 6 ones 9 | 0 — 3×3 tens = 9 tens 9 | 6

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Work through the same example (from step 2:3), now progressing to the 'compact' layout.

As you work through each stage of laying out and applying the algorithm, draw attention to the following key points, encouraging children to repeat the instructions at each step:

'First, write the largest factor: thirty-four.'
'Then write the smallest factor below, lining up the digits: two.'
'Now multiply, starting with the ones: three times two ones is equal to six ones; write "6" in the ones column.'
'Then move to the tens: three times three tens is equal to nine tens; write "9" in the tens column.'

Explain that we call this 'short multiplication'.

Work through a variety of problems, gradually removing the place-value headings.
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Example 1 – compact layout with place-value headings:

$$\begin{array}{r|c|c} & 10s & 1s \\ \hline & 3 & 2 \\ \times & & 3 \\ \hline & 9 & 6 \\ \hline \end{array}$$

3×2 ones $= 6$ ones
'Write "6" in the ones column.'
3×3 tens $= 9$ tens
'Write "9" in the tens column.'

Example 2 – compact layout without place-value headings:

$$\begin{array}{rcc} & 2 & 1 \\ \times & & 4 \\ \hline & 8 & 4 \\ \hline \end{array}$$

4×1 one $= 4$ ones
'Write "4" in the ones column.'
4×2 tens $= 8$ tens
'Write "8" in the tens column.'

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<tr>
<td>2:5</td>
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At this point, give children practice laying out and completing calculations for factors that do not involve regrouping.
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Laying out short multiplication calculations:

'Write these as short multiplication calculations.'

23×3 2×41 4×22 2×44

Applying the short multiplication algorithm:

'Complete the calculations.'

$$\begin{array}{rcc} & 3 & 1 \\ \times & & 3 \\ \hline & & \\ \hline \end{array} \quad \begin{array}{rcc} & 4 & 3 \\ \times & & 2 \\ \hline & & \\ \hline \end{array} \quad \begin{array}{rcc} & 1 & 3 \\ \times & & 3 \\ \hline & & \\ \hline \end{array}$$
Dòng nǎo jīn:

'Fill in the missing numbers.'

$$\begin{array}{rcc} & 4 & 1 \\ \times & & \square \\ \hline & 8 & 2 \\ \hline \end{array} \quad \begin{array}{rcc} & \square & 2 \\ \times & & 4 \\ \hline & 4 & 8 \\ \hline \end{array} \quad \begin{array}{rcc} & \square & \square \\ \times & & 4 \\ \hline & 8 & 8 \\ \hline \end{array}$$
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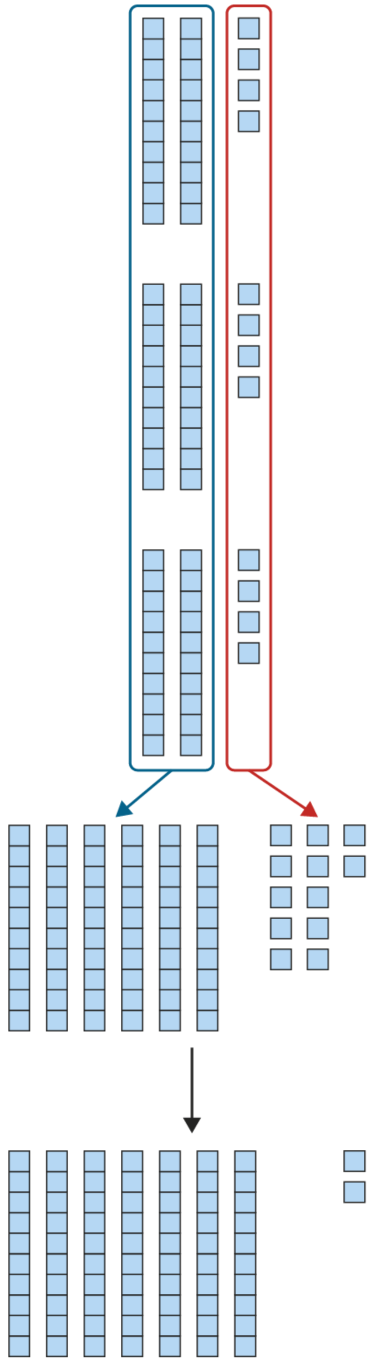
2:6	Once children have mastered the basic principles of short multiplication without regrouping, introduce regrouping of ones into tens. Use the example from step *1:3* (*3* × *24*), building up the expanded form of the calculation, supported by Dienes. Draw attention to the regrouping of twelve ones into one ten and two ones: we don't write '12' in the ones column, instead we write '1' in the tens column and '2' in the ones column. Remind children of the stem sentence from step *1:3*: ***'If there are ten or more ones, we must regroup the ones into tens and ones.'***	*'There are three rows, each with twenty-four chairs. How many chairs are there altogether?'* Dienes representation: 

Multiplication algorithm – expanded layout:

	10s	1s	
	2	4	
×		3	
	1	2	3×4 ones = 12 ones = 1 ten + 2 ones
	6	0	3×2 tens = 6 tens
	7	2	

2:7

Quickly move to using the 'compact' layout. As in step *2:4*, encourage children to describe the steps in laying out and applying the algorithm, now also including the <u>regrouping of the ones</u>:

- *'First, write the largest factor: twenty-four.'*
- *'Then write the smallest factor below, lining up the digits: three.'*
- *'Now multiply, starting with the ones: three times four ones is equal to twelve ones…'*
- *'<u>and regroup: twelve ones is equal to one ten and two ones; write "1" below the tens column and "2" in the ones column</u>.'*
- *'Then move to the tens: three times two tens is equal to six tens…'*
- *'<u>and add the ten from regrouping</u>: six tens plus one ten is equal to seven tens; write "7" in the tens column.'*

Draw attention to:

- recording the '1' (that represents the ten ones regrouped into ten) *below* the line because we still have the tens to multiply
- remembering to add the extra '1' ten to the rest of the tens.

Work through a variety of examples, gradually removing the place-value headings, including examples with a zero in the ones column of the product.

Example 1 – compact layout *with* place-value headings:

Step 1 – write the factors:

	10s	1s
	2	4
×		3

Step 2 – multiply the single-digit number by the ones and regroup:

	10s	1s
	2	4
×		3
		2
	1	

3×4 ones = 12 ones
= 1 ten + 2 ones

'Write "1" <u>below</u> the tens column and "2" in the ones column.'

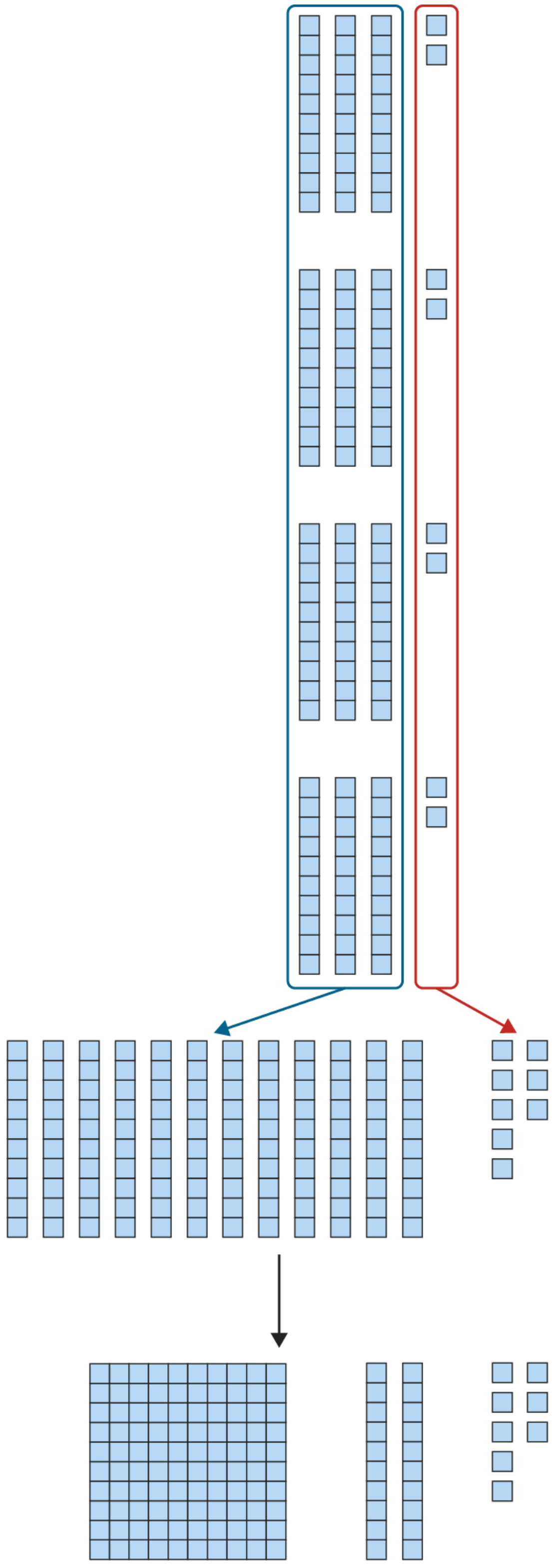
Step 3 – multiply the single-digit number by the tens and add the tens from regrouping:

	10s	1s
	2	4
×		3
	7	2
	1	

3×2 tens = 6 tens

6 tens + 1 ten = 7 tens
'Write "7" in the tens column.'

		Example 2 – compact layout *without* place-value headings: $\begin{array}{rcc} & 1 & 8 \\ \times & & 5 \\ \hline & 9 & 0 \\ \hline & 4 & \end{array}$ • 5 × 8 ones = 40 ones; 40 ones = 4 tens and 0 ones *'Write "4" below the tens column and '0' in the ones column.'* • 5 × 1 ten = 5 tens 5 tens + 4 tens = 9 tens *'Write "9" in the tens column.'*
2:8	At this point, provide children with some practice laying out and completing calculations for factors that result in the need to regroup the ones. When presenting problems not already laid out as short multiplication, remember to vary the order of the factors (*single-digit* × *two-digit* and *two-digit* × *single-digit*). Also include examples where the product has a zero in the ones column.	Applying the short multiplication algorithm: *'Complete the calculations.'* $\begin{array}{rcc} & 1 & 3 \\ \times & & 5 \\ \hline & & \\ \hline \end{array}$ $\quad$ $\begin{array}{rcc} & 1 & 7 \\ \times & & 4 \\ \hline & & \\ \hline \end{array}$ Laying out and applying the short multiplication algorithm: *'Use short multiplication to do these calculations.'* 27 × 3 $\quad$ 5 × 14 $\quad$ 2 × 46 29 × 3 $\quad$ 16 × 5 $\quad$ 4 × 23

2:9	Now introduce regrouping of tens into hundreds within the short multiplication algorithm. Use the example from step *1:4* (*4* × *32*), building up the expanded form of the calculation, supported by Dienes. Draw attention to the regrouping of twelve tens into one hundred and two tens: we don't write '12' in the tens column, instead we write '1' in the hundreds column and '2' in the tens column. Remind children of the stem sentence from step *1:4*: ***'If there are ten or more tens, we must regroup the tens into hundreds and tens.'***	*'There are four rows, each with thirty-two chairs. How many chairs are there altogether?'* Dienes representation: 

<table>
<tr>
<td></td>
<td></td>
<td>
Multiplication algorithm – expanded layout:
<table>
<tr><td></td><td>100s</td><td>10s</td><td>1s</td></tr>
<tr><td></td><td></td><td>3</td><td>2</td></tr>
<tr><td>×</td><td></td><td></td><td>4</td></tr>
<tr><td></td><td></td><td></td><td>8</td></tr>
<tr><td></td><td>1</td><td>2</td><td>0</td></tr>
<tr><td></td><td>1</td><td>2</td><td>8</td></tr>
</table>

4 × 2 ones = 8 ones
4 × 3 tens = 12 tens = 1 hundred + 2 tens

</td>
</tr>
<tr>
<td>2:10</td>
<td>
As before, quickly move to using the ‘compact’ layout. Continue to encourage children to describe the steps in laying out and applying the algorithm, now including the regrouping of the tens:

‘First, write the largest factor: thirty-two.’
‘Then write the smallest factor below, lining up the digits: four.’
‘Now multiply, starting with the ones: four times two ones is equal to eight ones; write “8” in the ones column.’
‘Then move to the tens: four times three tens is equal to twelve tens…’
‘and regroup: twelve tens is equal to one hundred and two tens; write “1” in the hundreds column and “2” in the tens column.’

Work through a variety of examples, gradually removing the place-value headings, including examples with a zero in the tens column of the product.
</td>
<td>
Example 1 – compact layout with place-value headings:
<table>
<tr><td></td><td>100s</td><td>10s</td><td>1s</td></tr>
<tr><td></td><td></td><td>3</td><td>2</td></tr>
<tr><td>×</td><td></td><td></td><td>4</td></tr>
<tr><td></td><td>1</td><td>2</td><td>8</td></tr>
</table>

4 × 2 ones = 8 ones
‘Write “8” in the ones column.’
4 × 3 tens = 12 tens = 1 hundred + 2 tens
‘Write “1” in the hundreds column and “2” in the tens column.’

Example 2 – compact layout without place-value headings:
<table>
<tr><td></td><td></td><td>2</td><td>1</td></tr>
<tr><td>×</td><td></td><td></td><td>5</td></tr>
<tr><td></td><td>1</td><td>0</td><td>5</td></tr>
</table>

5 × 1 one = 5 ones
‘Write “5” in the ones column.’
5 × 2 tens = 10 tens; 10 tens = 1 hundred and 0 tens
‘Write “1” in the hundreds column and “0” in the tens column.’

</td>
</tr>
</table>

2:11

At this point, provide children with some practice similar to that in step *2:8*, but now with regrouping of the tens. When presenting problems not already laid out as short multiplication, remember to vary the order of the factors (*single-digit* × *two-digit* and *two-digit* × *single-digit*). Also include examples where the product has a zero in the tens column.

Applying the short multiplication algorithm:

'Complete the calculations.'

$$\begin{array}{r} 7\ 4 \\ \times\quad 2 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 4\ 1 \\ \times\quad 5 \\ \hline \\ \hline \end{array}$$

Laying out and applying the short multiplication algorithm:

'Use short multiplication to do these calculations.'

3×52 62×4 3×72 71×8

Dòng nǎo jīn:

'Without completing the calculations, circle the ones that involve regrouping in the tens.'

$$\begin{array}{r} 4\ 1 \\ \times\quad 2 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 4\ 1 \\ \times\quad 4 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 6\ 1 \\ \times\quad 2 \\ \hline \\ \hline \end{array}$$

$$\begin{array}{r} 4\ 2 \\ \times\quad 2 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 5\ 2 \\ \times\quad 2 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 6\ 2 \\ \times\quad 2 \\ \hline \\ \hline \end{array}$$

2:12

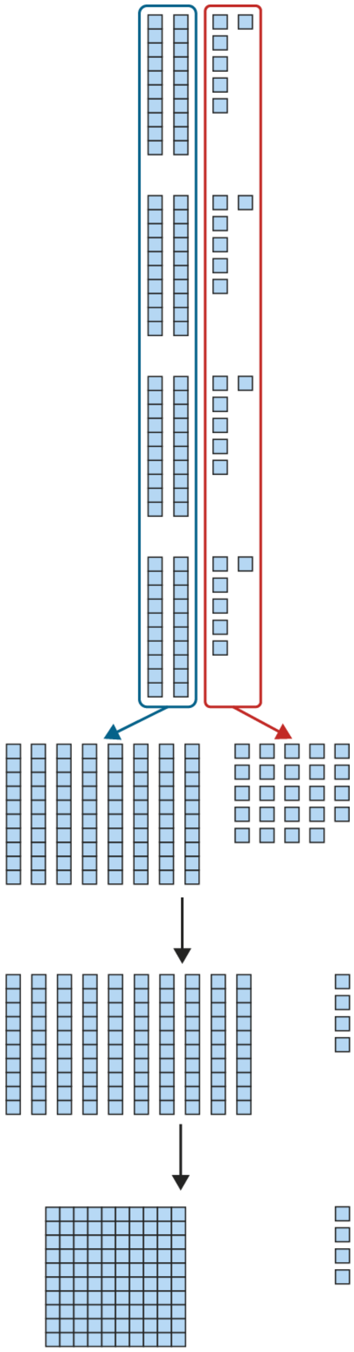
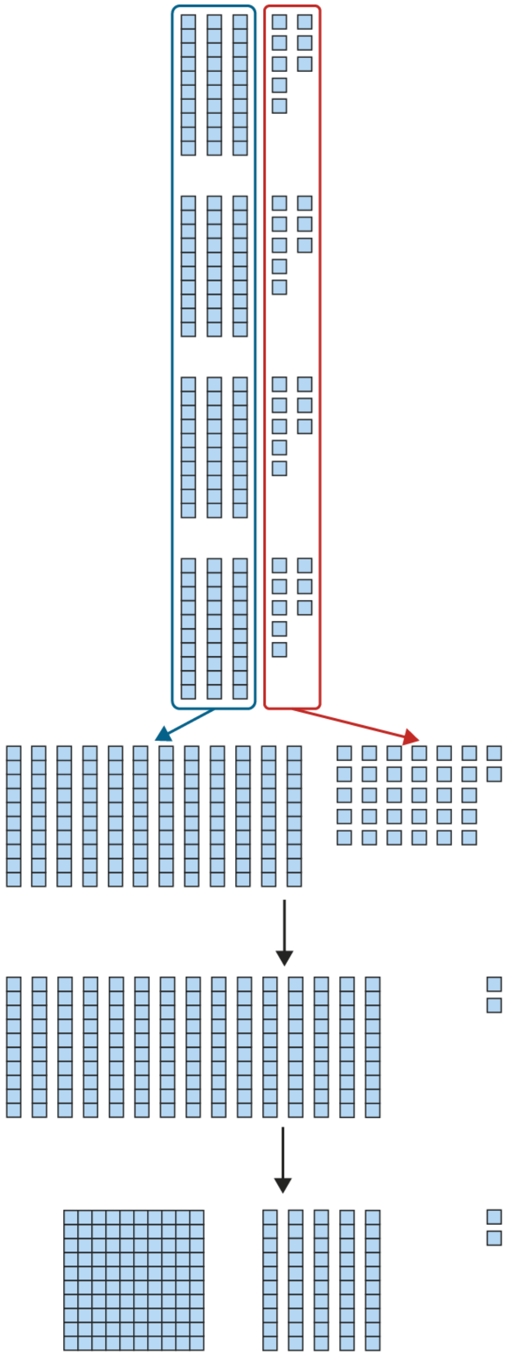
Finally, look at examples where:

- regrouping of the ones creates the need to regroup the tens (using the example from step *1:5*)
- regrouping is required in *both* the tens *and* ones as a result of multiplying the parts (see *Example 2* on the next page).

Note that, over time, children will begin to shorten the descriptive language that they use to reason through application of the algorithm (compact layout); i.e. in *Example 1* on the next page (*4 × 26*), children may eventually say in their heads:

- *'four sixes are twenty-four'*
  (writing down '2' below the tens column and '4' in the ones column)
- *'four twos are eight…; plus two is ten'* (writing down '1' in the hundreds column and '0' in the tens column)

However, for now, encourage children to keep using the descriptive language outlined in steps *2:4, 2:7 and 2:10*, since it is important that they 'see' the place value of the digits that they have built up with the Dienes.

	Example 1 – regrouping the ones creates the need to regroup the tens during addition: *'There are four rows, each with twenty-six chairs. How many chairs are there altogether?'* Dienes representation: 	Example 2 – regrouping required in both tens and ones due to multiplication: *'A gardener has four rows of thirty-eight carrot plants. How many carrot plants does he have altogether?'* Dienes representation: 

Multiplication algorithm – expanded layout:

	100s	10s	1s
		2	6
×			4
		2	4
		8	0
	1	0	4

- 4 × 6 ones = 24 ones = 2 tens + 4 ones
- 4 × 2 tens = 8 tens
- 8 tens + 2 tens + 4 ones = 10 tens + 4 ones
  = 1 hundred + 4 ones

Multiplication algorithm – compact layout:

		2	6
×			4
	1	0	4
		2	

Multiplication algorithm – expanded layout:

	100s	10s	1s
		3	8
×			4
		3	2
	1	2	0
	1	5	2

- 4 × 8 ones = 32 ones = 3 tens + 2 ones
- 4 × 3 tens = 12 tens = 1 hundred + 2 tens
- 1 hundred + 2 tens + 3 tens + 2 ones
  = 1 hundred + 5 tens + 2 ones

Multiplication algorithm – compact layout:

		3	8
×			4
	1	5	2
		3	

2:13

Again, provide children with some practice, similar to that in steps *2:8* and *2:11*, but now for calculations that involve regrouping of both the tens and the ones.

The examples opposite include:

- pairs of problems that use the same digits; within each pair, ask children to comment on how the products are affected by the swap in digits
- calculations that have a product with:
  - '0' in the ones place (e.g. *4 × 75*)
  - '0' in both the tens and the ones places (e.g. *80 × 5*)

If children struggle to place the digits correctly in the answer row, ask them to return to the expanded layout briefly, before re-doing the problem in the compact layout.

Applying the short multiplication algorithm:

'Complete the calculations.'

	4	8			6	4
×		3		×		5
	___	___			___	___
	3	7			7	5
×		7		×		4
	___	___			___	___

Laying out and applying the short multiplication algorithm:

'Use short multiplication to do these calculations.'

43 × 8   54 × 6   73 × 7   80 × 5

2:14

Before providing some general practice on applying the short multiplication algorithm (step *2:15*), explore how estimation can be used before or after carrying out a full calculation in order to spot errors. Use learning from segment *2.13 Calculation: multiplying and dividing by 10 or 100* and from *Teaching point 1* above to support estimation and error checking, as exemplified opposite. *Example 1* uses discursive reasoning, while *Example 2* uses inequality symbols to represent the reasoning.

Provide children with some practice using inequalities and making estimations, as shown opposite.

Estimation – example 1:

24×3

- *'Twenty-four is between twenty and thirty.'*

$20 \times 3 = 60$

$30 \times 3 = 90$

- *'So, twenty-four times three must be between sixty and ninety.'*

Estimation – example 2:

4×38

$$30 < 38 < 40$$

$$4 \times 30 < 4 \times 38 < 4 \times 40$$

$$120 < 4 \times 38 < 160$$

Using inequalities and estimating – example practice:

- *'Fill in the missing numbers to complete this estimation.'*

48×6

48 is between 40 and 50.

$40 \times 6 =$ ☐

$50 \times 6 =$ ☐

So 48×6 must be between ___ and ___.

- *'Decide whether each inequality is true or false.'*

	True (✓) or false (×)?
$6 \times 30 < 6 \times 35$	
$180 < 6 \times 35$	
$5 \times 32 < 200$	
$8 \times 49 > 400$	
$7 \times 75 > 490$	

- *'Estimate the answer to each of these calculations.'*

33×9 4×67 8×56

2:15

Now provide children with general practice applying the short multiplication algorithm, in the form of:

- completing calculations (see examples in steps *2:5*, *2:8*, *2:11* and *2:13*)
- contextual problems, varying the order in which the factors (single-digit and two-digit) are presented, such as the examples opposite and here:
  - *'Ellen has three bags of twenty-six marbles. How many marbles does she have altogether?'*
  - *'A shop has forty-three packs of bread rolls in stock. There are nine rolls in each pack. How many rolls are there altogether?'*

Ensure that you include examples that:

- require no regrouping
- require regrouping *only* of the ones *or* of the tens
- require regrouping *both* of the ones *and* of the tens
- have products with no zero digits
- have products with a zero in *only* the ones place *or* the tens place
- have products with a zero in *both* the ones place *and* the tens place.

For all problems, children should estimate the value of the product first, perform the calculation using the algorithm, and then use their estimate to sense-check their answers.

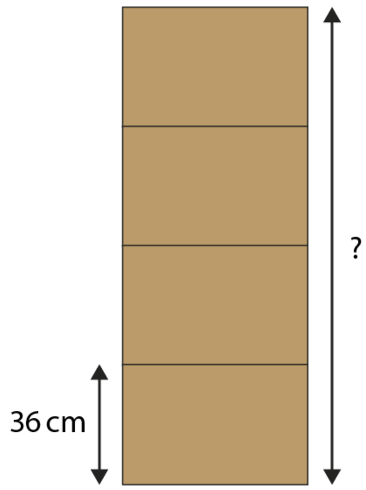
Use calculations with missing digits to deepen children's understanding of the algorithm.

For the dòng nǎo jīn missing-digit problems opposite, encourage children to reason carefully:

- Problem *a* has one solution, which children should be able to determine by finding the value of the product of the ones ('35').

Contextual problem:

'If all of these boxes are the same height, how tall is the stack of boxes?'



Calculations with missing digits:

'Fill in the missing digits.'

```
   5 □            7 2
×    3         ×    □
 -----         ------
 □ □ 9          1 4 □
```

Dòng nǎo jīn:

'Fill in the missing digits.'

```
a     3 □        b     3 □
  ×     7          ×     □
  -------          -------
    2 4 5            2 2 8

c     7 □        d     □ □
  ×     6          ×     4
  -------          -------
    □ □ 4            □ 8 0
```


	• Problem *b* also has one solution, but children will need to try different digits in the ones column that have a product with a ones digit of '8'; here the key is to think about the possible values of multiples of '30' (or '3') that are less than '220' (or '22') – children should not expect to solve such problems at the first attempt. • Problem *c* has two possible solutions, since there are two single-digit numbers ('4' and '9') that when multiplied by six give a product with a ones digit of four. • Problem *d* has two possible solutions; first look at the ones – there is only one single-digit number (5) that when multiplied by four gives a number with a ones-digit of zero; this allows us to determine that we need a digit in tens column of the two-digit factor that when multiplied by four gives a ones digit of '6' – there are two possible single-digit numbers ('4' and '9') that satisfy this criterion.	• *'Jade and Hamid are doing this calculation:'* 6×20 *'Whose method do you think is most efficient? Why?'* • Jade's method: *'I'll use short multiplication.'* $\begin{array}{r} 2\ 0 \\ \times \quad\ 6 \\ \hline 1\ 2\ 0 \\ \hline \end{array}$ • Hamid's method: *'Six times two tens is twelve tens.'* 120

Teaching point 3:

The distributive law can be applied to multiply any three-digit number by a single-digit number, by partitioning the three-digit number into hundreds, tens and ones, multiplying the parts by the single-digit number, then adding the partial products.

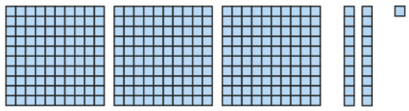
Steps in learning

<table>
<tr><th></th><th>Guidance</th><th>Representations</th></tr>
<tr><td>3:1</td><td>Before using the short-multiplication algorithm to multiply three-digit numbers by single-digit numbers (Teaching point 4), return to the use of Dienes and informal written methods (as in Teaching point 1) to demonstrate that the partitioning approach already used for two-digit numbers can be extended to three-digit numbers.

Begin with an example that does not require any regrouping (e.g. 321 × 3), following a similar sequence of learning to that described in step 1:2, as shown opposite. (Note that the representation step numbers used opposite, and in step 3:2 below, reflect the representation step numbers in step 1:2 for easy comparison).

Repeat the process for a range of three-digit × single-digit calculations, without regrouping, until children are confident. Use examples where the three-digit number is sometimes presented as the first factor and sometimes as the second factor.</td><td>321 × 3

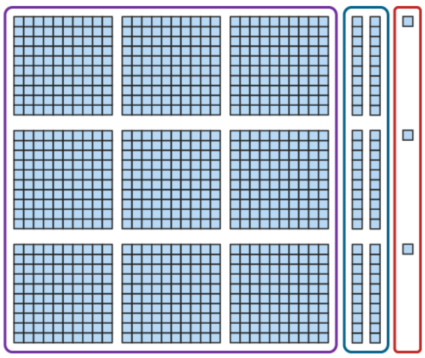
Step 1 – partition 321



321 = 300 + 20 + 1

321 = 3 hundreds + 2 tens + 1 one

Steps 2 and 3 – gather three sets of 321, multiply the hundreds, tens and ones, and recombine



3 hundreds × 3 = 9 hundreds

2 tens × 3 = 6 tens

1 one × 3 = 3 ones

321 × 3 = 300 × 3 + 20 × 3 + 1 × 3
= 900 + 60 + 3
= 60</td></tr>
<tr><td>3:2</td><td colspan="2">Now move on to an example that requires regrouping of hundreds into thousands.

Follow the same sequence as for step 3:1, then regroup the hundreds.

Work through several examples, working towards the generalisation: ‘If there are ten or more hundreds, we must regroup the hundreds into thousands and hundreds.’</td></tr>
</table>

As the numbers involved get larger, you can continue to use Dienes, or start to use place-value counters. Place-value counters will be used in *Teaching point 4*, to support application of the short multiplication algorithm to *three-digit* × *single-digit* calculations.

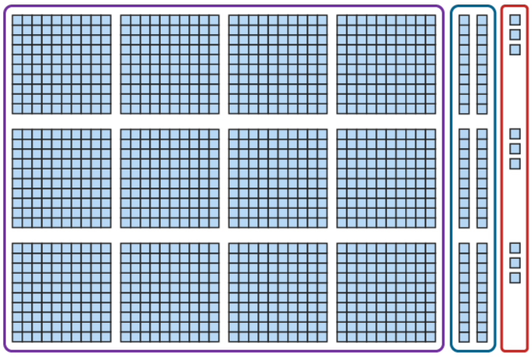
Dienes representation of 423 × 3:

Step 1 – partition 423

$423 = 400 + 20 + 3$

$423 = 4$ hundreds + 2 tens + 3 ones

Steps 2 and 3 – gather three sets of 423, multiply the hundreds, tens and ones and recombine



4 hundreds × 3 = 12 hundreds

2 tens × 3 = 6 tens

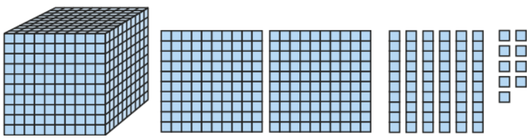
3 ones × 3 = 9 ones

$$\begin{aligned} 423 \times 3 &= 400 \times 3 + 20 \times 3 + 3 \times 3 \\ &= 1200 + 60 + 9 \end{aligned}$$

Step 4 - regroup the hundreds into thousands and hundreds

12 hundreds = 1 thousand + 2 hundreds

$$\begin{aligned} 423 \times 3 &= 1000 + 200 + 60 + 9 \\ &= 1269 \end{aligned}$$



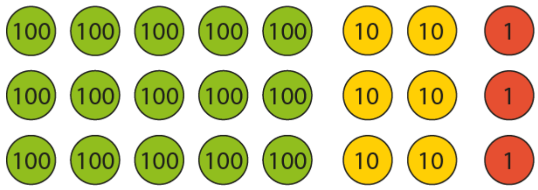
Place-value counter representation of 521 × 3:

Step 1 – partition 521:

$521 = 500 + 20 + 1$

$521 = 5$ hundreds + 2 tens + 1 one

Steps 2 and 3 – gather three sets of 521, multiply the hundreds, tens and ones and recombine:



5 hundreds × 3 = 15 hundreds

2 tens × 3 = 6 tens

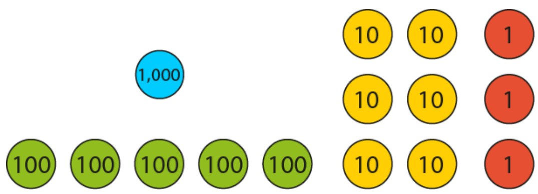
1 one × 3 = 3 ones

$$\begin{aligned} 521 \times 3 &= 500 \times 3 + 20 \times 3 + 1 \times 3 \\ &= 1500 + 60 + 1 \end{aligned}$$

Step 4 - regroup the hundreds into thousands and hundreds

15 hundreds = 1 thousand + 5 hundreds

$$\begin{aligned} 521 \times 3 &= 1000 + 500 + 60 + 3 \\ &= 1563 \end{aligned}$$



<table>
<tr>
<td>3:3</td>
<td>Work through some examples without the support of Dienes or place-value counters, until children can confidently partition the three-digit number into hundreds, tens and ones, multiply the resulting parts by the single-digit number and add the partial products. Two different methods of showing working are exemplified opposite; as long as children can confidently and correctly show their working (including correct use of the '=' sign), the method chosen is not important. Note that the regrouping is now 'hidden' within addition across tens/hundreds/ thousands boundaries. Regrouping will be apparent again in Teaching point 4, when children use the short multiplication algorithm.

Include a selection of problems, such as those:

without regrouping
with regrouping only of hundreds
with regrouping of hundreds and with regrouping of any combination of tens and ones
where the three-digit factor has a zero in either the ones or tens place
that have products with no zero digits
that have products with a zero in only one position (ones, tens or hundreds)
that have products with a zero in more than one position.

</td>
<td>Example 1:

$201 \times 4 = 804$
(201 partitioned into 200 and 1)

$200 \times 4 = 800$
$1 \times 4 = 4$
$800 + 4 = 804$

Example 2:

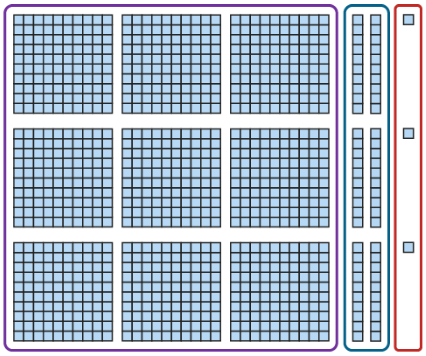
$427 \times 3 = 400 \times 3 + 20 \times 3 + 7 \times 3$
$= 1200 + 60 + 21$
$= 1281$</td>
</tr>
</table>

3:4	To complete this teaching point, provide children with practice multiplying three-digit numbers by single-digit numbers, using the informal written methods outlined above. Children can initially use Dienes or place-value counters for support, but should progress to working with jottings/equations only.	Matching multiplication expressions with partial products: *'Draw a line to match each multiplication expression with the correct addition expression.'* 4×203 $800 + 40 + 12$ 3×204 $800 + 12$ 4×213 $600 + 30 + 12$ 3×214 $600 + 12$ Missing-number problems: *'Fill in the missing numbers.'* $122 \times 4 = 100 \times 4 + 20 \times 4 + 2 \times 4$ $= \square + \square + \square$ $= \square$ $257 \times 6 = \square \times 6 + 50 \times 6 + \square \times 6$ $= \square + \square + \square$ $= \square$ $102 \times 3 = \square$ $9 \times 132 = \square$ $4 \times 810 = \square$ $208 \times 6 = \square$

Teaching point 4:

Any three-digit number can be multiplied by a single-digit number using the short multiplication algorithm.

Steps in learning

	Guidance	Representations
4:1	This teaching point extends the short multiplication algorithm to *three-digit* × *single-digit* calculations, improving calculation efficiency. As in *Teaching point 2*, familiar examples are chosen (in this case from *Teaching point 3*) and manipulatives are used initially to support use of the 'expanded layout'. As in *Teaching point 2*, the expanded layout is used only to expose the structure of the algorithm, and children should progress quickly to using the 'compact layout'. Begin with an example that does not require regrouping (such as *3 × 321*, from step *3:1*). Use Dienes (as in step *3:1*) or place-value counters to support working through the expanded layout, then repeat the calculation using the compact layout. As in *Teaching point 2*, when using the compact layout, encourage children to describe the process at each step: • *'First, write the largest factor: "321".'* • *'Then write the smallest factor below, lining up the digits: "3".'* • *'Now multiply, starting with the ones: three times one one is equal to three ones; write "3" in the ones column.'* • *'Then move to the tens: three times two tens is equal to six tens; write "6" in the tens column.'* • *'Then move to the hundreds: three times three hundreds is equal to nine hundreds; write "9" in the hundreds column.'*	321 × 3 Dienes representation:  Multiplication algorithm – expanded layout: (see below) Multiplication algorithm – compact layout: (see below)

Multiplication algorithm – expanded layout:

	100s	10s	1s	
	3	2	1	
×			3	
			3	3 × 1 ones = 3 ones
		6	0	3 × 2 tens = 6 tens
	9	0	0	3 × 3 hundreds = 9 hundreds
	9	6	3	

Multiplication algorithm – compact layout:

	3	2	1
×			3
	9	6	3

4:2

Give children practice laying out and completing *three-digit* × *single-digit* calculations, without regrouping.

Laying out short multiplication calculations:

'Write these as short multiplication calculations.'

232 × 3   2 × 413   3 × 321   2 × 331

Applying the short multiplication algorithm:

'Complete the calculations.'

$$\begin{array}{r} 2\;1\;2 \\ \times \quad\;\; 4 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 4\;2\;4 \\ \times \quad\;\; 2 \\ \hline \\ \hline \end{array}$$

4:3

Now work through an example that requires regrouping of the hundreds. Use the example from step *3:2* (*521 × 3*).

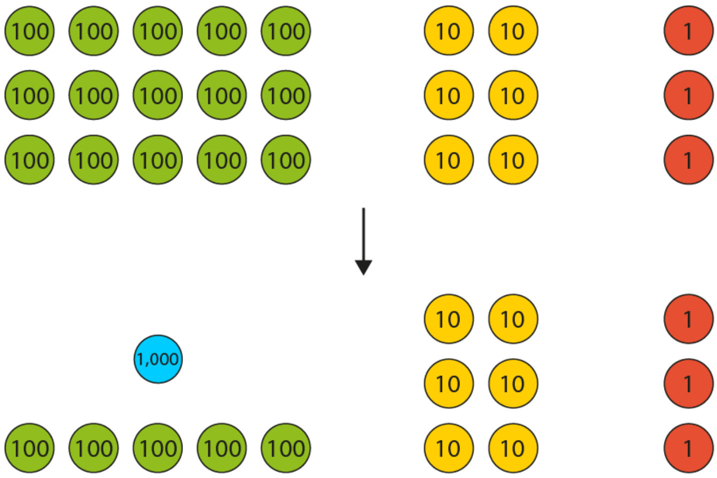
Use place-value counters (as in step *3:2*) to support working through the expanded layout, then repeat the calculation using the compact layout.

As in step *4:1*, when using the compact layout encourage children to describe the steps in laying out and applying the algorithm, now also including the <u>regrouping</u>:

- *'First, write the largest factor: "521".'*
- *'Then write the smallest factor below, lining up the digits: "3".'*
- *'Now multiply, starting with the ones: three times one one is equal to three ones; write "3" in the ones column.'*
- *'Then move to the tens: three times two tens is equal to six tens; write "6" in the tens column.'*
- *'Then move to the hundreds: three times five hundreds is equal to fifteen hundreds…'*
- *'…<u>and regroup: fifteen hundreds is equal to one thousand and five hundreds; write "1" in the thousands column and "5" in the hundreds column</u>.'*

521 × 3

Place-value counters:



Multiplication algorithm – expanded layout:

	1,000s	100s	10s	1s
		5	2	1
×				3
				3
			6	0
	1	5	0	0
	1	5	6	3

3 × 1 ones = 3 ones

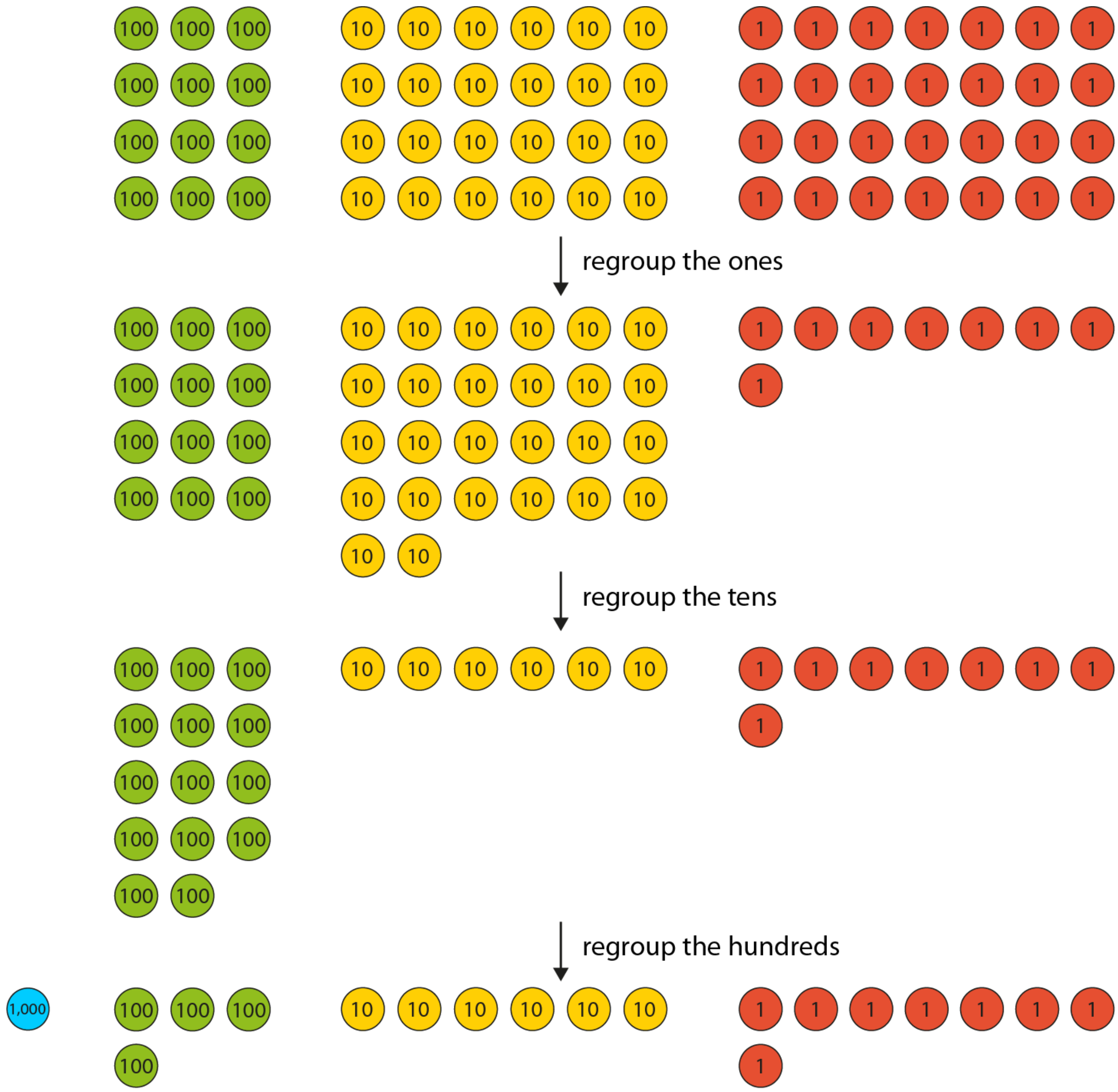
3 × 2 tens = 6 tens

3 × 5 hundreds = 15 hundreds

= 1 thousand + 5 hundreds

	Work through a range of examples that require regrouping only of the hundreds, until children can confidently work with just the compact layout of the algorithm.	Multiplication algorithm – compact layout: $\begin{array}{rrrrr} & & 5 & 2 & 1 \\ \times & & & & 3 \\ \hline & 1 & 5 & 6 & 3 \\ \hline \end{array}$
4:4	Now, use place-value counters to support children in combining their understanding from *Teaching point 2* and steps *4:1–4:3* to solve examples that involve regrouping in all columns (e.g. *367* × *4*). As in step *4:3*, when using the compact layout encourage children to describe the steps in laying out and applying the algorithm (shown alongside the compact layout below). Draw particular attention to: • recording the '2' (that represents the twenty ones regrouped into two tens) *below* the tens column, because we still have the tens to multiply • remembering to add the extra '2' tens to the rest of the tens • recording the '2' (that represents the twenty tens regrouped into two hundreds) *below* the hundreds column, because we still have the hundreds to multiply • remembering to add the extra '2' hundreds to the rest of the hundreds. As discussed in step *2:12*, over time, children will begin to shorten the descriptive language that they use to reason through application of the algorithm; for example, for *367* × *4* below, children may eventually say in their heads: • *'Four sevens are twenty-eight.'* (writing down '2' below the tens column and '8' in the ones column) • *'Four sixes are twenty-four; plus two is twenty-six.'* (writing down '2' below the hundreds column and '6' in the tens column) • *'Four threes are twelve; plus two is fourteen.'* (writing down '1' in the thousands column and '4' in the hundreds column). $\begin{array}{rrrrr} & & 3 & 6 & 7 \\ \times & & & & 4 \\ \hline & 1 & 4 & 6 & 8 \\ \hline & & 2 & 2 & \end{array}$ However, for now, encourage children to keep using the full descriptive language, as exemplified on the next page, since it is important that they 'see' the place value of the digits at this early stage.	

	367 × 4 Place-value counters:



Multiplication algorithm – expanded layout:

	1,000s	100s	10s	1s
		3	6	7
×				4
			2	8
		2	4	0
	1	2	0	0
	1	4	6	8

4×7 ones = 28 ones
= 2 tens + 8 ones

4×6 tens = 24 tens
= 2 hundreds + 4 tens

4×3 hundreds = 12 hundreds
= 1 thousand + 2 hundreds

Multiplication algorithm – compact layout:

		3	6	7
×				4
	1	4	6	8
		2	2	

- *'First, write the largest factor: "367".'*
- *'Then write the smallest factor below, lining up the digits: "4".'*
- *'Now multiply, starting with the ones: four times seven ones is equal to twenty-eight ones…'*
- *'…and regroup: twenty-eight ones is equal to two tens and eight ones; write "8" in the ones column and "2" below the tens column.'*
- *'Then move to the tens: four times six tens is equal to twenty-four tens…'*
- *'…and regroup: twenty-four tens is equal two hundreds and four tens…'*
- *'…and add the two tens from regrouping to give two hundreds and six tens: write "6" in the tens column and "2" below the hundreds column.'*
- *'Then move to the hundreds: four times three hundreds is equal to twelve hundreds…'*
- *'…and regroup: twelve hundreds is equal to one thousand and two hundreds…'*
- *'…and add the two hundreds from regrouping to give one thousand and four hundreds; write "1" in the thousands column and "4" in the hundreds column.'*

<table>
<tr>
<td>4:5</td>
<td>Give children practice laying out and completing three-digit × single-digit calculations, now with regrouping in various places.

Include examples with zero digits in the three-digit factor (e.g. 9 × 608) or in the product, and calculations with repeated digits (e.g. 446 × 4).</td>
<td>Applying the short multiplication algorithm:
'Complete the calculations.'

  3 6 0
×   7
______

  6 0 8
×   9
______

Laying out and applying the short multiplication algorithm:
'Use short multiplication to do these calculations.'

446 × 4   8 × 787

Dòng nǎo jīn:
'Without completing the calculations, which of the following involve:
• regrouping of the hundreds?
• regrouping of the tens?
• regrouping of the ones?'

  4 2 6
×   2
______

  4 2 3
×   3
______

  4 6 2
×   2
______

  4 2 3
×   5
______</td>
</tr>
</table>

4:6

Before providing some general practice, explore how estimation can be used in the context of *three-digit* × *single-digit* calculations. In a similar way to step *2:14*, use learning from segment *2.13 Calculation: multiplying and dividing by 10 or 100* and from *Teaching point 3* above to support estimation and error checking, as exemplified opposite.

Provide children with some practice using inequalities and making estimations, as shown opposite.

Estimation – example 1:

242×3

$$200 < 242 < 300$$

$$200 \times 3 < 242 \times 3 < 300 \times 3$$

$$600 < 242 \times 3 < 900$$

Estimation – example 2:

4×383

$$300 < 383 < 400$$

$$4 \times 300 < 4 \times 383 < 4 \times 400$$

$$1200 < 4 \times 383 < 1600$$

Using inequalities and estimating – example practice:

- *'Decide whether each inequality is true or false.'*

	True (✓) or false (✗)?
$6 \times 300 < 6 \times 350$	
$1800 < 6 \times 354$	
$5 \times 328 < 1500$	
$8 \times 494 > 5000$	
$7 \times 758 > 5600$	

- *'Estimate the answer to each of these calculations.'*

332×7 4×617 8×506

4:7

Now provide children with general practice applying the short multiplication algorithm to both *two-digit* × *single-digit* and *three-digit* × *single-digit* calculations, including:

- completing calculations (see examples in *Teaching points 2* and *4*, and opposite)
- identifying errors

Completing and laying out calculations:

- *'Complete the calculations.'*

$$\begin{array}{r} 4\ \ 5 \\ \times\quad\ 3 \\ \hline \\ \hline \end{array} \qquad\qquad \begin{array}{r} 3\ \ 4\ \ 2 \\ \times\quad\quad\ 6 \\ \hline \\ \hline \end{array}$$

- *'Use short multiplication to do these calculations.'*

46×7 3×475

- contextual problems, including multistep problems, varying the order in which the factors (single-digit and either two-digit or three-digit) are presented; for example:
  - *'Ellen has one large bag of ninety-six marbles, and four smaller bags each containing seventy-six marbles. How many marbles does she have altogether?'*
  - *'Harvey has five hundred and ninety-four packs of buns for a party. Each pack contains three buns. When Harvey is preparing for the party, he drops eighteen buns. How many buns are left?'*

Each time children use the short multiplication algorithm, encourage them to estimate the value of the product, then perform the calculation, and finally use their estimate to sense-check their answers.

Use calculations with missing digits to deepen children's understanding of the algorithm.

Identifying errors:

'Spot the mistakes.'

$$\begin{array}{rcccc} & & & 7 & 3 \\ \times & & & & 4 \\ \hline & 2 & 8 & 1 & 2 \\ \hline \end{array} \qquad \begin{array}{rccc} & 1 & 0 & 3 \\ \times & & & 4 \\ \hline & 5 & 0 & 2 \\ \hline \end{array}$$

$$\begin{array}{rccc} & 5 & 2 & 4 \\ \times & & & 3 \\ \hline & 5 & 6 & 2 \\ \hline \end{array} \qquad \begin{array}{rcccc} & & 4 & 7 & 2 \\ \times & & & & 6 \\ \hline & 2 & 9 & 2 & 2 \\ \hline \end{array}$$

Dòng nǎo jīn:

- *'Fill in the missing digits. How many solutions are there for each problem?'*

$$\begin{array}{rccc} & 1 & \square & 7 \\ \times & & & \square \\ \hline & 8 & 2 & 2 \\ \hline \end{array} \qquad \begin{array}{rccc} & 1 & 5 & \square \\ \times & & & 6 \\ \hline & \square & \square & 4 \\ \hline \end{array}$$

- *'For each calculation, discuss whether it would be more efficient to use a mental method or short multiplication.'*

200×4	201×4	207×4
210×4	217×4	250×4